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RESEARCH ARTICLE

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ON PRE GENERALIZED PRE REGULAR WEAKLY CONTINUOUS MAPS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, a new class of maps called Pgprw-Continuous maps are introduced and investigated. A map f: X→Y is called pgprw-continuous maps if the inverse image of every closed set in Y is pgprw closed set in X and some of their properties are studied.

Key words: Pgprw closed sets, Pgprw open sets, Pgprw- continuous maps.

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1.INTRODUCTION

The Concept of semi-continuous maps was introduced and studied by Levine [1], Mishra et.al [2], Benchalli and Wali [3], Abd El-Monsef, El-deeb and Mahmoud [4], Jayalakshmi and Janaki [5], Vadivel and Vairamanickam [6], Joshi,Gupta,Bharadwaj Kumar andKumar [7], Bhattacharya [8], Janaki and Renu Thomas [9], Shlya Isac Mary and Thangvely [10], Mashhour, Abd El-Monsef, El-deeb & El-deeb [11], Maki,Umehara and Noiri [12], Navalagi, Chandrashakarappa and Gurushatanavar [13],Gnanambal [14], Bhattacharya and Lahiri [15], Arya and Gupta [16] introduced and studied rgw-continuous maps, rw-continuous maps, β-continuous maps,wgrα-continuous,rα-continuous maps,gprw-continuous maps,gr-continuous maps, R*-continuous maps, rps-continuous maps, p-continuous maps, gp-continuousmaps, gspr-continuous maps, gpr-continuous maps, sg-continuous maps, completely continuous maps respectively. In this paper, a new class of maps called pgprw continuous maps are introduced and invetigated.

2.PRELIMINARIES

Throughout this paper space (X, τ),(Y, σ) &(Z,μ)(or simply X,Y&Z) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c,P-Cl(A) and P-int(A) denote the Closure of A, Interior of A , Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

Definition 2.1:[6] A subset A of topological space (X, τ) is called a rgα-closed set if α-Cl(A)$\subseteq$ U, Whenever A$\subseteq$ U and U is regular α- open in X.

Definition 2.2:[17]A subset A of topological space (X, τ) is called a pre generalized pre regularωeakly closed set(briefly pgprω-closed set) if pCl(A)⊆U whenever A⊆U and U is rgα-open in (X, τ).

Theorem 2.3:[17] If a subset A of (X, τ) is pre-closed set ,then it is pre generalized pre regular weakly closed set but not conversely.

Theorem 2.4:[17] If a subset A of (X, τ) is Closed , then it is pgprw-closed but not conversely.

Theorem 2.5:[17] If a subset A of (X, τ) is pgprw closed set ,then it is gpr-closed set but not conversely.

Theorem 2.6:[17] If a subset A of (X, τ) is pgprw closed set ,then it is gspr-closed set but not conversely

Theorem 2.7:[17] If a subset A of (X, τ) is pgprw cosed set,then it is gp closed set but not conversely.

Theorem 2.8:[18] If a subset A of (X, τ) is called pre generalized pre regular weakly open set if $A^c$is a pgprw closed.

Theorem 2.9:[19] Let A be a subset of (X, τ) Then pgprw-cl(A) of A is defined to be the intersection of all pgprw-closed sets containing A and is denoted by pgprw-cl(A).

Definition 2.10:[20] A subset A of topological space (X, τ) is called Regular openset if A = int(clA)) and a regular closed set if A = cl(int(A)).

Definition 2.11:A map f: (X, τ) → (Y,σ) is said to be

(i)Compeletely continuous map[16]if f^{-1}(V)is a regular closed set of (X, τ)for every closed set V of (Y, σ).

(ii)semi-continuousmap[1]if f^{-1}(V)is a semi-closed set of (X, τ)for every closed set V of (Y, σ).

(iii)Rgw-continuous map[2] if f^{-1}(V)is a rgw-closed set of (X, τ)for every closed set V of (Y,σ).

(iv)Rw-continuousmap[3] if f^{-1}(V)is a rw-closed set of (X, τ)for every closed set V of (Y, σ).

(v)β-continuous map[4]if f^{-1}(V)is a β-closed set of (X, τ)for every closed set V of (Y, σ).

(vi)wgrα- continuousmap[5] if f^{-1}(V)is a wgrα-closed set of(X, τ)for every closed set V of (Y, σ).

(vii)Rgα-continuous map[6]if f^{-1}(V)is aRgα-closed set of(X, τ)for every closed set V of (Y,σ).

(viii)gprw continuous map[7]if f^{-1}(V)is a gprw-closed set of(X, τ)for every closed set V of (Y, σ).

(ix)gr continuousmap[8] if f^{-1}(V)is a gr closed set of(X, τ)for every closed set V of (Y,σ).

(x)R* continuous map[9]if f^{-1}(V)is a R*-closed set of (X, τ)for every closed set V of (Y, σ).

(xi)Rpscontinuousmap[10]if f^{-1}(V)is a rps-closed set of (X, τ)for every closed set V of (Y, σ).

(xii)p- continuous map[11]if f^{-1}(V)is a p- closed set of (X, τ)for every closed set V of (Y,σ).

(xiii)gp- continuousmap[12] if f^{-1}(V)is a gp closed set of (X, τ)for every closed set V of (Y, σ).

(xiv)gspr-continuous map[13]if f^{-1}(V)is a gspr closed set of (X, τ)for every closed set V of (Y, σ).

(xv)gpr-continuous map[14]if f^{-1}(V)is a gpr-closed set of (X, τ)for every closed set V of (Y, σ).

(xvi)sg-continuous map[15]if f^{-1}(V)is a sg closed set of (X, τ)for every closed set V of (Y,σ).

3. PGPRW-CONTINOUS MAPS AND THEIR BASIC PROPERTIES

Definition 3.1: A map f: (X, τ) → (Y, σ) is said to be pgprw continuous maps(briefly pgprw-continuous) if the inverse image of every closed set in Y is pgprw closed set in X.

Theorem 3.2: If a map f: (X, τ) → (Y,σ) is continuous map then it is pgprw-continuous map,but not conversely.

Proof: It is cleary by theorem [2.4].

Example.3.3: Let, X=Y={a,b,c,d}, τ={X, ∅,{a},{b},{a,b},{a,b,c}}and σ = {Y, ∅,{a},{b},{a,b},{a,b,c}. Let a map f: X →Y defined by f(a)=c,f(b)=a,f(c)=b,f(d)=d, then f is pgprw continuous map, but not continuous map,as closed set f={c,d} in Y, then f^{-1}(F)={a,d} in X which is not closed set in X.

Theorem.3.4: If a map f: (X, τ) → (Y, σ) is p-continuous map, then it is pgprw-continuous map,but not conversely.

Proof: Follow from the fact that every p-closed set is pgprw closed set[Theorem 2.3].

Example 3.5: Let, X=Y={a,b,c,d}, τ={X, ∅,{a},{b},{a,b},{a,b,c}}and

σ = {Y, ∅,{a},{b},{a,b},{a,b,c}. Let a map f: X→Y defined by f(a)=c,f(b)=a,f(c)=b,f(d)=d,

Then f is pgprw-continuous map,but not p-continuous map as closed set F = {c,d} in Y then

f^{-1}(F)={a,d} in X which is not p-closed set in X.

Theorem.3.6: If a map f: (X, τ) → (Y, σ) is pgprw-continuous map, then f is gpr-continuous map,but not conversely.

Proof: Follow from the fact that every pgprw-closed set is gpr closed set[Theorem 2.5].

Example 3.7: Let, X=Y={a,b,c}, τ={X, ∅,{a},{b,c}}and

σ = {Y, ∅,{a}}. Let a map f: X→Y defined by f(a)=b,f(b)=a,f(c)=c.

Then f is gpr-continuous map but not pgprw-continuous map as closed set F = {b,c} in Y then ;

f^{-1}(F)={a,c} in X which is not pgprw-closed set in X.

Theorem.3.8: If a map f: (X, τ) → (Y,σ) is pgprw-continuous map, then it is gspr-continuous,but not conversely.

Proof: Follow from the fact that every pgprw-closed set is gspr closed set[Theorem 2.6].

Example 3.9: Let, X=Y={a,b,c}, τ={X, ∅,{a},{b,c}}and

σ = {Y, ∅,{a}}. Let a map f: X→Y defined by f(a)=b,f(b)=a,f(c)=c.

Then f is gspr-continuous map but not pgprw-continuous map as closed set F = {b,c} in Y then,

f^{-1}(F)={a,c} in X which is not pgprw-closed set in X.

Theorem.3.10: If a map f: (X, τ) → (Y,σ) is pgprw-continuous map, then it is gp-continuous map,but not conversely.

Proof: Follow from the fact that every pgprw-closed set is gp closed set[Theorem 2.7].

Example 3.11: Let, X=Y={a,b,c}, τ={X, ∅,{a},{b,c}} and σ = {Y, ∅,{a}}. Let a map f: X→Y defined by f(a)=b,f(b)=a,f(c)=c.

Then f is gp-continuous map but not pgprw-continuous map as closed set F = {b,c} in Y ,then

f^{-1}(F)={a,c} in X which is not pgprw-closed set in X.

Theorem.3.12: Let f: X→Y be a map, then the following statements are equivalent

(i)f is pgprw-continuous map

(ii)The inverse image of each open set in Y is pgprw-open in X.

Proof: Assume that f : X→Y is pgprw-continuous map,Let G be open in Y, ThenG^c is closed in Y. Since f is pgprw-continuous map,$f^{-1}(G^c)$ is pgprw closed in X. But $f^{-1}(G^c)$ = X- f^{-1}(G). Thus f^{-1}(G) is pgprw-open in X.

Conversely, assume that inverse image of each open set in Y is pgprw-open in X.Let F be any closed set in Y, by assumption $f^{-1}(F^c)$ is pgprw-open in X. But $f^{-1}(F^c)$= X- f^{-1}(F).Thus X- f^{-1}(F) is pgprw-open in X and so f^{-1}(F) is pgprw closed in X. Therefore f is pgprw-continuous map; hence (i) & (ii) are equivalent.

Theorem.3.13: If a map f:X→Y is completely continuous map, then it is pgprw-continuous map.

Proof: Suppose that a map f: (X, τ) → (Y, σ) is completely continuous map; Let F be closed set in Y then f^{-1}(F) is regular closed in X and f^{-1}(F) is pgprw closed in X. Thus f is pgprw-continuous map.

Remark: The following examples shows that pgprw-continuous maps are independent ofβ-continuousmap,rw-continuousmap,rgw-continuousmap,wgrα-continuous-map,rα continuousmap, gprw-continuousmap, gr-continuousmap, R*-continuousmap, rps-continuousmap, Semi-continuousmap.

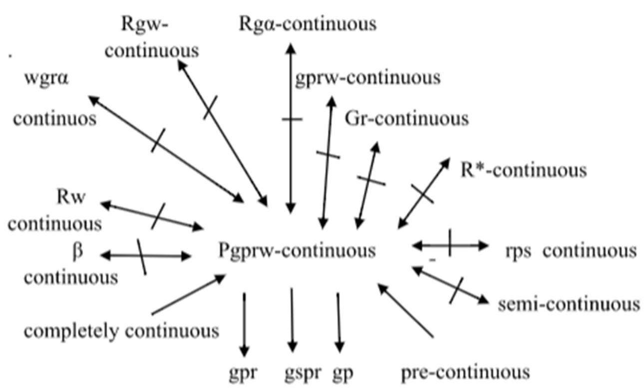
Example 3.14: Let, X=Y={a,b,c}, τ={X, ∅,{a},{b,c}} and σ = {Y, ∅,{a}}. Let a map f: X→Y defined by f(a)=b,f(b)=a,f(c)=c. then f is β-continuousmap,rw-continuousmap,rgw-continuousmap,wgrα-continuousmap,rα-continuousmap,gprw-continuousmap,gr-continuousmap,R*-continuousmap,rps-continuousmap,Semi-continuousmap but f is not pgprw-continuous as closed set F = {b,c} in Y then f^{-1}(F)={a,c} in X which is not pgprw-closed set in X.

Example3.15: Let X={a,b,c,d}, Y={a,b,c},τ={X, ∅,{a},{b},{a,b},{a,b,c}}and σ = {Y, ∅,{a}}.Let a map f: X→ Y defined by f(a)=b,f(b)=a,f(c)=a,f(d)=c, Then f is pgprw-continuous mapbut f is not-rw-continuous-map,rgw-continuous-map,wgrα-continuous-map,
rgα-continuous-map,gprw-continuousmap,R*continuousmaps,rps-continuousmap respectively as closed set F= {b,c} in Y, then f^{-1}(F)={a,d} in X which is not rw-closed,rgw-closed, wgrα-closed, rgα-closed,gprw-closed,R*-closed,rps closed set in X.

Example3.16: Let X=Y={a,b,c,d} and τ ={X,∅,{a},{b},{a,b},{a,b,c}}
and σ = {Y, ∅,{b,c},{b,c,d},{a,c,d}}. Let map f: X→ Y defined by f(a)=c,f(b)=b,f(c)=a,f(d)=d, then f is pgprw-continuousmap but f is not gr-continuousmap as closed set F= {a} in Y, then f^{-1}(F)={c}in X,which is not gr-closed set in X.

Example3.17: Let X={a,b,c,d},Y={a,b,c} and τ ={X,∅,{a},{b},{a,b},{a,b,c}} σ = {Y, ∅,{a}}
Let map f: X→Y defined by f(a)=b,f(b)=b,f(c)=a,f(d)=c, then f is pgprw-continuous map but f is not Semi-continuous map, β-continous map,sg-continuousmaps as closed set F={b,c} in Y then
f^{-1}(F)={a,b,d}in X which is not semi-closed, β-closed, sg-closed set in X.

Remark 3.18:From the above discussion and known results we have the following implication



Theorem 3.19: The Composition of two pgprw-continuous maps need not be pgprw-continuous maps and this can be shown by following example.

Example 3.20: Let X=Y=Z={a,b,c} and τ ={X,∅,{a},{b},{a,b},{a,b,c}}
and σ = {Y, ∅,{a}},μ = { Z,∅,{a},{a,b},{a,c}} and a maps f: X→Y, g: Y→ Z & gof:X→ Zare identity maps both f & g are pgprw continuous maps but gof not pgprw-continuous map.Sinceclosed set V={b} in Z, f^{-1}(V)={b},which is not pgprw closed set in X.

Theorem 3.21: Let f: X →Y is pgprw continuousmap function and g: Y→Z is continuous map then gof:X→Z is pgprw-continuous map.

Proof: Let g be continuous map and V be any open set in Z then g^{-1}(v) is open in y,Since f is Pgprw-continuous map $f^{-1}(g^{-1}(v))=(gof)^{-1}(V)$ is pgprw-open; hence gof is pgprw-continuous map.

Theorem 3.22: Let f:X → Y be a function from a topological space X in to t.s Y.
Iff:X → Yis pgprw-continuous map then f(pgprw-cl(A))⊆ cl(f(A)), for every subset A of X.

Proof: Since f(A)⊆Cl(f(A), implies that A⊆ f^{-1}(cl(f(A)). Since Cl(f(A)) is a closed set in Y and
f is pgprw-continuous map then by defintion f^{-1}(cl(f(A)) is a pgprw closed set in X containing A ; hence pgprw-cl(A) ⊆f^{-1}(cl(f(A)). Therefore f (pgprw-cl(A)) ⊆ cl(f(A)).

The converse of the above theorem need not be true as seen from the following example

Example 3.23: Let X=Y={a,b,c,d} and τ ={X,∅,{a},{c,d},{a,c,d}}

and σ = {Y, ∅,{b,c},{b,c,d},{a,c,d}}. Let map f: X→Y defined by f(a)=d,f(b)=b,f(c)=c,f(d)=d, for every subset of X, f(pgprw-cl(A))⊆ cl(f(A)), holds ;but f is not pgprw-continuous map. Since closed set V={d} in Y, f^{-1}(V)={a,d} which is not pgprw closed set in X.

Theorem 3.24: Let A be a subset of a topological space X. Then x ∈pgprw-cl(A) if and only if for any pgprw–open set U containing x, A∩U≠ φ.

Proof: Let x ∈ pgprω-cl(A) and suppose that, there is a pgprw–open set U in X such that x ∈ U and A∩U= φ implies that A ⊂ U^c which is pgprw–closed in X implies pgprw-cl(A) ⊆pgprw-cl(U^c) = U^c. Since x ∈ U implies that x ∉ U^c implies that x ∉pgprw-cl(A), this is a contradiction. Converserly, Suppose that, for any pgprw–open set U containing x, A∩U≠ φ. To prove that x ∈pgprw-cl(A). Suppose that x ∉pgprw-cl(A),then there is a pgprw–closed set F in X such that x ∉ F and A ⊆ F. Since x ∉ F implies that x ∈ F^c which is pgprw–open in X. Since A ⊆ F implies that

A ∩ F^c = φ, this is a contradiction. Thus x ∈pgprw-cl(A).

Theorem 3.25 : Let f: X→Y be a function from a topological space X into a topological space Y. Then the following statements are equivalent:

(i) For each point x in X and each open set V in Y with f(x) ∈V, there is a pgprw–open set U in X such that x ∈ U and f(U) ⊆ V

(ii) For each subset A of X, f(pgprw-cl(A)) ⊆ cl(f(A)).

(iii) For each subset B of Y, pgprw-cl(f^{-1}(B)) ⊆ f^{-1}(cl(B)).

(iv) For each subset B of Y, f^{-1}(int(B)) ⊆ pgprw-int(f^{-1}(B)).

Proof:(i) → (ii) Suppose that (i) holds and let y ∈f(pgprw-cl(A)) and let V be any open set of Y. Since y ∈f(pgprw-cl(A)) implies that there exists x ∈pgprw-cl(A) such that f(x) = y.

Since f(x) ∈ V, then by (i) there exists a pgprw–open set U in X such that x∈U and f(U)⊆V. Since x∈f(pgprw-cl(A)), then by theorem 3.24 U∩A≠ φ. φ ≠ f(U∩A)⊆f(U)∩f(A) ⊆V∩f(A), then V∩f(A) ≠ φ.Therefore we have y = f(x) ∈cl(f((A)). Hence f(pgprw-cl(A)) ⊆ cl(f(A)).

(ii) → (i) Let if (ii) holds and let x ∈ X and V be any open set in Y containing f(x). Let A = f^{-1}(V^c) this implies that x ∉A. Since f(pgprw-cl(A)) ⊆ cl(f(A)) ⊆ V^c this implies that

pgprw-cl(A) ⊆ f^{-1}(V^c) = A. Since x ∉ A implies that x ∉pgprw-cl(A) and by theorem 3.24 there exists a pgprw–open set U containing x such that U∩A= φ ,then U⊆A^c and

hence f(U) ⊆ f(A^c) ⊆ V.

(ii) → (iii) Suppose that (ii) holds and Let B be any subset of Y. Replacing A by f^{-1}(B)

we get from (ii) f(pgprw-cl(f^{-1}(B))) ⊆ cl(f(f^{-1}(B))) ⊆ cl(B). Hence pgprw-cl(f^{-1}(B)) ⊆ f^{-1}(cl(B)).

(iii) → (ii) Suppose that (iii) holds, let B = f(A) where A is a subset of X. Then we get from (iii) , pgprw-cl(f^{-1}(f(A)) ⊆ f^{-1}(cl(f(A))) implies pgprw-cl(A)⊆ f^{-1}(cl(f(A))).

Therefore f(pgprw-cl(A)) ⊆ cl(f(A)).

(iii) → (iv) Suppose that (iii) holds. Let B ⊆Y, then Y–B ⊆Y. By (iii) ,

Pgprw-cl(f^{-1}(Y–B)) ⊆ f^{-1}(cl(Y–B)) this implies X– pgprw-int(f–1(B)) ⊆X– f^{-1}(int(B)) .

Therefore f^{-1}(int(B)) ⊆ pgprw-int(f^{-1}(B)).

(iv) → (iii) Suppose that (iv) holds Let B ⊆Y, then Y–B ⊆Y. By (iv),

f^{-1}(int(Y–B)) ⊆pgprw-int(f^{-1}(Y–B)) this implies that X– f^{-1}(cl(B)) ⊆X–pgprw-cl(f^{-1}(B)). Therefore pgprw-cl(f^{-1}(B)) ⊆ f^{-1}(cl(B)).

Conclusion : In this paper, a new class of maps called Pgprw-Continuous maps are introduced and investigated and we observed that the composition of two pgprw-continuous maps need not be pgprw-continuous maps.in future the same process will be analyzed for pgprw-properties.

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