International Research Journal of Pure Algebra-6(2), 2016, 255-259

Available online through www.rjpa.info ISSN 2248-9037

ON PRE GENERALIZED PRE REGULAR WEAKLY INTERIOR AND PRE GENERALIZED PRE REGULAR WEAKLY CLOSURE IN TOPOLOGICAL SPACES

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(Received On: 19-02-16; Revised & Accepted On: 29-02-16)

ABSTRACT

In this paper, the notation of pgprw-interior and pgprw-closure are introduced and studied in topological spaces. It is proved that the complement of pgprw-interior of A is the closure of the complement of A and some properties of the new concepts have been studied.

Keywords: Pgprw closed sets, Pgprw open sets, Pgprw-closure, Pgprw-interior.

Mathematics subject classification (2010): 54A05.

1. INTRODUCTION

N.Levine[1] introduced the concept of generalized sets of a topological space in 1970. Dunham[2] defined generalized closure operator in 1982.Mashhour, Abd El-Monsef and Deeb[3] introduced the concept of pre-closed sets in 1982. Gnanambal[4] introduced and studied the concept of gpr closed sets in topological space. Gnanambal and Balachandran[5], introduced and studied the concept of gpr- interior and gpr-closure operator in topological space in 1999. Recently in the year 2015 Wali and Vivekananda Dembre [6] introduced and studied Pre generalized pre regular weakly closed sets (briefly-pgprw)and Pre generalized pre regular weakly open [7] sets in topological spaces.

2. PRELIMINARIES

A subset A of a topological space (X, τ) is called

- (i) Generalized closed set(briefly g-closed) if $cl(A) \subset U$ whenever $A \subset U$ and U is open in X.
- (ii) Pre-open set if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.
- (iii) Generalized pre regular closed set(briefly gpr-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
- (iv) A subset A of topological space (X,τ) is called a pre generalized pre regular weakly closed set(briefly pgprw-closed set) if pCl(A) \subseteq U whenever A \subseteq U and U is rg α open in (X,τ) .
- (v) A subset A in (X,τ) is called Pre generalized pre regular weakly open set in X if A^c is Pre generalized pre regular weakly closed set in X.

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c , P-Cl(A) and P-int(A) denote the Closure of A, Interior of A, Compliment of A, pre closure of A and pre-interior of A in X respectively.

3. PGPRW-CLOSURE AND PGPRW- INTERIOR IN TOPOLOGICAL SPACES

In this section the notation of pgprw-closure and pgprw-Interior is defined and some of its basic properties are studied.

Definition 3.1: For a subset A of (X, τ) , pgprw-closure of A is denoted by pgprw-cl(A) and defined as pgprw-cl(A) = $\cap \{G: A \subseteq G, G \text{ is pgprw-closed in } (X, \tau)\}$ or $\cap \{G: A \subseteq G, G \in \text{pgprw-C}(X)\}$

Theorem 3.2: If A and B are subsets of space (X, τ) then

- (i) pgprw-cl (X) = X, pgprw-cl(ϕ) = ϕ
- (ii) $A \subset pgprw-cl(A)$.
- (iii) If B is any pgprw-closed set containing A, then pgprw-cl(A) \subseteq B.
- (iv) If $A \subseteq B$ then $pgprw-cl(A) \subseteq pgprw-cl(B)$
- (v) pgprw-cl(A) = pgprw-cl(pgprw-cl(A))
- (vi) $pgprw-cl(A \cup B) = Pgprw-cl(A) \cup pgprw-cl(B)$.

Proof:

- (i) By definition of pgprw-closure, X is only pgprw-closed set containing X. Therefore pgprw-cl(X) = Intersection of all the pgprw-closed set containing $X = \bigcap \{X\} = X$ therefore pgprw-cl (X) = X and again by definition of pgprw-closure. Pgprw-cl(ϕ) = Intersection of all pgprw-closed sets containing $\phi = \phi \cap A$ and pgprw-closed set containing $\phi = A$ and pgprw-closed set containing $\phi = A$ and pgprw-closed sets containing $\phi = A$ and pgprw-
- (ii) By definition of pgprw-closure of A, it is obvious that $A \subseteq pgprw-cl(A)$.
- (iii) Let B be any pgprw-closed set containing A. Since pgprw-cl(A) is the intersection of all pgprw-closed set containing A, pgprw-cl(A) is contained in every pgprw-closed set containing A. Hence in particular pgprw-cl(A) ⊂B
- (iv) Let A and B be subsets of (X,τ) such that $A\subseteq B$ by definition of pgprw-closure, $Pgprw-cl(B)=\cap \{F:B\subseteq F\in pgprw-C(X)\}.$ If $B\subseteq F\in pgprw-C(X)$, then $pgprw-cl(B)\subseteq F$. since $A\subseteq B$, $A\subseteq B\subseteq F\in pgprw-C(X)$, we have $pgprw-cl(A)\subseteq F$, $pgprw-cl(A)\subseteq \cap \{F:B\subseteq F\in pgprw-C(X)\}=pgprw-cl(B)$. Therefore $pgprw-cl(A)\subseteq pgprw-cl(B)$.
- (v) Let A be any subset of X by definition of pgprw-closure , pgprw-cl(A) = \cap {F : A \subseteq F ϵ pgprw-C(X)}. If A \subseteq F ϵ pgprw-C(X) then pgprw-cl(A) \subseteq F, since F is pgprw-closed set containing pgprw-cl(A) by (iii) pgprw-cl(pgprw-cl(A)) \subseteq F;Hence pgprw-cl(pgprw-cl(A))= \cap {F:A \subseteq F ϵ pgprw-C(X)}=pgprw-cl(A). Therefore; pgprw-cl(pgprw-cl(A))=pgprw-cl(A).
- (vi) Let A and B be subsets of X, Clearly $A \subseteq A \cup B$, $B \subseteq A \cup B$ from (iv) $pgprw-cl(A) \subseteq pgprw-cl(A \cup B)$, $pgprw-cl(B) \subseteq pgprw-cl(A \cup B)$; hence, $pgprw-cl(A) \cup pgprw-cl(B) \subseteq pgprw-cl(A \cup B)$(1) Now we have to prove that $pgprw-cl(A \cup B) \subseteq pgprw-cl(A) \cup pgprw-cl(B)$. Suppose $x \notin pgprw-cl(A) \cup pgprw-cl(B)$ then $\exists pgprw-closed$ set A_1 and B_1 with $A \subseteq A_1$, $B \subseteq B_1$ and $x \notin A_1 \cup B_1$. We have $A \cup B \subseteq A_1 \cup B_1$ and $A_1 \cup B_1$ is the pgprw-closed set (we know that union of two pgprw closed subsets of X is pgprw closed set in X) such that $x \notin A_1 \cup B_1$. Thus $x \notin pgprw-cl(A \cup B)$.
- (vii)Hence pgprw-cl(AUB) \subseteq pgprw-cl(A)U pgprw-cl(B)-----(2). From (1) and (2) we have pgprw-cl(AUB)= pgprw-cl(A)Upgprw-cl(B).

Theorem 3.3: If $A \subset X$ is pgprw-closed set then pgprw-cl(A) =A

Proof: Let A be pgprw-closed subset of X. we know that $A \subseteq pgprw-cl(A) - (1)Also A \subseteq A$ and A is pgprw-closed set by theorem3.2 (iii) pgprw-cl(A) \subseteq A - (2)Hence pgprw-cl(A) = A.

The Converse of the above need not be true as seen from the following example.

Example 3.4: Let $X = \{a,b,c,d\}$, $\tau = \{X,\phi,\{a\},\{c,d\},\{a,c,d\}\}\}$. $A = \{a,c\}$ pgprw-cl(A)= $\{a,c\}$ =A then A is not pgprw-closed set.

Theorem 3.5: If A and B are subsets of space X then pgprw-cl(A \cap B) \subseteq pgprw-cl(A) \cap pgprw-cl(B).

Proof: Let A and B be subsets of X, Clearly $A \cap B \subseteq A$, $A \cap B \subseteq B$ by theorem 3.2(iv) Pgprw-cl($A \cap B$) \subseteq ppprw-cl($A \cap B$

Remark 3.6: In-general; pgprw-cl(A) \cap pgprw-cl(B) \not pgprw-cl(A \cap B) as seen from the following example.

Example 3.7: Consider $X=\{a, b, c, d\}$, $\tau=\{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$, $A=\{b, c\}$, $B=\{c, d\}$, $A\cap B=\{c\}$, $pgprw-cl(A) = \{b, c, d\}$, $pgprw-cl(B) = \{c, d\}$, $pgprw-cl(A\cap B) = \{c\}$ and $pgprw-cl(A) \cap pgprw-cl(B) = \{c, d\}$ therefore $pgprw-cl(A) \cap pgprw-cl(B) \not \equiv pgprw-cl(A\cap B)$.

Theorem 3.8: For an $x \in X$, $x \in pgprw-cl(A)$ if and if $A \cap V \neq \phi$ for every pgprw-open set V containing x.

Proof: Let $x \in \text{pgprw-cl}(A)$. To prove $A \cap V \neq \emptyset$ for every pgprw-open set V containing x by contradiction.

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Suppose $\exists pgprw$ -open set V containing x s.t $A \cap V = \emptyset$. Then $A \subseteq X$ -V, X-V is pgprw-closed set, pgprw-cl(A) $\subseteq X$ -V. This shows that $x \notin pgprw$ -cl(A) which is contradiction. Hence $A \cap V \neq \emptyset$ for every pgprw-open set V containing x. Conversely: Let $A \cap V \neq \emptyset$ for every pgprw-open set V containing x. To prove $x \in pgprw$ -cl(A). we prove the result by contradiction. Suppose $x \notin pgprw$ -cl(A) then there exist a pgprw-closed subset Fcontaining A s.t $x \notin F$. Then $x \in X$ -F is pgprw-open. Also $(X - F) \cap A = \emptyset$ which is contradiction. Hence $x \in pgprw$ -cl(A).

Theorem 3.9: If Ais subset of space X then

- (i) Pgprw-cl(A) \subseteq cl(A).
- (ii) Pgprw-cl(A) \subseteq pcl(A).

Proof:

- (i) Let A be subset of space X by definition of Closure $Cl(A) = \subseteq \cap \{F: A \subseteq F \in C(X)\}$. If $A \subseteq F \in C(X)$ then $A \subseteq F \in C(X)$ because every closed set is pgprw-closed that is pgprw-cl(A) $\subseteq F$ therefore pgprw-cl(A) $\subseteq \cap \{F: A \subseteq F \in C(X)\} = cl(A)$. Hence pgprw-cl(A) $\subseteq cl(A)$.
- (ii) Let A be subset of space X by definition of p-closure $pcl(A) = \subseteq \cap \{F: A \subseteq F \in p-C(X)\}$. If $A \subseteq F \in p-C(X)$ then $A \subseteq F \in pcl(X)$ because every p-closed set is pgprw-closed that is pgprw-cl(A) $\subseteq F$ therefore $pcl(A) \subseteq \cap \{F: A \subseteq F \in p-C(X)\} = pcl(A)$. Hence $pcl(A) \subseteq p-cl(A)$

Remark 3.10: Containment relation in the above theorem 3.9 may be proper as seen from following example.

Example 3.11: Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$, $A = \{a\}$ $cl(A) = \{a, c, d\}$ $pgprw-cl(A) = \{a, c, d\}$ It follows that $pgprw-cl(A) \subset cl(A)$ and $pgprw-cl(A) \subset pcl(A)$

Theorem 3.12: If A is subset of space X then $gpr-cl(A) \subseteq pgprw-cl(A)$ where $gpr-cl(A) = \bigcap \{F: A \subseteq F \in GPR-C(X)\}$

Proof: Let A be a subset of X by definition of pgprw-closure, pgprw-cl(A) = $\cap \{F: A \subseteq F \in pgprw-C(X)\}$ If $A \subseteq F \in pgprw-C(X)$ then $A \subseteq F \in GPR-C(X)$, because every pgprw-closed is gpr-closed i.e gpr-cl(A) $\subseteq F$ therefore gpr-cl(A) $\subseteq \cap \{F: A \subseteq F \in pgprw-C(X)\}$ = pgprw-cl(A). Hence gpr-cl(A) $\subseteq pgprw-cl(A)$.

Theorem 3.13: Pgprw-closure is a Kuratowski closure operator on a space X.

Proof: Let A and B be the subsets space X.

- (i) $pgprw-cl(X) = X, pgprw-cl(\phi) = \phi$
- (ii) $A \subseteq pgprw-cl(A)$
- (iii) pgprw-cl(A) = pgprw-cl(pgprw-cl(A))
- (iv) pgprw-cl(AUB) = Pgprw-cl(A)Upgprw-cl(B) by theorem 3.2 Hence pgprw-closure is a Kuratowski closure operator on a space X.

Definition 3.14: For a subset A of (X, τ) , pgprw-interior of A is denoted by pgprw-int(A) and defined as pgprw-int(A)= $\cup \{G: G \subseteq A \text{ and } G \text{ is pgprw-open in } X\}$ or $\cup \{G: G \subseteq A \text{ and } G \text{ } \epsilon \text{pgprw-O}(X)\}$ i.e pgprw-int(A) is the union of all pgprw-open set contained in A.

Theorem 3.15: Let A and B be subset of space X then

- (i) pgprw-int(X) = X, $pgprw-int(\phi) = \hat{\phi}$
- (ii) $pgprw-int(A) \subseteq A$
- (iii) If B is any pgprw-open set contained in A, then B⊆ pgprw-int(A)
- (iv) If $A \subseteq B$ then pgprw-int(A) \subseteq pgprw-int(B)
- (v) pgprw-int(A)= pgprw-int(pgprw-int(A))
- (vi) pgprw-int(A \cap B)=pgprw-int(A) \cap pgprw-int(B)

Proof: (i) and (ii) by definition of pgprw-interior of A, it is obvious.

(iii) Let B be any pgprw-open set s.t $B \subseteq A$. Let $x \in B$, B is an pgprw-open set contained in A, x is an pgprw-interior of A i.e. $x \in B$ for $B \subseteq B$ for $B \subseteq B$ is an pgprw-interior as theorem 3.2 and definition of pgprw-interior.

Theorem 3.16: If a subset A of X is pgprw-open then pgprw-int(A) = A

Proof: Let A be pgprw-open subset of X. We know that $pgprw-int(A) \subseteq A$ -(1) Also A is pgprw-open set contained in Afrom Theorem 3.15 (iii) $A \subseteq pgprw-int(A)$ --(2). Hence From(1) and (2) pgprw-int(A) = A

Theorem 3.17: If A and B are subsets of space X then $pgprw-int(A) \cup pgprw-int(B) \subset pgprw-int(A \cup B)$

Proof: We know that $A \subseteq A \cup B$ and $B \subseteq A \cup B$, We have Theorem 3.15(iv) pgprw-int($A \cup B$) and pgprw-int($A \cup B$). This implies that pgprw-int($A \cup B$) pgprw-int($A \cup B$). This implies that pgprw-int($A \cup B$) pgprw-int($A \cup B$) and pgprw-int($A \cup B$).

Remark 3.18: The converse of the above theorem need not be true as seen from the following example.

Example 3.19: Let X={a, b, c, d} and τ ={X, ϕ , {a},{b},{a, b}, {a, b, c}}, A={b, c}, B={a, d}, A\cup B={a, b, c, d}, pgprw-int(A)={b, c}, pgprw-int(B)={a}, pgprw-int(A\cup B)=X, pgprw-int(A) \cup pgprw-int(B) = {a, b, c}; therefore pgprw-int(A\UB) \neq pgprw-int(A\UB) \neq pgprw-int(A\UB) \neq pgprw-int(B).

Theorem 3.20: If A is a subset of X then (i) $int(A) \subseteq pgprw-int(A)(ii) p-int(A) \subseteq pgprw-int(A)$.

Proof:

- (i) Let A be a subset of a space X. Let $x \in Int(A) => x \in U\{G: G \text{ is open, } G \subseteq A\} => \exists \text{ an open set } G \text{ s.t. } x \in G \subseteq A => \exists \text{ an pgprw-open set } G \text{ s.t. } x \in G \subseteq A \text{ as every open set is an pgprw-open set in } X => x \in U\{G: G \text{ is pgprw-open set in } X\} => x \in pgprw-int(A). Thus <math>x \in Int(A) => x \in pgprw-int(A)$. Hence $Int(A) \subseteq pgprw-int(A)$.
- (ii) Let A be a subset of a space X. Let $x \in p \text{int}(A) => x \in U\{G : G \text{ is p-open, } G \subseteq A\} => \exists$ an p-open set G s.t. $x \in G \subseteq A => \exists$ an pgprw-open set G s.t. $x \in G \subseteq A$, as every p-open set is an pgprw-open set in $X => x \in U\{G : G \text{ is pgprw-open set in } X\} => x \in pgprw-int(A)$. Thus $x \in p-int(A) \subseteq pgprw-int(A)$. Hence $p-int(A) \subseteq pgprw-int(A)$.

Remark 3.21: Containment relation in the above theorem may be proper as seen from the following example

Example 3.22 Let $X = \{a, b, c, d\}$ $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}; A = \{b, c\}, Int(A) = \{b\}, p-int(A) = \{b\}, p-int(A) = \{b, c\}, therefore int(A) = pgprw-int(A) and p-int(A) = pgprw-int(A)$

Theorem 3.23: If A issubset of X, then pgprw-int(A) \subseteq gpr-int(A), where gpr-int(A) is given by gpr-int(A) = $\bigcup \{G \subseteq X : G \text{ is gpr-open, } G \subseteq A\}$

Proof: Let A be a subset of a space X. Let $x \in \operatorname{pgprw-int}(A) => x \in \bigcup \{G : G \text{ is pgprw-open }, G \subseteq A\} => \exists \text{ an pgprw-open set } G \text{ s.t. } x \in G \subseteq A, \text{ as every pgprw-open set is an gpr-open set in } X => x \in \bigcup \{G : G \text{ is gpr-open, } G \subseteq A\} => x \in \operatorname{gpr-int}(A). \text{ Thus } x \in \operatorname{pgprw-int}(A) => x \in \operatorname{gpr-int}(A) \text{ Hence pgprw-int}(A) \subseteq \operatorname{gpr-int}(A).$

Theorem 3.24: For any subset A of X

- (i) X pgprw-int(A) = pgprw-cl(X A)
- (ii) pgprw-int(A) = X pgprw-cl(X A)
- (iii) pgprw-cl(A) = X pgprw-int(X A)
- (iv) X pgprw-cl(A) = pgprw-int(X A)

Proof:

- (i) $x \in X$ -pgprw-int(A) then x is not in pgprw-int(A) i.e. every pgprw-open set G containing x s.t. $G \nsubseteq A$. This implies every pgprw open set G containing x intersects (X A) i.e. $G \cap (X A) \neq \emptyset$. Then by theorem 3.8 $x \in \text{pgprw-cl}(X-A)$ therefore X-pgprw-int(A) $\subseteq \text{pgprw-cl}(X-A)$ and Let, $x \in \text{pgprw-cl}(X-A)$, then every $x \in \text{pgprw}$ open set, G containing x intersects X A i.e. $G \cap (X A) \neq \emptyset$, i.e. every pgprw-open G containing x s.t. $G \subseteq A$. Then by definition 3.14, x is not in pgprw-int(A), i.e. $x \in X$ -pgprw-int(A) and so pgprw-cl $(X-A) \subseteq X$ -pgprw-int(A)- (2). Thus X pgprw-int(A) = pgprw-cl(X A).
- (ii) Follows by taking complements in (i).
- (iii) Follows by replacing A by X-A in (i)
- (iv) Follows by taking complements in (iii).

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Source of Support: Nil, Conflict of interest: None Declared

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