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ON THE LINE-CUT TRANSFORMATION GRAPHS G^{xy}

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ABSTRACT

In this paper, we introduce line-cut transformation graphs. We investigate some basic properties such as order, size, connectedness, graph equations and diameters of the line-cut transformation graphs.

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Keywords: *cutpoint, line graph, line-cut transformation graphs* G^{xy}.

1. INTRODUCTION

By a graph $G=(V,E)$, we mean a simple, finite, undirected graphs without isolated points. For any graph G, let $V(G)$, $E(G)$, $W(G)$ and $U(G)$ denote the point set, line set, cutpoint set and block set of G, respectively. The lines and cutpoints of a graph are called its members.

Eccentricity of a point $u \in V(G)$ is defined as $e(u) = max\{d_G(u,v) : v \in V(G)\}$, where $d_G(u,v)$ is the distance between u and v in G. The minimum and maximum eccentricities are the *radius* $r(G)$ and *diameter* $diam(G)$ of G, respectively.

A *cutpoint* of a connected graph G is the one whose removal increases the number of components. A *nonseparable graph* is connected, nontrivial and has no cutpoints. A *block* of a graph G is a maximal nonseparable subgraph. A block is called *endblock* of a graph if it contains exactly one cutpoint of G. The *line graph* $L(G)$ of G is the graph whose point set is $E(G)$ in which two points are adjacent if and only if they are adjacent in G. The *jump graph* $J(G)$ of G is the graph whose point set is $E(G)$ in which two points are adjacent if and only if they are nonadjacent in G [4]. For graph theoretic terminology, we refer to [5, 7].

2. LINE-CUT TRANSFORMATION GRAPHS G^{xy}

Inspired by the definition of total transformation graphs [10] and block-transformation graphs [3], we introduce the graph valued functions namely line-cut transformation graphs and we define as follows.

Definition: Let $G=(V,E)$ be a graph, and let α, β be two elements of $E(G)\cup W(G)$. We say that the associativity of α and β is $+$ if they are adjacent or incident in G, otherwise is $-$. Let xy be a 2-permutation of the set $\{+,-\}$. We say that α and β correspond to the first term x of xy if both α and β are in $E(G)$ and α and β correspond to the second term y of xy if one of α and β is in $E(G)$ and the other is in $W(G)$. The *line-cut transformation graph* G^{xy} of G is defined on the point set $E(G)\cup W(G)$. Two points α and β of G^{xy} are joined by a line if and only if these associativity in G is consistent with corresponding term of xy. Since there are four distinct 2-permutations of $\{+,-\}$, we obtain four line-cut transformations of G namely G^{++}, G^{+-}, G^{-+} and G^{--}.

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In other words, let G be a graph, and x, y be two variables taking values $+$ or $-$. The *line-cut transformation graph* G^{xy} is the graph having $E(G) \cup W(G)$ as the point set, and for α, $\beta \in E(G) \cup W(G)$, α and β are adjacent in G^{xy} if and only if one of the following holds:

(i) α, $\beta \in E(G)$. α and β are adjacent in G if $x = +$; α and β are nonadjacent in G if $x = -$.

(ii) $\alpha \in E(G)$, $\beta \in W(G)$. α and β are incident in G if $y = +$; α and β are nonincident in G if $y = -$.

It is interesting to see that G^{++} is exactly the lict graph of G [6]. It is also called as line-cut graph of G [1]. Many papers are devoted to lict graph [1, 2, 6, 8].

The point c_i' (e_i') of G^{xy} corresponding to a cutpoint c_i (line e_i) of G and is referred to as cutpoint (line) vertex.

A graph G and all its four line-cut transformation graphs are shown in Fig 1. In line-cut transformation graphs the line vertices are denoted by dark circles and the cutpoint vertices are denoted by light circles.

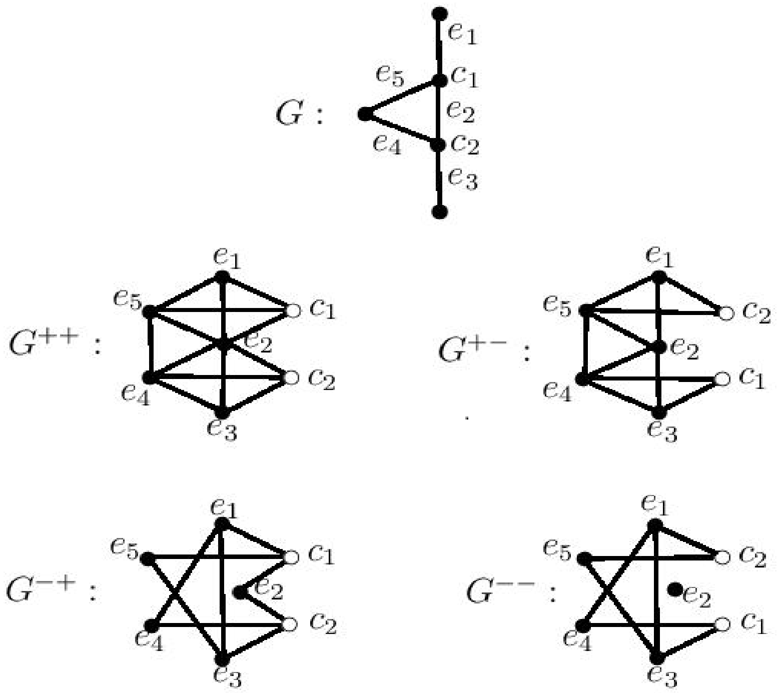

Figure 1

The following will be useful in the proof of our results.

Remark: 2.1 $L(G)$ *is an induced subgraph of* G^{++} *and* G^{+-}.

Remark: 2.2 $J(G)$ *is an induced subgraph of* G^{-+} *and* G^{--}.

Theorem: 2.1 *[5] If* G *is connected, then* $L(G)$ *is connected.*

Theorem: 2.2 *[11] Let* G *be a graph of size* $q \geq 1$. *Then* $J(G)$ *is connected if and only if* G *contains no line that is adjacent to every other lines of* G *unless* $G = K_4$ *or* C_4.

Theorem: 2.3 *[6] A connected graph* G *is isomorphic to its* G^{++} *if and only if* G *is a cycle.*

The following theorem determines the order and size of a line-cut transformation graphs G^{xy}.

Theorem: 2.4 *Let G be a nontrivial connected (p,q) -graph with point set $V(G)=\{v_1,v_2,...,v_p\}$, line set $E(G)=\{e_1,e_2,...,e_q\}$, cutpoint set $W(G)=\{c_1,c_2,...,c_m\}$ and block set $U(G)=\{B_1,B_2,...,B_n\}$, the points of G have degree d_i and L_i be the number of lines to which cutpoint c_i belongs in G and $C(B_i)$ be the number of cutpoints of a connected graph G which are the points of the block B_i. Then the order of G^{xy} is $q+1+\sum_{i=1}^{n}(C(B_i)-1)$ and*

1. The size of G^{+-} = $-q+\frac{1}{2}\sum_{i=1}^{p}d_i^2+\sum_{i=1}^{m}(q-L_i)$.
2. The size of G^{-+} = $\binom{q+1}{2}-\frac{1}{2}\sum_{i=1}^{p}d_i^2+\sum_{i=1}^{m}L_i$.
3. The size of G^{--} = $\binom{q+1}{2}-\frac{1}{2}\sum_{i=1}^{p}d_i^2+\sum_{i=1}^{m}(q-L_i)$.

Proof: If G is a connected graph with p points and q lines, then $L(G)$ has q points. Let $C(B_i)$ be the number of cutpoints of a connected graph G which are the points of the block B_i. Then the number of points in the cutpoint graph $C(G)$ is given by $1+\sum_{i=1}^{n}(C(B_i)-1)$. Since $L(G)$ and $J(G)$ have same number of points.

Therefore the order of G^{xy} = $q+1+\sum_{i=1}^{n}(C(B_i)-1)$.

1. The number of lines in G^{+-} is the sum of the number of lines in $L(G)$ and sum of the number of lines nonincident with the cutpoints in G.
   Thus the size of G^{+-}= $-q+\frac{1}{2}\sum_{i=1}^{p}d_i^2+\sum_{i=1}^{m}(q-L_i)$.
2. The number of lines in G^{-+} is the sum of the number of lines in $J(G)$ and sum of the number of lines incident with the cutpoints in G.
   Thus the size of G^{-+} = $\binom{q+1}{2}-\frac{1}{2}\sum_{i=1}^{p}d_i^2+\sum_{i=1}^{m}L_i$.
3. The number of lines in G^{--} is the sum of the number of lines in $J(G)$ and sum of the number of lines nonincident with the cutpoints in G.
   Thus the size of G^{--} = $\binom{q+1}{2}-\frac{1}{2}\sum_{i=1}^{p}d_i^2+\sum_{i=1}^{m}(q-L_i)$.

3. CONNECTEDNESS OF G^{xy}

The first theorem is well-known.

Theorem: 3.1 *For a given graph G, G^{++} is connected if and only if G is connected.*

Theorem: 3.2 *For any graph G with $q\geq 2$, G^{+-} is connected if and only if*

(i) $G\neq K_{1,p}$

(ii) $G\neq K_{1,p}\cup K_{1,r}$

(iii) $G \neq \bigcup_{i=2}^{n} B_i$

(iv) $G \neq K_{1,p} \cup (\bigcup_{i=1}^{n} B_i)$.

Proof: Suppose a graph G satisfies conditions (i), (ii), (iii) and (iv). We prove the result by following cases.

Case-1. If G is connected, then we have the following subcases.

Subcase-1.1: If G is a block, then clearly $G^{+-} = L(G)$ is connected.

Subcase-1.2: If G has at least one cutpoint, then $L(G)$ is connected subgraph of G^{+-} and also each cutpoint vertex is adjacent to at least one line vertex because every cutpoint is nonincident with at least one line in G. Hence G^{+-} is connected.

Case-2: If G is disconnected with G_1, G_2 ,..., G_n components. By conditions (ii), (iii) and (iv) one of the component G_i is not a star with at least one cutpoint c_i. For every pair of line vertex e_i' and e_j' whose corresponding lines e_i and e_j respectively are non adjacent in G are connected by cutpoint vertex c_i' and for every pair of line vertex e_x' and e_y' whose corresponding lines e_x and e_y respectively are adjacent in G are adjacent in G^{+-}. Therefore G^{+-} is connected.

The converse is obvious.

Theorem: 3.3 *For any graph* G *with* $q \geq 2$, G^{-+} *is connected if and only if* $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to a cutpoint.

Proof: let G be a connected graph with $q \geq 2$, $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to a cutpoint. Then to prove G^{-+} is connected. We consider the following cases.

Case 1. If G is connected then we have the following subcases.

Subcase-1.1: If G is block and $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4), then clearly $G^{-+} = J(G)$ is connected.

Subcase-1.2: If G has atleast one cutpoint then we have the following subsubcases.

Subsubcase-1.2.1: If G contains no line which is adjacent to all other lines, then by Theorem 2.2, $J(G)$ is connected subgraph of G, hence G^{-+} is connected.

Subsubcase-1.2.2: If G contains at least one line e which is adjacent to all other lines, clearly e is incident with a cutpoint c in G, then line vertices and cutpoint vertices are connected in G^{-+}. Therefore G^{-+} is connected.

Case-2: If G is not connected then $J(G)$ is connected subgraph of G^{-+} and each cutpoint vertex is adjacent to atleast one line vertex because every cutpoint is incident with atleast one line in G. Hence G^{-+} is connected.

Conversely, clearly G^{-+} is connected for any graph G of size $q \geq 2$, $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to a cutpoint.

Theorem: 3.4 *For any graph* G *with* $q \geq 2$, G^{--} *is connected if and only if* $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ *(where* x *is any line in* K_4*) has no line which is adjacent to all other lines and is incident to a cutpoint.*

Proof: let G be a connected graph with $q \geq 2$, $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to a cutpoint. Then to prove G^{--} is connected. We consider the following cases.

Case-1. If G is connected then we have the following subcases.

Subcase-1.1: If G is block and $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4), then clearly $G^{--} = J(G)$ is connected.

Subcase-1.2: If G has atleast one cutpoint then we have the following subsubcases.

Subsubcase-1.2.1: If G contains no line which is adjacent to all other lines, then by Theorem 2.2, $J(G)$ is connected subgraph of G. Hence G^{--} is connected.

Subsubcase-1.2.2: If G contains at least one line e which is adjacent to all other lines, since $G \neq K_{1,p}$ therefore there is atleast one line which is nonincident with cutpoint in G, then line vertices and cutpoint vertices are connected in G^{--}. Therefore G^{--} is connected.

Case-2: If G is not connected, then $J(G)$ is connected subgraph of G^{--} and each cutpoint vertex is adjacent to atleast one line vertex because every cutpoint is nonincident with atleast one line in G. Hence G^{--} is connected.
Conversely, clearly G^{--} is connected for any graph G of size $q \geq 2$, $G \neq K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to a cutpoint.

4. GRAPH EQUATIONS AND ITERATIONS OF G^{xy}

For a given graph operator Φ, which graph is fixed under the operator Φ ?, that is $\Phi(G) \cong G$? It was known that for a connected graph G, $L(G) \cong G$ if and only if G is a cycle [9].

For a given line-cut transformation graph G^{xy}, we define the iteration of G^{xy} as follows:

1. $G^{(xy)^1} = G^{xy}$
2. $G^{(xy)^n} = [G^{(xy)^{n-1}}]^{xy}$ for $n \geq 2$.

The isomorphism of G and G^{++} is shown in [6].

Theorem: 4.1 *Let G be a connected graph. Then $L(G) = G^{+-}$ if and only if G is a block.*

Proof: Suppose G is a block. It is known that G has no cutpoints. Then G^{+-} has q points. By definition of $L(G)$ it has q points. Clearly $L(G) = G^{+-}$.

Conversely, suppose $L(G) = G^{+-}$. Assume G is not a block. Then there exist at least one cutpoint. It is known that $L(G)$ has q points where as the number of points of G^{+-} are the sum of the number of lines and cutpoints of G. Thus $L(G)$ has less number of points than G^{+-}. Clearly $G^{+-} \neq L(G)$, a contradiction.

Theorem: 4.2 *A connected graph G is isomorphic to its G^{+-} if and only if G is a cycle.*

Proof: We known that a connected graph G is isomorphic to its line graph if and only if it is a cycle. Also from Theorem 4.1, $L(G) = G^{+-}$ if and only if G is a block. Therefore a connected graph G is isomorphic to its G^{+-} if and only if G is a cycle.

Corollary: 4.3 *For a nontrivial connected graph G, $G = G^{(+-)^n}$ if and only if G is a cycle.*

Theorem: 4.4 *Let G be a connected graph. Then $J(G) = G^{-+}$ if and only if G is a block.*

Proof: Suppose G is a block. It is known that G has no cutpoints. Then G^{-+} has q points. By definition of $J(G)$ it has q points. Clearly $J(G)=G^{-+}$.

Conversely, suppose $J(G)=G^{-+}$. Assume G is not a block. Then there exist at least one cutpoint. It is known that $J(G)$ has q points where as the number of points of G^{-+} are the sum of the number of lines and cutpoints of G. Thus $J(G)$ has less number of points than G^{-+}. Clearly $G^{-+} \neq J(G)$, a contradiction.

Theorem: 4.5 *A connected graph G is isomorphic to its G^{-+} if and only if G is $K_{1,p}$ or C_5.*

Proof: Suppose $G^{-+} = G$. Assume $G \neq C_5$, $K_{1,p}$. We consider the following cases.

Case-1: Suppose G is a block. If $G \neq C_5$, then $G^{-+} \neq J(G)$, a contradiction.

Case-2: Suppose G is not block. If $G \neq K_{1,p}$, then there exists atleast one line which is nonincident with cutpoint in G. Therefore $G^{-+} \neq G$, a contradiction.

Conversely, if G is $K_{1,p}$ or C_5, then clearly $G^{-+} = G$.

Therefore a connected graph G is isomorphic to its G^{-+} if and only if G is $K_{1,p}$ or C_5.

Corollary: 4.6 *For a nontrivial connected graph G, $G = G^{(-+)^n}$ if and only if G is $K_{1,p}$ or C_5.*

Theorem: 4.7 *Let G be a connected graph. Then $G^{--} = J(G)$ if and only if G is a block.*

Proof: Suppose G is a block. It is known that G has no cutpoints. Then G^{--} has q points. By definition of $J(G)$ it has q points. Clearly $J(G)=G^{--}$.

Conversely, suppose $J(G)=G^{--}$. Assume G is not a block. Then there exist at least one cutpoint. It is known that $J(G)$ has q points where as the number of points of G^{--} are the sum of the number of lines and cutpoints of G. Thus $J(G)$ has less number of points than G^{--}. Clearly $G^{--} \neq J(G)$, a contradiction.

Theorem: 4.8 *A connected graph G is isomorphic to its G^{--} if and only if G is C_5.*

Proof: We known that a connected graph G is isomorphic to its jump graph if and only if it is C_5. Also from Theorem 4.7, $J(G)=G^{--}$ if and only if G is a block. Therefore a connected graph G is isomorphic to its G^{--} if and only if G is C_5.

Corollary: 4.9 *For a nontrivial connected graph G, $G = G^{(--)^n}$ if and only if G is C_5.*

5. DIAMETERS OF G^{xy}

Theorem: 5.1 *For any nontrivial connected graph G such that G^{++} is connected,*

$$diam(G^{++}) \leq diam(G) + 1.$$

Proof: Let G be a connected graph. We consider the following three cases.

Case-1: Assume G is a tree, then clearly $diam(G^{++}) < diam(G)+1$.

Case-2: Assume G is a cycle C_p, $p \geq 3$, then from Theorem 2.3, $G^{++} = L(G)$. Therefore $diam\,(G^{++}) \leq diam(G)+1$.

Case-3: Assume G contains a cycle C_p, $p \geq 3$ corresponding to a cycle C_p, $L(C_p)$ is a subgraph in G^{++}. Therefore $diam\,(G^{++}) \leq diam(G)+1$.

From all the above cases, $diam\,(G^{++}) \leq diam(G)+1$.

Theorem: 5.2 *For any nontrivial connected graph G with atleast one cutpoint and $G \neq K_{1,p}$ such that G^{+-} is connected, $diam\,(G^{+-})$ is atmost 4.*

Proof: Let G be a nontrivial connected graph with atleast one cutpoint and $G \neq K_{1,p}$, such that G^{+-} is connected. We consider the following cases.

Case-1: Let e_1' and e_2' be line vertices of G^{+-}. If the lines e_1 and e_2 are adjacent in G then $d_{G^{+-}}(e_1', e_2') = 1$. If the lines e_1 and e_2 are nonadjacent in G then there exists a line e in G adjacent to both the lines e_1 and e_2 in G or there exists a cutpoint in G nonincident with both the lines e_1 and e_2 in G. In both the cases $d_{G^{+-}}(e_1', e_2') \leq 2$, so that, the distance between any two line vertices in G^{+-} is atmost 2.

Case-2: Let c_1' and c_2' be cutpoint vertices of G^{+-}. We consider the following subcases.

Subcase-2.1: If the cutpoints c_1 and c_2 are nonadjacent in G and e is a line in G nonincident with both c_1 and c_2 in G, then $(c_1' e' c_2')$ is a path of length 2 in G^{+-}, hence $d_{G^{+-}}(c_1', c_2') = 2$.

Subcase-2.2: If the cutpoints c_1 and c_2 are nonadjacent in G and e is a line in G incident with c_1 but nonincident with c_2, then c_1' and c_2' are connected by a path of length 2 in G^{+-}, hence $d_{G^{+-}}(c_1', c_2') = 2$.

Subcase-2.3: Let c_1 and c_2 be adjacent in G. If all the lines of G are incident with c_1 and c_2, then

$$d_{G^{+-}}(c_1', c_2') = \begin{cases} 4 \text{ if non endblock is } K_2. \\ 3 \text{ if non endblock is } K_3. \end{cases}$$

If c_1 and c_2 are adjacent and there exists a line e which is nonincident with c_1 and c_2 in G, then the cutpoint vertices c_1' and c_2' are connected by line vertex e' in G^{+-}, hence $d_{G^{+-}}(c_1', c_2') = 2$.
In all the cases the distance between any two cutpoint vertices in G^{+-} is atmost 4.

Case-3: Let c_1' and e_1' be cutpoint vertex and line vertex respectively of G^{+-}. If the cutpoint c_1 is nonincident with a line e_1 in G, then $d_{G^{+-}}(c_1', e_1') = 1$. If the cutpoint c_1 is incident with a line e_1 in G, then

$$d_{G^{+-}}(c_1', e_1') = \begin{cases} 2 \text{ if } e_1 \text{ is not a pendant line in } G. \\ 3 \text{ if } e_1 \text{ is a pendant line in } G. \end{cases}$$

Therefore the distance between cutpoint vertex and line vertex in G^{+-} is atmost 4.

Hence from all the above cases, $diam\,(G^{+-})$ is atmost 4.

Theorem: 5.3 *For any graph G of size $q \geq 2$, $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to cutpoint such that G^{-+} is connected, $diam\,(G^{-+})$ is atmost 3.*

Proof: Let G be a graph of size $q \geq 2$, $G \neq K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is nonincident to cutpoint such that G^{-+} is connected. We consider the following cases.

Case-1: Let e_1' and e_2' be the line vertices of G^{-+}. If the lines e_1 and e_2 are nonadjacent in G, then $d_{G^{-+}}(e_1',e_2') = 1$. If the lines e_1 and e_2 are adjacent in G, then there exists a line e_1 in G which is nonadjacent to both the lines e_1 and e_2 in G or there exists a cutpoint c in G incident to both the lines e_1 and e_2 in G, then $d_{G^{-+}}(e_1',e_2') = 2$. Otherwise $d_{G^{-+}}(e_1',e_2') = 3$. Therefore $d_{G^{-+}}(e_1',e_2') \le 3$, so that the distance between any two line vertices in G^{-+} is atmost 3.

Case-2: Let c_1' and c_2' be cutpoint vertices in G^{-+}. We consider the following subcases.

Subcase-2.1: If the cutpoints c_1 and c_2 are adjacent in G, then the cutpoint vertices c_1' and c_2' in G^{-+} are connected by an line vertex e_1' corresponding to a line e_1 which is incident with both cutpoints c_1 and c_2 in G, hence $d_{G^{-+}}(c_1',c_2') = 2$.

Subcase-2.2: If the cutpoints c_1 and c_2 are nonadjacent in G and there exists lines e_1 and e_2 in G such that a line e_1 is incident with a cutpoint c_1 and a line e_2 is incident with a cutpoint c_2 in G, then c_1' and c_2' are connected by a path of length 3 in G^{-+}, hence $d_{G^{-+}}(c_1',c_2') = 3$.
In both subcases the distance between cutpoint vertices in G^{-+} is atmost 3.

Case-3: Let e_1' and c_1' be line vertex and cutpoint vertex respectively of G^{-+}. If a line e_1 is incident with cutpoint c_1 in G, then $d_{G^{-+}}(e_1',c_1') = 1$. If a line e_1 is nonincident with cutpoint c_1 in G and there exist a line e_1 in G which is incident with cutpoint c_1 and nonadjacent to a line e_1 in G, then e_1' and c_1' are connected by a path of length 2 in G^{-+}, hence $d_{G^{-+}}(e_1',c_1') = 2$.

Hence from all the above cases, $diam(G^{-+})$ is atmost 3.

Theorem: 5.4 *For any graph G of size $q \ge 2$, $G \ne K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to cutpoint such that G^{--} is connected, $diam(G^{--})$ is atmost 4.*

Proof: Let G be a graph of size $q \ge 2$, $G \ne K_{1,p}, K_3, C_4, K_4, K_4 - x$ (where x is any line in K_4) has no line which is adjacent to all other lines and is incident to cutpoint such that G^{--} is connected. We consider the following cases.

Case-1: Let e_1' and e_2' be the line vertices of G^{--}. If the lines e_1 and e_2 are nonadjacent in G, then $d_{G^{--}}(e_1',e_2') = 1$. If the lines e_1 and e_2 are adjacent in G, then there exists a line e_1 in G which is nonadjacent to both the lines e_1 and e_2 in G or there exists a cutpoint c in G nonincident to both the lines e_1 and e_2 in G, then $d_{G^{--}}(e_1',e_2') = 2$. If there is another cutpoint in G which is incident with either e_1 or e_2, then $d_{G^{--}}(e_1',e_2') = 3$. Otherwise $d_{G^{--}}(e_1',e_2') = 4$. Therefore $d_{G^{--}}(e_1',e_2') \le 4$, so that the distance between any two line vertices in G^{--} is atmost 4.

Case-2: Let c_1' and c_2' be cutpoint vertices in G^{--}. We consider the following subcases.

Subcase-2.1: If the cutpoints c_1 and c_2 are adjacent in G, then the cutpoint vertices c_1' and c_2' in G^{--} are connected by an line vertex e_1' corresponding to the line e_1 which is nonincident with both cutpoints c_1 and c_2 in G, hence $d_{G^{--}}(c_1',c_2') = 2$.

Subcase-2.2: If the cutpoints c_1 and c_2 are nonadjacent in G. If there exists a line e which is nonincident with both c_1 and c_2 in G, then $d_{G^{--}}(c_1',c_2') = 2$. Otherwise $d_{G^{--}}(c_1',c_2') = 3$.

Case-3: Let e_1' and c_1' be line vertex and cutpoint vertex respectively of G^{--}. If a line e_1 is nonincident with cutpoint c_1 in G, then $d_{G^{--}}(e_1', c_1') = 1$. If a line e_1 is incident with cutpoint c_1 in G and there exist a line e_1 in G which is nonincident with cutpoint c_1 in G and nonadjacent to a line e_1 in G, then e_1' and c_1' are connected by a path of length 2 in G^{--}, hence $d_{G^{--}}(e_1', c_1') = 2$.

Hence from all the above cases, $diam\,(G^{--})$ is atmost 4.

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