Journal of Computer and Mathematical Sciences, *Vol.6(7),371-387, July 2015* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

On the Total Line-cut Transformation Graphs G^{xyz}

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(Received on: July 15, 2015)

ABSTRACT

In this paper, we introduce total line-cut transformation graphs. We investigate some basic properties such as connectedness, graph equations and diameters of the total line-cut transformation graphs.

2010 Mathematics Subject Classification: 05C12

Keywords: cutpoint, line graph, cutpoint graph, total line-cut transformation graphs G^{xyz}.

1. INTRODUCTION

By a graph G=(V, E), we mean a simple, finite, undirected graphs without isolated points. For any graph G, let $V(G)$, $E(G)$, $W(G)$ and $U(G)$ denote the point set, line set, cutpoint set and block set of G, respectively. The lines and cutpoints of a graph are called its members.

Eccentricity of a point $u \in V(G)$ is defined as $e(u)$=$max\{d_G(u, v): v \in V(G)\}$, where $d_G(u, v)$ is the distance between u and v in G. The minimum and maximum eccentricities are the *radius* $r(G)$ and *diameter* $diam(G)$ of G, respectively.

A *cutpoint* of a connected graph G is the one whose removal increases the number of components. A *nonseparable graph* is connected, nontrivial and has no cutpoints. A *block* of a graph G is a maximal nonseparable subgraph. The *line graph* $L(G)$ of G is the graph whose point set is $E(G)$ in which two points are adjacent if and only if they are adjacent in G[11]. The *jump graph* $J(G)$ of G is the graph whose point set is $E(G)$ in which two points are adjacent if and only if they are nonadjacent in G[4]. If a block is incident with cutpoints $c_1, c_2, \ldots, c_r$, $r \geq 2$, we say that c_i and c_j are *coadjacent* where $i \neq j$ and $1 \leq i, j \leq r$. The *cutpoint graph* $C(G)$ of a graph G is the graph whose point set corresponds to the cutpoints of G and in which two points of $C(G)$ are adjacent if the cutpoints of G to which they correspond lie on a common block[5]. Let $\lceil x \rceil$ ($\lfloor x \rfloor$) denote the least (greatest) integer greater (less) than or equal to x. For graph theoretic terminology, we refer to[6,8].

2. TOTAL LINE-CUT TRANSFORMATION GRAPHS G^{xyz}

In[12], Wu and Meng generalized the concept of total graph and introduced the total transformation graphs and defined as follows:

Definition: Let $G = (V, E)$ be a graph, and x, y, z be three variables taking values $+$ or $-$. The *transformation graph* G^{xyz} is the graph having $V(G) \cup E(G)$ as the point set, and for α, $\beta \in V(G) \cup E(G)$, α and β are adjacent in G^{xyz} if and only if one of the following holds:

(i) α, $\beta \in V(G)$. α and β are adjacent in G if $x = +$; α and β are nonadjacent in G if $x = -$.

(ii) α, $\beta \in E(G)$. α and β are adjacent in G if $y = +$; α and β are nonadjacent in G if $y = -$.

(iii) $\alpha \in V(G)$, $\beta \in E(G)$. α and β are incident in G if $z = +$; α and β are nonincident in G if $z = -$.

Let $G = (V, E)$ be a graph with block set $U(G)$=$\{B_i : B_i \ is \ a \ block \ of \ G\}$. If $B \in U(G)$ with point set $\{u_1, u_2, \ldots, u_r; r \geq 2\}$, then we say that the point u_i and block B are incident with each other where $1 \leq i \leq r$. Two blocks B_i and B_j in $U(G)$ are said to be adjacent if they are incident with a common cutpoint. In [3], B. Basavanagoud et. al generalized the concept of total-block graph and introduced the block-transformation graphs and defined as follows:

Definition: Let $G = (V, E)$ be a graph with block set $U(G)$, and let α, β, γ be three variables having values 0 or 1. The *block-transformation graph* $G^{\alpha\beta\gamma}$ is the graph having $V(G) \cup U(G)$ as the point set. For any two vertices x and $y \in V(G) \cup U(G)$ we define α, β, γ as follows:

(i) Suppose x, y are in $V(G)$. α=1 if x and y are adjacent in G. α=0 if x and y are nonadjacent in G.

(ii) Suppose x, y are in $U(G)$. β=1 if x and y are adjacent in G. β=0 if x and y are nonadjacent in G.

(iii) $x \in V(G)$ and $y \in U(G)$. γ=1 if x and y are incident with each other in G. γ=0 if x and y are nonincident with each other in G.

In[7], Kulli *et al.*, introduced the concept of lict and litact graph. In[1], the lict graph is also called line-cut graph of G. Now we call litact graph as total line-cut graph of G. Inspired by the definition of total transformation graphs[12] and block-transformation graphs[3], we generalize the concept of total line-cut graph[7] and introduce the graph valued functions namely total line-cut transformation graphs and we define as follows.

Definition: Let $G = (V, E)$ be a graph with cutpoint set W(G)=$\{c_1, c_2, \ldots, c_r\}$, and x, y, z be three variables taking values $+$ or $-$. The *total line-cut transformation graph* G^{xyz} is the graph having $E(G) \cup W(G)$ as the point set, and for α, $\beta \in E(G) \cup W(G)$, α and β are adjacent in G^{xyz} if and only if one of the following holds:

(i) $\alpha, \beta \in E(G)$. α and β are adjacent in G if $x = +$; α and β are nonadjacent in G if $x = -$.

(ii) $\alpha, \beta \in W(G)$. α and β are adjacent or coadjacent in G if $y = +$; α and β are nonadjacent or noncoadjacent in G if $y = -$.

(iii) $\alpha \in E(G)$, $\beta \in W(G)$. α and β are incident in G if $z = +$; α and β are nonincident in G if $z = -$.

Thus, we obtain eight kinds of total line-cut transformation graphs, in which G^{+++} is the total line-cut graph of G [7], and G^{---} is its complement. Also, G^{--+}, G^{-+-} and G^{-++} are the complements of G^{++-}, G^{+-+} and G^{+--}, respectively. Many papers are devoted to total line-cut graph[7,9].

The point c_i' (e_i') of G^{xyz} corresponding to a cutpoint c_i (line e_i) of G and is referred to as cutpoint (line) vertex.

The following will be useful in the proof of our results.

Remark 2.1 L(G) is an induced subgraph of G^{+yz}.

Remark 2.2 J(G) is an induced subgraph of G^{-yz}.

Remark 2.3 If G is connected, then C(G) is connected.

Theorem 2.1 [6] If G is connected, then L(G) is connected.

Theorem 2.2 [13] Let G be a graph of size $q \geq 1$. Then J(G) is connected if and only if G contains no line that is adjacent to every other lines of G unless $G = K_4$ or C_4.

Theorem 2.3 [2] *For a given graph G, $J(G)$=G if and only if $G \cong C_5$ or cor(K_3), where cor(K_3)is defined as follows: Let K_3=$v_1v_2v_3$ be a triangle, and let $V(cor(K_3))$=$\{u_1, u_2, u_3\} \cup \{v_1, v_2, v_3\}$ and $E(cor(K_3))$=$E(K_3) \cup \{u_1v_1, u_2v_2, u_3v_3\}$.*

Theorem 2.4 [7] *A connected graph G is isomorphic to its G^{+++} if and only if G is a cycle.*

Theorem 2.5 *For any nontrivial graph G, G^{+yz}=$L(G)$ if and only if G is a block.*

Proof. Suppose G is a block. It is known that G has no cutpoints. Then G^{+yz} has q points. By definition of $L(G)$ it has q points. Clearly G^{+yz}=$L(G)$.

Conversely, suppose G^{+yz}=$L(G)$. Assume G is not a block. Then there exist at least one cutpoint. It is known that $L(G)$ has q points where as the number of points of G^{+yz} are the sum of the number of lines and cutpoints of G. Thus $L(G)$ has less number of points than G^{+yz}. Clearly $G^{+yz} \neq L(G)$, a contradiction.

Theorem 2.6 *For any nontrivial graph G, G^{-yz}=$J(G)$ if and only if G is a block.*

Proof. Suppose G is a block. It is known that G has no cutpoints. Then G^{-yz} has q points. By definition of $J(G)$ it has q points. Clearly G^{-yz}=$L(G)$.

Conversely, suppose G^{-yz}=$J(G)$. Assume G is not a block. Then there exist at least one cutpoint. It is known that $J(G)$ has q points where as the number of points of G^{-yz} are the sum of the number of lines and cutpoints of G. Thus $J(G)$ has less number of points than G^{-yz}. Clearly $G^{-yz} \neq J(G)$, a contradiction.

3. CONNECTEDNESS OF G^{xyz}

The first theorem is well-known.

Theorem 3.1 *For a given graph G, G^{+++} is connected if and only if G is connected.*

Theorem 3.2 *Let G=(p,q) be a nontrivial graph with $q \geq 2$ lines. Then G^{++-} is connected if and only if G satisfies following conditions*

(i) $G \neq K_{1,p}$

(ii) $G \neq K_{1,p} \cup K_{1,r}$

(iii) $G \neq \bigcup_{i=2}^{n} B_i$

(iv) $G \neq K_{1,p} \cup (\bigcup_{i=1}^{n} B_i)$.

Proof. Suppose a graph G satisfies conditions (i), (ii), (iii) and (iv). We prove the result by following cases.

Case 1. If G is connected, then we have the following subcases.

Subcase 1.1. If G is a block, then from Theorem 2.5, G^{++-}=$L(G)$. Therefore by Theorem 2.1, G^{++-} is connected.

Subcase 1.2. If G has at least one cutpoint, then by Theorem 2.1 and Remark 2.3, $L(G)$ and $C(G)$ are connected subgraphs of G^{++-} and also each cutpoint vertex is adjacent to at least one line vertex because every cutpoint is nonincident with at least one line in G. Hence G^{++-} is connected.

Case 2. If G is disconnected with G_1, G_2,..., G_n, $n \geq 2$ components. By conditions (ii), (iii) and (iv), one of the component G_i is not a star with at least one cutpoint c_i. If the lines e_i and e_j are nonadjacent in G, then the line vertices e_i' and e_j' are connected by cutpoint vertex c_i' in G^{++-}. If the lines e_x and e_y are adjacent in G, then the line vertices e_x' and e_y' are adjacent in G^{++-}. Therefore every pair of line vertices are connected in G^{++-}. Also if the cutpoints c_x and c_y are adjacent or coadjacent in G, then cutpoint vertices c_x' and c_y' are adjacent in G^{++-}. If the cutpoints c_1 and c_2 are nonadjacent or noncoadjacent in G, then cutpoint vertices c_1' and c_2' are connected by a line vertex e_1' (cutpoint vertex c_1') corresponding to the line e (cutpoint c) which is nonincident (adjacent or coadjacent) with both the cutpoints c_1 and c_2 in G or cutpoint vertices c_1' and c_2' are connected by line vertices e_1' and e_2', such that e_1 is nonincident with c_1 and e_2 is nonincident with c_2 in G. Therefore every pair of cutpoint vertices are connected in G^{++-}. And also each cutpoint vertex is adjacent to at least one line vertex because every cutpoint is nonincident with at least one line in G. Hence G^{++-} is connected.

Conversely, (i) If $G = K_{1,p}$, then $G^{++-} = K_p \cup K_1$ is disconnected, a contradiction.

(ii) If $G = K_{1,p} \cup K_{1,r}$, then $G^{++-} = K_{p+1} \cup K_{r+1}$ is disconnected, a contradiction.

(iii) If $G = \bigcup_{i=2}^{n} B_i$, then $G^{++-} = \bigcup_{i=2}^{n} L(B_i)$ is disconnected, a contradiction.

(iv) If $G = K_{1,p} \cup (\bigcup_{i=1}^{n} B_i)$, then $G^{++-} = K_p \cup (\bigcup_{i=1}^{n} L(B_i) + K_1)$ is disconnected, a contradiction.

Theorem 3.3 *Let G=(p,q) be a nontrivial graph with $q \geq 2$ lines. Then G^{+-+} is connected if and only if G satisfies following conditions*
(i) $G \neq \bigcup_{i=2}^{n} B_i$
(ii) $G \neq G_1 \cup G_2$, where G_1 is a graph with at least one cutpoint and G_2 is a union of blocks.
Proof. Suppose a graph G satisfies conditions (i) and (ii). We prove the result by following cases.

Case 1. If G is connected, then we have the following subcases.

Subcase 1.1. If G is a block, then by Theorem 2.5, G^{+-+}=$L(G)$. Therefore by Theorem 2.1, G^{+-+} is connected.

Subcase 1.2. If G has at least one cutpoint, then by Theorem 2.1, $L(G)$ is connected subgraph of G^{+-+}. Therefore every pair of line vertices are connected. If the cutpoints c_x and c_y are nonadjacent or noncoadjacent in G, then cutpoint vertices $c_x{}'$ and $c_y{}'$ are adjacent in G^{+-+}. If the cutpoints c_1 and c_2 are adjacent or coadjacent in G, then cutpoint vertices $c_1{}'$ and $c_2{}'$ are connected by a line vertex $e_1{}'$ (cutpoint vertex $c_1{}'$) corresponding to the line e (cutpoint c) which is incident (nonadjacent or noncoadjacent) with both the cutpoints c_1 and c_2 in G or cutpoint vertices $c_1{}'$ and $c_2{}'$ are connected by line vertices $e_1{}'$ and $e_2{}'$, such that e_1 is incident with c_1 and e_2 is incident with c_2 in G. Therefore every pair of cutpoint vertices are connected in G^{+-+}. Also each cutpoint vertex is adjacent to at least one line vertex because each cutpoint is incident with at least one line in G. Hence G^{+-+} is connected.

Case 2. If G is disconnected with G_1, G_2,..., G_n, $n \geq 2$ components. By conditions (i) and (ii), $\overline{C(G)}$ is connected subgraph of G^{+-+}, therefore every pair of cutpoint vertices are connected in G^{+-+}. If the lines e_x and e_y are adjacent in G, then line vertices $e_x{}'$ and $e_y{}'$ are adjacent in G^{+-+}. If the lines e_i and e_j are nonadjacent in G, then line vertices $e_i{}'$ and $e_j{}'$ are connected by cutpoint vertex $c_i{}'$. Therefore every pair of line vertices are connected. Also each cutpoint vertex is adjacent to at least one line vertex because each cutpoint is incident with at least one line in G. Hence G^{+-+} is connected.
Conversely, (i) If $G = \bigcup_{i=2}^{n} B_i$, then G^{+-+}=$\bigcup_{i=2}^{n} L(B_i)$ is disconnected, a contradiction.
(ii) If $G = G_1 \cup G_2$, where G_1 is a graph with at least one cutpoint and G_2 is a union of blocks, $G^{+-+} = G_1^{+-+} \cup L(G_2)$ is disconnected, a contradiction.

Theorem 3.4 *Let G=(p, q) be a nontrivial graph with $q \geq 2$ lines. Then G^{+--} is connected if and only if G satisfies the following conditions*
(i) $G \neq K_{1,p}$
(ii) $G \neq \bigcup_{i=2}^{n} B_i$
(iii) $G \neq K_{1,p} \cup (\bigcup_{i=1}^{n} B_i)$.
Proof. Suppose a graph G satisfies conditions (i), (ii) and (iii). We prove the result by following cases.

Case 1. If G is connected, then we have the following subcases.

Subcase 1.1. If G is a block, then by Theorem 2.5, G^{+--}=$L(G)$. Therefore by Theorem 2.1, G^{+--} is connected.

Subcase 1.2. If G has at least one cutpoint, then by Theorem 2.1, $L(G)$ is connected subgraph of G^{+--}. Therefore every pair of line vertices are connected in G^{+--}. If the cutpoints c_1 and c_2 are nonadjacent or noncoadjacent in G, then cutpoint vertices c_1' and c_2' are adjacent in G^{+--}. If the cutpoints c_x and c_y are adjacent or coadjacent in G, then cutpoint vertices c_x' and c_y' are connected by line vertex e_1' (cutpoint vertex c_1') corresponding to the line e_1 (cutpoint c_1) which is nonincident (nonadjacent or noncoadjacent) with both cutpoints c_x and c_y in G or c_x' and c_y' are connected by line vertices e_x' and e_y', such that e_x is nonincident with c_x and e_y is nonincident with c_y in G. Therefore every pair of cutpoint vertices are connected in G^{+--}. Also each cutpoint vertex is adjacent to at least one line vertex because each cutpoint is nonincident with at least one line in G. Hence G^{+--} is connected.

Case 2. If G is disconnected with G_1, G_2,..., G_n, $n \geq 2$ components. By conditions (ii) and (iii), $\overline{C(G)}$ is connected subgraph of G^{+--}, therefore every pair of cutpoint vertices are connected in G^{+--}. If the lines e_i and e_j are nonadjacent in G, then line vertices e_i' and e_j' are adjacent in G. If the lines e_x and e_y are adjacent in G, then line vertices e_x' and e_y' are connected by line vertex e_i' (cutpoint vertex c_i') corresponding to the line e_i (cutpoint c_i) which is adjacent (nonincident) with both the lines e_x and e_y in G or line vertices e_x' and e_y' are connected by cutpoint vertices c_x' and c_y' such that c_x is nonincident with e_x and c_y is nonincident with e_y in G. Therefore every pair of line vertices are connected in G^{+--}. Also each cutpoint vertex is adjacent to at least one line vertex because each cutpoint is nonincident with at least one line in G. Hence G^{+--} is connected.

Conversely, (i) If $G = K_{1,p}$, then $G^{+--} = K_p \cup K_1$ is disconnected, a contradiction.

(ii) If $G = \bigcup_{i=2}^{n} B_i$, then $G^{+--} = \bigcup_{i=2}^{n} L(B_i)$ is disconnected, a contradiction.

(iii) If $G = K_{1,p} \cup (\bigcup_{i=1}^{n} B_i)$, then $G^{+--} = K_p \cup (\bigcup_{i=1}^{n} L(B_i) + K_1)$ is disconnected, a contradiction.

Theorem 3.5 *Let G be a nontrivial (p, q) graph with $q \geq 2$ lines. Then G^{-++} is connected if and only if $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to a cutpoint.*

Proof. Let G be a nontrivial (p, q) graph with $q \geq 2$, $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to a cutpoint. Then to prove G^{-++} is connected. We consider the following cases.

Case 1. If G is connected, then we have the following subcases.

Subcase 1.1. If G is a block and $G \neq K_3, C_4, K_4, K_4 - x$(where x is any line in K_4), then by Theorem 2.6, $G^{-++} = J(G)$. Therefore by Theorem 2.2, G^{-++} is connected.

Subcase 1.2. If G has at least one cutpoint. If G contains no line which is adjacent to all other lines, then by Theorem 2.2, $J(G)$ is connected subgraph of G^{-++} or if G contains at least one line e which is adjacent to all other lines, clearly e is incident with a cutpoint c in G.

Therefore every pair of line vertices are connected in G^{-++}. And from Remark 2.3, $C(G)$ is connected subgraph of G^{-++}. Therefore every pair of cutpoint vertices are connected in G^{-++}. Also each cutpoint vertex is adjacent to at least one line vertex because every cutpoint is incident with at least one line in G, hence G^{-++} is connected.

Case 2. If G is not connected, then $J(G)$ is connected subgraph of G^{-++}. Therefore every pair of line vertices are connected. If the cutpoints c_1 and c_2 are adjacent or coadjacent in G, then cutpoint vertices $c_1{}'$ and $c_2{}'$ are adjacent in G^{-++}. If the cutpoints c_x and c_y are nonadjacent or noncoadjacent in G, then the cutpoint vertices $c_x{}'$ and $c_y{}'$ are connected by line vertex $e_1{}'$ (cutpoint vertex $c_1{}'$) corresponding to a line e_1 (cutpoint c_1) which is incident (adjacent or coadjacent) with both cutpoints c_x and c_y in G or cutpoint vertices $c_x{}'$ and $c_y{}'$ are connected by line vertices $e_x{}'$ and $e_y{}'$, such that e_x is incident with c_x and e_y is incident with c_y in G. Therefore every pair of cutpoint vertices are connected in G^{-++}. Also each cutpoint vertex is adjacent to at least one line vertex because every cutpoint is incident with at least one line in G, hence G^{-++} is connected.

Conversely, clearly G^{-++} is connected for any graph G of size $q \geq 2$, $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to a cutpoint.

Theorem 3.6 *Let G be a nontrivial (p, q) graph with $q \geq 2$ lines. Then G^{--+} is connected if and only if $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to a cutpoint.*

Proof. Let G be a nontrivial (p, q) graph with $q \geq 2$, $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to a cutpoint. Then to prove G^{--+} is connected. We consider the following cases.

Case 1. If G is connected, then we have the following subcases.

Subcase 1.1. If G is a block and $G \neq K_3, C_4, K_4, K_4 - x$(where x is any line in K_4), then by Theorem 2.6, $G^{--+} = J(G)$. Therefore by Theorem 2.2, G^{--+} is connected.

Subcase 1.2. If G has at least one cutpoint. If G contains no line which is adjacent to all other lines, then by Theorem 2.2, $J(G)$ is connected subgraph of G^{--+} or if G contains at least one line e which is adjacent to all other lines, clearly e is incident with a cutpoint c in G. Therefore every pair of line vertices are connected in G^{--+}. If the cutpoints c_x and c_y are nonadjacent or noncoadjacent in G, then cutpoint vertices $c_x{}'$ and $c_y{}'$ are adjacent in G^{--+}. If the cutpoints c_x and c_y are adjacent or coadjacent in G, then the cutpoint vertices $c_x{}'$ and $c_y{}'$ are connected by line vertex $e_1{}'$ (cutpoint vertex $c_1{}'$) corresponding to the line e_1 (cutpoint c_1) which is incident (nonadjacent or noncoadjacent) with both cutpoints c_x and c_y in G or cutpoint vertices $c_x{}'$ and $c_y{}'$ are connected by line vertices $e_x{}'$ and $e_y{}'$, such that e_x is incident with c_x and e_y is incident with c_y in G. Therefore every pair of cutpoint vertices are connected in G^{--+}. Also each cutpoint vertex is adjacent to at least one line vertex because each cutpoint is incident with at least one line in G. Hence G^{--+} is connected.

Case 2. If G is not connected, then $J(G)$ is connected subgraph of G^{--+}. Therefore every pair of line vertices are connected. If the cutpoints c_x and c_y are nonadjacent or noncoadjacent in G, then the cutpoint vertices c_x' and c_y' are adjacent in G^{--+}. If the cutpoints c_x and c_y are adjacent or coadjacent in G, then the cutpoint vertices c_x' and c_y' are connected by line vertex e_1' (cutpoint vertex c_1') corresponding to the line e_1 (cutpoint c_1) which is incident (nonadjacent or noncoadjacent) with both cutpoints c_x and c_y in G or cutpoint vertices c_x' and c_y' are connected by line vertices e_x' and e_y', such that e_x is incident with c_x and e_y is incident with c_y in G. Therefore every pair of cutpoint vertices are connected in G^{--+}. Also each cutpoint vertex is adjacent to at least one line vertex because each cutpoint is incident with at least one line in G. Hence G^{--+} is connected.

Conversely, clearly G^{--+} is connected for any graph G of size $q \geq 2$, $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to a cutpoint.

Theorem 3.7 *Let G be a nontrivial (p, q) graph with $q \geq 2$ lines. Then G^{-+-} is connected if and only if $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to a cutpoint.*

Proof. Let G be a nontrivial (p, q) graph with $q \geq 2$, $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to a cutpoint. Then to prove G^{-+-} is connected. We consider the following cases.

Case 1. If G is connected, then we have the following subcases.

Subcase 1.1. If G is a block and $G \neq K_3, C_4, K_4, K_4 - x$(where x is any line in K_4), then by Theorem 2.6, $G^{-+-} = J(G)$. Therefore by Theorem 2.2, G^{-+-} is connected.

Subcase 1.2. If G has at least one cutpoint. If G contains no line which is adjacent to all other lines, then by Theorem 2.2, $J(G)$ is connected subgraph of G^{-+-} or if G contains at least one line e which is adjacent to all other lines, clearly e is incident with a cutpoint c in G. Therefore every pair of line vertices are connected in G^{-+-}. Also from Remark 2.3, $C(G)$ is connected subgraph of G^{-+-}. Therefore every pair of cutpoint vertices are connected in G^{-+-}. Also each cutpoint vertex is adjacent to at least one line vertex because each cutpoint is nonincident with at least one line in G. Hence G^{-+-} is connected.

Case 2. If G is not connected, then $J(G)$ is connected subgraph of G^{-+-}. If the cutpoints c_x and c_y are adjacent or coadjacent in G, then the cutpoint vertices c_x' and c_y' are adjacent in G^{-+-}. If the cutpoints c_x and c_y are nonadjacent or noncoadjacent in G, then the cutpoint vertices c_x' and c_y' are connected by line vertex e_1' (cutpoint vertex c_1') corresponding to the line e_1 (cutpoint c_1) which is nonincident (adjacent or coadjacent) with both cutpoints c_x and c_y in G or cutpoint vertices c_x' and c_y' are connected by line vertices e_x' and e_y', such that e_x is nonincident with c_x and e_y is nonincident with c_y in G. Therefore every pair of cutpoint vertices are connected in G^{-+-}. Also each cutpoint vertex is adjacent to at least one line vertex because each cutpoint is nonincident with at least one line in G. Hence G^{-+-} is connected.

Conversely, clearly G^{-+-} is connected for any graph G of size $q \geq 2$, $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to a cutpoint.

Theorem 3.8 *Let G be a nontrivial (p, q) graph with $q \geq 2$ lines. Then G^{---} is connected if and only if $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to a cutpoint.*

Proof. Let G be a nontrivial (p, q) graph with $q \geq 2$, $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to a cutpoint. Then to prove G^{---} is connected. We consider the following cases.

Case 1. If G is connected, then we have the following subcases.

Subcase 1.1. If G is a block and $G \neq K_3, C_4, K_4, K_4 - x$(where x is any line in K_4), then by Theorem 2.6, $G^{---} = J(G)$. Therefore by Theorem 2.2, G^{---} is connected.

Subcase 1.2. If G has at least one cutpoint. If G contains no line which is adjacent to all other lines, then by Theorem 2.2, $J(G)$ is connected subgraph of G^{---} or if G contains at least one line e which is adjacent to all other lines, clearly e is incident with a cutpoint c in G. Therefore every pair of line vertices are connected in G^{---}. If the cutpoints c_x and c_y are nonadjacent or noncoadjacent in G, then the cutpoint vertices $c_x{}'$ and $c_y{}'$ are adjacent in G^{---}. If the cutpoints c_x and c_y are adjacent or coadjacent in G, then the cutpoint vertices $c_x{}'$ and $c_y{}'$ are connected by line vertex $e_1{}'$ (cutpoint vertex $c_1{}'$) corresponding to the line e_1 (cutpoint c_1) which is nonincident (nonadjacent or noncoadjacent) with both cutpoints c_x and c_y in G or cutpoint vertices $c_x{}'$ and $c_y{}'$ are connected by line vertices $e_x{}'$ and $e_y{}'$, such that e_x is nonincident with c_x and e_y is nonincident with c_y in G. Therefore every pair of cutpoint vertices are connected in G^{---}. Also each cutpoint vertex is adjacent to at least one line vertex because each cutpoint is nonincident with at least one line in G. Hence G^{---} is connected.

Case 2. If G is not connected, then $J(G)$ is connected subgraph of G^{---}. Therefore every pair of line vertices are connected in G^{---}. If the cutpoints c_x and c_y are nonadjacent or noncoadjacent in G, then the cutpoint vertices $c_x{}'$ and $c_y{}'$ are adjacent in G^{---}. If the cutpoints c_x and c_y are adjacent or coadjacent in G, then the cutpoint vertices $c_x{}'$ and $c_y{}'$ are connected by line vertex $e_1{}'$ (cutpoint vertex $c_1{}'$) corresponding to the line e_1 (cutpoint c_1) which is nonincident (nonadjacent or noncoadjacent) with both cutpoints c_x and c_y in G or cutpoint vertices $c_x{}'$ and $c_y{}'$ are connected by line vertices $e_x{}'$ and $e_y{}'$, such that e_x is nonincident with c_x and e_y is nonincident with c_y in G. Therefore every pair of cutpoint vertices are connected in G^{---}. Also each cutpoint vertex is adjacent to at least one line vertex because each cutpoint is nonincident with at least one line in G. Hence G^{---} is connected.

Conversely, clearly G^{---} is connected for any graph G of size $q \geq 2$, $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to a cutpoint.

4. GRAPH EQUATIONS AND ITERATIONS OF G^{xyz}

For a given graph-operator Φ and a graph G, we define the iterations of Φ as follows: 1. $\Phi^1(G) = \Phi(G)$ 2. $\Phi^n(G) = \Phi(\Phi^{n-1}(G))$ for $n \geq 2$.

For a given total line-cut transformation graph G^{xyz}, we define the iteration of G^{xyz} as follows:

1. $G^{(xyz)^1} = G^{xyz}$ 2. $G^{(xyz)^n} = [G^{(xyz)^{n-1}}]^{xyz}$ for $n \geq 2$.

The isomorphism of G and G^{+++} is shown in [7].

Theorem 4.1 *For any nontrivial (p,q) graph G, G^{+yz}=G if and only if G is a cycle C_p, $p \geq 3$.*

Proof. We known that a connected graph G is isomorphic to its line graph if and only if it is a cycle. Also from Theorem 2.5, $G^{+yz} = L(G)$ if and only if G is a block. Therefore a connected graph G is isomorphic to its G^{+yz} if and only if G is a cycle.

Corollary 4.2 *For any nontrivial (p,q) graph G, $G^{(+yz)^n} = G$ for $n \geq 2$ if and only if G is a cycle C_P, $p \geq 3$.*

Theorem 4.3 *For any nontrivial (p,q) graph G, G^{-++}=G if and only if G is C_5 or $K_{1,p}$ or $K_{1,p} \cup K_{1,r}$, for $p \geq 2$, $r \geq 2$.*

Proof. Suppose G^{-++}=G. Assume $G \neq C_5$. We consider the following cases.

Case 1. Suppose G is a block. Then G^{-++}=$J(G)$. By Theorem 2.3, $G \neq J(G) = G^{-++}$, a contradiction.

Case 2. Suppose a non block G is union of star. Then we consider following subcases.

Subcase 2.1. If $G = K_{1,p}$, then G^{-++}=G.

Subcase 2.2. If G=$K_{1,p} \cup K_{1,r}$, then G^{-++}=$K_{1,p} \cup K_{1,r}$.

Subcase 2.3. If G is a union of more than two stars, then by Theorem 3.5, G^{-++} is connected, a contradiction.

Case 3. Suppose a non block G is not a union of star. Then we consider the following subcases.

Subcase 3.1. If G is either union of cycles and stars with q lines, then G^{-++} has at least $q+1$ lines, a contradiction.

Subcase 3.2. If G is neither union of cycles nor union of cycles and stars, then by Theorem 3.5, G^{-++} is connected, a contradiction.

Conversely, if G is C_5 or $K_{1,p}$ or $K_{1,p} \cup K_{1,r}$, then clearly G^{-++}=G.

Corollary 4.4 *For any nontrivial (p,q) graph G, $G^{(-++)^n}$=G for $n \geq 2$ if and only if G is C_5 or $K_{1,p}$ or $K_{1,p} \cup K_{1,r}$, for $p \geq 2$, $r \geq 2$.*

Theorem 4.5 *For any nontrivial (p,q) graph G, $G^{--+} = G$ if and only if G is C_5 or $K_{1,p}$, for $p \geq 2$.*

Proof. Suppose G^{--+}=G. Assume $G \neq C_5$. We consider the following cases.

Case 1. Suppose G is a block. Then G^{--+}=$J(G)$. By Theorem 2.3, $G \neq J(G) = G^{--+}$, a contradiction.

Case 2. Suppose a non block G is union of star. Then we consider following subcases.
Subcase 2.1. If $G = K_{1,p}$, then G^{--+}=G.
Subcase 2.2. If G is a union of more than two stars, then by Theorem 3.6, G^{--+} is connected, a contradiction.

Case 3. Suppose a non block G is not a union of star. Then we consider the following subcases.
Subcase 3.1. If G is either union of cycles and stars with q lines, then G^{--+} has at least $q + 1$ lines, a contradiction.
Subcase 3.2. If G is neither union of cycles nor union of cycles and stars, then by Theorem 3.6, G^{--+} is connected, a contradiction.
Conversely, if G is C_5 or $K_{1,p}$, then clearly G^{--+}=G.
Corollary 4.6 *For any nontrivial* (p, q) *graph* G, $G^{(--+)^n} = G$ *for* $n \geq 2$ *if and only if* G *is* C_5 *or* $K_{1,p}$, *for* $p \geq 2$.

Theorem 4.7 *For any nontrivial* (p, q) *graph* G, $G^{-+-} = G$ *if and only if* G *is* C_5 *or* $K_{1,p} \cup K_{1,r}$, *for* $p \geq 2$, $r \geq 2$.
Proof. Suppose G^{-+-}=G. Assume $G \neq C_5$. We consider the following cases.
Case 1. Suppose G is a block. Then G^{-+-}=$J(G)$. By Theorem 2.3, $G \neq J(G) = G^{-+-}$, a contradiction.

Case 2. Suppose a non block G is union of star. Then we consider following subcases.
Subcase 2.1. If $G = K_{1,p}$, then G^{-+-}=$(p + 1)K_1$, a contradiction.
Subcase 2.2. If G=$K_{1,p} \cup K_{1,r}$, then G^{-+-}=$K_{1,p} \cup K_{1,r}$.
Subcase 2.3. If G is a union of more than two stars, then by Theorem 3.7, G^{-+-} is connected, a contradiction.
Case 3. Suppose a non block G is not a union of star. Then we consider the following subcases.
Subcase 3.1. If G is either union of cycles and stars with q lines, then G^{-+-} has at least $q + 1$ lines, a contradiction.
Subcase 3.2. If G is neither union of cycles nor union of cycles and stars, then by Theorem 3.7, G^{-+-} is connected, a contradiction.
Conversely, if G is C_5 or $K_{1,p} \cup K_{1,r}$, then clearly G^{-+-}=G.
Corollary 4.8 *For any nontrivial* (p, q) *graph* G, $G^{(-+-)^n}$=G *for* $n \geq 2$ *if and only if* G *is* C_5 *or* $K_{1,p} \cup K_{1,r}$, *for* $p \geq 2$, $q \geq 2$.
Theorem 4.9 *For any nontrivial graph* G, $G^{---} = G$ *if and only if* G *is* C_5.
Proof. Suppose G^{---}=G. Assume $G \neq C_5$. We consider the following cases.
Case 1. Suppose G is a block. Then G^{---}=$J(G)$. By Theorem 2.3, $G \neq J(G) = G^{---}$, a contradiction.
Case 2. Suppose a non block G is union of star. Then we consider following subcases.
Subcase 2.1. If $G = K_{1,p}$, then G^{---}=$(p + 1)K_1$, a contradiction.

Subcase 2.2. If G is a union of more than two stars, then by Theorem 3.8, G^{---} is connected, a contradiction.
Case 3. Suppose a non block G is not a union of star. Then we consider the following subcases.
Subcase 3.1. If G is either union of cycles and stars with q lines, then G^{---} has at least $q+1$ lines, a contradiction.
Subcase 3.2. If G is neither union of cycles nor union of cycles and stars, then by Theorem 3.8, G^{---} is connected, a contradiction.
Conversely, if G is C_5, then clearly G^{---}=G.
Corollary 4.10 *For any nontrivial graph* G, $G^{(---)^n} = G$ *if and only if* G *is* C_5.

5. DIAMETERS OF G^{xyz}

Theorem 5.1 *For any nontrivial connected graph* G,
$diam(G^{+++}) \leq diam(G) + 1$.
Proof. Let G be a connected graph. We consider the following three cases.
Case 1. Assume G is a tree. It is easy to see that $diam(G^{+++}) < diam(G) + 1$.

Case 2. Assume G is a cycle C_p, $p \geq 3$, then from Theorem 2.4, $G^{+++} = L(G)$. Therefore $diam(G^{+++}) = diam(L(G)) \leq diam(G) + 1$.

Case 3. Assume G contains a cycle C_p, $p \geq 3$ corresponding to a cycle C_p, $L(C_p)$ is a subgraph in G^{+++}. Therefore $diam(G^{+++}) \leq diam(G) + 1$.
From all the above cases, $diam(G^{+++}) \leq diam(G) + 1$.

Theorem 5.2 *If* $G = (p,q)$ *is nontrivial graph with* $q \geq 2$, $G \neq C_p$ *and satisfying following conditions*
(i) $G \neq K_{1,p}$
(ii) $G \neq K_{1,p} \cup K_{1,r}$
(iii) $G \neq \bigcup_{i=2}^{n} B_i$
(iv) $G \neq K_{1,p} \cup (\bigcup_{i=1}^{n} B_i)$, then $diam(G^{++-}) \leq 4$.
Proof. Let G be a nontrivial graph with $q \geq 2$, $G \neq C_p$ and satisfying conditions (i), (ii), (iii) and (iv), such that G^{++-} is connected. We consider the following cases.
Case 1. Let e_1' and e_2' be line vertices of G^{++-}. If the lines e_1 and e_2 are adjacent in G, then $d_{G^{++-}}(e_1', e_2')$=1. If the lines e_1 and e_2 are nonadjacent in G and there exists a line e (cutpoint c) in G which is adjacent (nonincident) to both the lines e_1 and e_2 in G, then $d_{G^{++-}}(e_1', e_2')$=2. Otherwise $d_{G^{++-}}(e_1', e_2')$=3 or 4. Therefore the distance between any two line vertices in G^{++-} is at most 4.

Case 2. Let c_1' and c_2' be cutpoint vertices of G^{++-}. If the cutpoints c_1 and c_2 are adjacent or coadjacent in G, then $d_{G^{++-}}(c_1', c_2')$=1. If the cutpoints c_1 and c_2 are nonadjacent or noncoadjacent in G and there exists a line e (cutpoint c) in G which is nonincident (adjacent or coadjacent) to both the cutpoints c_1 and c_2 in G, then $d_{G^{++-}}(c_1', c_2')$=2. Therefore the distance between any two cutpoint vertices in G^{++-} is at most 2.

Case 3. Let e_1' and c_1' be line vertex and cutpoint vertex respectively of G^{++-}. If c_1 is nonincident with e_1 in G, then $d_{G^{++-}}(e_1', c_1') = 1$. Otherwise $d_{G^{++-}}(e_1', c_1') = 2$ or 3. Therefore the distance between line vertices and cutpoint vertices in G^{++-} is at most 3.

Hence from all the above cases, $diam(G^{++-}) \leq 4$.

Theorem 5.3 *If a non trivial connected* (p, q) *graph* G *is a cycle* C_p*, then* $diam(G^{++-}) = \left\lceil \frac{p}{2} \right\rceil$.

Proof. If $G = C_p$, then by Theorem 4.1, $G^{++-} = G$.

Therefore $diam(G^{++-}) = diam(C_p) = \left\lceil \frac{p}{2} \right\rceil$.

Theorem 5.4 *If* G *is nontrivial* (p, q) *graph with* $q \geq 2$ *lines,* $p \neq q$ *and satisfying following conditions*

(i) $G \neq \bigcup_{i=2}^{n} B_i$

(ii) $G \neq G_1 \cup G_2$, where G_1 is a graph with at least one cutpoint and G_2 is a union of blocks, then $diam(G^{+-+}) \leq 4$

Proof. Let G be a nontrivial (p, q) graph with $q \geq 2$, $p \neq q$ and satisfying conditions (i) and (ii), such that G^{+-+} is connected. We consider the following cases.

Case 1. Let e_1' and e_2' be line vertices of G^{+-+}. If the lines e_1 and e_2 are adjacent in G, then $d_{G^{+-+}}(e_1', e_2')$=1. If the lines e_1 and e_2 are nonadjacent in G and there exists a line e (cutpoint c) in G which is adjacent (incident) to both the lines e_1 and e_2 in G, then $d_{G^{+-+}}(e_1', e_2')$=2. Otherwise $d_{G^{+-+}}(e_1', e_2')$=3 or 4. Therefore the distance between any two line vertices in G^{+-+} is at most 4.

Case 2. Let c_1' and c_2' be cutpoint vertices of G^{+-+}. If the cutpoints c_1 and c_2 are nonadjacent or noncoadjacent in G, then $d_{G^{+-+}}(c_1', c_2')$=1. If the cutpoints c_1 and c_2 are adjacent or coadjacent in G and there exists a line e (cutpoint c) in G which is incident (nonadjacent or noncoadjacent)with both the cutpoints c_1 and c_2 in G, then $d_{G^{+-+}}(c_1', c_2')$=2. Otherwise $d_{G^{+-+}}(c_1', c_2')$=3. Therefore the distance between any two cutpoint vertices in G^{+-+} is at most 3.

Case 3. Let e_1' and c_1' be line vertex and cutpoint vertex respectively of G^{+-+}. If c_1 is incident with e_1 in G, then $d_{G^{+-+}}(e_1', c_1') = 1$. Otherwise $d_{G^{+-+}}(e_1', c_1') = 2$ or 3. Therefore the distance between line vertices and cutpoint vertices in G^{+-+} is at most 3.

Hence from all the above cases, $diam(G^{+-+}) \leq 4$.

Theorem 5.5 *If* G *is nontrivial connected graph with* $q \geq 2$, $G \neq C_p$ *and satisfying following conditions*

(i) $G \neq K_{1,p}$

(ii) $G \neq \bigcup_{i=2}^{n} B_i$

(iii) $G \neq K_{1,p} \cup (\bigcup_{i=1}^{n} B_i)$, then $diam(G^{+--}) \leq 4$.

Proof. Let G be a nontrivial graph with $q \geq 2$, $G \neq C_p$ and satisfying conditions (i), (ii) and (iii), such that G^{+--} is connected. We consider the following cases.
Case 1. Let e_1' and e_2' be line vertices of G^{+--}. If the lines e_1 and e_2 are adjacent in G, then $d_{G^{+--}}(e_1', e_2')$=1. If the lines e_1 and e_2 are nonadjacent in G and there exists a line e (cutpoint c) in G which is adjacent (nonincident) to both the lines e_1 and e_2 in G, then $d_{G^{+--}}(e_1', e_2')$=2. Otherwise $d_{G^{+--}}(e_1', e_2')$=3 or 4. Therefore the distance between any two line vertices in G^{+--} is at most 4.
Case 2. Let c_1' and c_2' be cutpoint vertices of G^{+--}. If the cutpoints c_1 and c_2 are nonadjacent or noncoadjacent in G, then $d_{G^{+--}}(c_1', c_2')$=1. If the cutpoints c_1 and c_2 are adjacent or coadjacent in G and there exists a line e (cutpoint c) in G which is nonincident (nonadjacent or noncoadjacent) to both the cutpoints c_1 and c_2 in G, then $d_{G^{+--}}(c_1', c_2')$=2. Otherwise $d_{G^{+--}}(c_1', c_2')$=3 or 4. Therefore the distance between any two cutpoint vertices in G^{+--} is at most 4.
Case 3. Let e_1' and c_1' be line vertex and cutpoint vertex respectively of G^{+--}. If c_1 is nonincident with e_1 in G, then $d_{G^{+--}}(e_1', c_1') = 1$. Otherwise $d_{G^{+--}}(e_1', c_1') = 2$ or 3. Therefore the distance between line vertices and cutpoint vertices in G^{+--} is at most 3.
Hence from all the above cases, $diam(G^{+--}) \leq 4$.

Theorem 5.6 *If a non trivial connected (p, q) graph G is a cycle C_p, then* $diam(G^{+--}) = \left\lceil \frac{p}{2} \right\rceil$
Proof. If $G = C_p$, then by Theorem 4.1, $G^{+--} = G$.
Therefore $diam(G^{+--}) = diam(C_p) = \left\lceil \frac{p}{2} \right\rceil$.

Theorem 5.7 *If G is a nontrivial graph with $q \geq 2$ lines, $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to a cutpoint, then* $diam(G^{-++}) \leq 3$.
Proof. Let G be a nontrivial graph with $q \geq 2$ lines, $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to a cutpoint, such that G^{-++} is connected. We consider the following cases.
Case 1. Let e_1' and e_2' be line vertices of G^{-++}. If the lines e_1 and e_2 are nonadjacent in G, then $d_{G^{-++}}(e_1', e_2')$=1. If the lines e_1 and e_2 are adjacent in G and there exists a line e (cutpoint c) in G which is nonadjacent (incident) to both the lines e_1 and e_2 in G, then $d_{G^{-++}}(e_1', e_2')$=2. Otherwise $d_{G^{-++}}(e_1', e_2')$=3. Therefore the distance between any two line vertices in G^{-++} is at most 3.
Case 2. Let c_1' and c_2' be cutpoint vertices of G^{-++}. If the cutpoints c_1 and c_2 are adjacent or coadjacent in G, then $d_{G^{-++}}(c_1', c_2')$=1. If the cutpoints c_1 and c_2 are nonadjacent or noncoadjacent in G and there exists a line e (cutpoint c) in G which is incident (adjacent or coadjacent) to both the cutpoints c_1 and c_2 in G, then $d_{G^{-++}}(c_1', c_2')$=2. Otherwise $d_{G^{-++}}(c_1', c_2')$=3. Therefore the distance between any two cutpoint vertices in G^{-++} is at most 3.

Case 3. Let e_1' and c_1' be line vertex and cutpoint vertex respectively of G^{-++}. If c_1 is incident with e_1 in G, then $d_{G^{-++}}(e_1', c_1') = 1$. Otherwise $d_{G^{-++}}(e_1', c_1') = 2$ or 3. Therefore the distance between line vertices and cutpoint vertices in G^{-++} is at most 3.

Hence from all the above cases, $diam(G^{-++}) \leq 3$.

Theorem 5.8 *If G is a nontrivial graph with $q \geq 2$ lines, $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to a cutpoint, then $diam(G^{--+}) \leq 3$.*

Proof. Let G be a nontrivial graph with $q \geq 2$ lines, $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to a cutpoint, such that G^{--+} is connected. We consider the following cases.

Case 1. Let e_1' and e_2' be line vertices of G^{--+}. If the lines e_1 and e_2 are nonadjacent in G, then $d_{G^{--+}}(e_1', e_2')$=1. If the lines e_1 and e_2 are adjacent in G and there exists a line e (cutpoint c) in G which is nonadjacent (incident) to both the lines e_1 and e_2 in G, then $d_{G^{--+}}(e_1', e_2')$=2. Otherwise $d_{G^{--+}}(e_1', e_2')$=3. Therefore the distance between any two line vertices in G^{--+} is at most 3.

Case 2. Let c_1' and c_2' be cutpoint vertices of G^{--+}. If the cutpoints c_1 and c_2 are nonadjacent or noncoadjacent in G, then $d_{G^{--+}}(c_1', c_2')$=1. If the cutpoints c_1 and c_2 are adjacent or coadjacent in G and there exists a line e (cutpoint c) in G which is incident (nonadjacent or noncoadjacent) to both the cutpoints c_1 and c_2 in G, then $d_{G^{--+}}(c_1', c_2')$=2. Otherwise $d_{G^{--+}}(c_1', c_2')$=3. Therefore the distance between any two cutpoint vertices in G^{--+} is at most 3.

Case 3. Let e_1' and c_1' be line vertex and cutpoint vertex respectively of G^{--+}. If c_1 is incident with e_1 in G, then $d_{G^{--+}}(e_1', c_1') = 1$. Otherwise $d_{G^{--+}}(e_1', c_1') = 2$ or 3. Therefore the distance between line vertices and cutpoint vertices in G^{--+} is at most 3.

Hence from all the above cases, $diam(G^{--+}) \leq 3$.

Theorem 5.9 *If G is a nontrivial graph with $q \geq 2$ lines, $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to a cutpoint, then $diam(G^{-+-}) \leq 4$.*

Proof. Let G be a nontrivial graph with $q \geq 2$ lines, $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to a cutpoint, such that G^{-+-} is connected. We consider the following cases.

Case 1. Let e_1' and e_2' be line vertices of G^{-+-}. If the lines e_1 and e_2 are nonadjacent in G, then $d_{G^{-+-}}(e_1', e_2')$=1. If the lines e_1 and e_2 are adjacent in G and there exists a line e (cutpoint c) in G which is nonadjacent (nonincident) to both the lines e_1 and e_2 in G, then $d_{G^{-+-}}(e_1', e_2')$=2. Otherwise $d_{G^{-+-}}(e_1', e_2')$=3 or 4. Therefore the distance between any two line vertices in G^{-+-} is at most 4.

Case 2. Let c_1' and c_2' be cutpoint vertices of G^{-+-}. If the cutpoints c_1 and c_2 are adjacent or coadjacent in G, then $d_{G^{-+-}}(c_1', c_2')$=1. If the cutpoints c_1 and c_2 are nonadjacent or noncoadjacent in G and there exists a line e (cutpoint c) in G which is nonincident (adjacent or coadjacent) to both the cutpoints c_1 and c_2 in G, then $d_{G^{-+-}}(c_1', c_2')$=2. Otherwise

$d_{G^{-+-}}(c_1', c_2')$=3. Therefore the distance between any two cutpoint vertices in G^{-+-} is at most 3.

Case 3. Let e_1' and c_1' be line vertex and cutpoint vertex respectively of G^{-+-}. If c_1 is nonincident with e_1 in G, then $d_{G^{-+-}}(e_1', c_1') = 1$. Otherwise $d_{G^{-+-}}(e_1', c_1') = 2$ or 3. Therefore the distance between line vertices and cutpoint vertices in G^{-+-} is at most 3.

Hence from all the above cases, $diam(G^{-+-}) \leq 4$.

Theorem 5.10 *If G is a nontrivial graph with $q \geq 2$ lines, $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to a cutpoint, then $diam(G^{---}) \leq 4$.*

Proof. Let G be a nontrivial graph with $q \geq 2$ lines, $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to a cutpoint, such that G^{---} is connected. We consider the following cases.

Case 1. Let e_1' and e_2' be line vertices of G^{---}. If the lines e_1 and e_2 are nonadjacent in G, then $d_{G^{---}}(e_1', e_2')$=1. If the lines e_1 and e_2 are adjacent in G and there exists a line e (cutpoint c) in G which is nonadjacent (nonincident) to both the lines e_1 and e_2 in G, then $d_{G^{---}}(e_1', e_2')$=2. Otherwise $d_{G^{---}}(e_1', e_2')$=3 or 4. Therefore the distance between any two line vertices in G^{---} is at most 4.

Case 2. Let c_1' and c_2' be cutpoint vertices of G^{---}. If the cutpoints c_1 and c_2 are nonadjacent or noncoadjacent in G, then $d_{G^{---}}(c_1', c_2')$=1. If the cutpoints c_1 and c_2 are adjacent or coadjacent in G and there exists a line e (cutpoint c) in G which is nonincident (nonadjacent or noncoadjacent) to both the cutpoints c_1 and c_2 in G, then $d_{G^{---}}(c_1', c_2')$=2. Otherwise $d_{G^{---}}(c_1', c_2')$=3. Therefore the distance between any two cutpoint vertices in G^{---} is at most 3.

Case 3. Let e_1' and c_1' be line vertex and cutpoint vertex respectively of G^{---}. If c_1 is nonincident with e_1 in G, then $d_{G^{---}}(e_1', c_1') = 1$. Otherwise $d_{G^{---}}(e_1', c_1') = 2$ or 3. Therefore the distance between line vertices and cutpoint vertices in G^{---} is at most 3.

Hence from all the above cases, $diam(G^{---}) \leq 4$.

6. ACKNOWLEDGEMENT

This research is supported by UGC-MRP, New Delhi, India: F. No. 41-784/2012 dated: 17-07-2012.

This research is supported by UGC- National Fellowship (NF) New Delhi. No. F./2014-15/NFO-2014-15-OBC-KAR-25873/(SA-III/Website) Dated: March-2015.

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