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ON Pgprw-LOCALLY CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper,we introduce three weaker forms of locally closed sets called PGPRW-LC sets,PGPRW-LC* set and PGPRW-LC** sets each of which is weaker than locally closed set and study some of their properties and their relationships with W-LC,θg-lc,G-LC and RG-LC sets.

Keywords: *pgprw-closed sets, pgprw-open sets,locally closed sets, pgprw-locally closed sets.*

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1. INTRODUCTION

Bourbaki[1] defined a subject of a topological space to be locally closed set if it is the intersection of an open set and a closed. Stone[2] has used the term FG for locally closed set and study some of their properties and their relationships with W-LC,θg-lc,G-LC, and RG-LC sets.

2.PRELİMİNARİES

A subset A of t.s (X,T) is called a

(i) Locally closed (briefly LC) set[3] if A=U∩F,where U is open and F is closed in X.
(ii) a pre generalized pre regular ωeakly closed set[4] (briefly pgprω-closed set) if pCl(A)⊆U whenever A⊆U and U is rgα-open in (X, τ).
(iii) pre generalized pre regular weakly open set if A^c is a [5] pgprw closed.
(iv) θg-lc set[6]if A=U∩F, where U is θg-open and F is θg-closed in X.
(v) θg-lc* set[6]if A=U∩F, where U is θg-open and F isclosed in X.
(vi) θg-lc **set [6]if A=U∩F, where U isopen and F is θg-closed in X.
(vii) G-LC set if A=U∩F[7] where U isg-open and F is g-closed in X.
(viii) G-LC* set if A=U∩F[7] where U isg-open and F isclosed in X.
(ix) G-LC**set if A=U∩F[7] where U isopen and F is g-closed in X.
(x) W-LC set if A=U∩F[8] where U isw-open and F is w-closed in X.
(xi) W-LC* set if A=U∩F[8] where U isw-open and F isclosed in X.
(xii) W-LC** set if A=U∩F[8] where U isopen and F is w-closed in X.
(xiii) RG-LC set if A=U∩F[9] where U is rg-open and F is rg-closed in X.
(xiv) RG-LC*set if A=U∩F[9] where U is rg-open and F isclosed in X.
(xv) RG-LC**set if A=U∩F[9] where U isopen and F is rg-closed in X.
(xvi) lδg-lc set if A=U∩F[10] where U is $l\delta$g-open and F is lδg-closed in X.
(xvii) lδg-lc* set if A=U∩F[10] where U is $l\delta$g-open and F isclosed in X.
(xviii) lδg-lc **setif A=U∩F[10] where U isopen and F is lδg-closed in X.

Theorem 2.1:[4]
(i) Every closed set is pgprw-closed set.

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3. Pgprw-LOCALLY CLOSED SETS İN TOPOLOGİCAL SPACES

Definition 3.1: A Subset A of t.s (X,T) is called pgprw-locally closed (briefly PGPRW-LC).

if A=U∩F where U is pgprw-open in (X,T) and F is pgprw-closed in (X,T).The set of all pgprw-locally closed sets of (X,T) is denoted by PGPRW-LC(X,T).

Example 3.2: Let X={a, b, c, d} and T ={X, ∅,{a},{b},{a,b},{a,b,c}}

Pgprwc(X,T)= { X, ∅,{c},{d},{a,d},{b,d},{c,d},{a,c,d},{b,c,d},{a,b,d}}.

PGPRW-LC-Set = {X, ∅,{a},{b},{c},{d},{a,b}{a,c}{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{b,c,d},{a,c,d}}.

Remark 3.3: The following are well known
- (i) A Subset A of (X,T) is PGPRW-LC set iff it's complement X-A is the union of a pgprw-open set and a pgprw-closed set.
- (ii) Every pgprw-open (resp. Pgprw-closed) subset of (X,T) is a PGPRW-LC set.

Theorem 3.4: Every locally closed set is a PGPRW-LC set but not conversely.

Proof: The proof follows from the two defintions[follows from theorem 2.1] and fact that every closed (resp.open) set is pgprw-closed (pgprw-open).

Example 3.5: Let X={a,b,c,d} and T ={X, ∅,{a},{b},{a,b},{a,b,c}} then {b,c} is PGPRW-LC but not a locally closed set in (X,T).

Remark 3.6: lδgc-sets and PGPRW-LC sets are independent of each other as seen from the following example

Example 3.7:
- (i) Let X={a,b,c} and T ={X, ∅,{a},{b,c}} then {b} is lδgc -lc but not PGPRW-LC set in (X,T).
- (ii) Let X={a,b,c}andT ={X, ∅,{a},{b},{a,c}}then {b} is PGPRW-LC but not lδgc-lc set in (X,T).

Remark 3.8: θ-g lc sets and PGPRW-LC sets are independent of each other as seen from the following example

Example: Let X={a,b,c} and T ={X, ∅,{a},{b,c}} then {c} is θ-g -lc but not PGPRW-LC set in (X,T).

Example: Let X={a,b,c} and T ={X, ∅,{a},{b},{a,c}} then {b} is PGPRW-LC but not θ-g-lc set in (X,T).

Definition 3.9: A subset A of (X,T) is called a PGPRW-LC* set if there exists a pgprw-open set G and a closed F of (X,T) s.t A=G∩F the collection of all PGPRW-LC* sets of (X,T) will be denoted by PGPRW-LC*(X,T).

Definition 3.10: A subset B of (X,T) is called a PGPRW-LC** set if there exists an open set G and pgprw closed set F of (X,T) s.t B=G∩F the collection of all PGPRW-LC** sets of (X,T) will be denoted by PGPRW-LC**(X,T).

From the above definition we have the following results.

Theorem 3.11:
- (i) Every locally closed set is a PGPRW-LC* set.
- (ii) Every locally closed set is a PGPRW-LC** set.
- (iii) Every PGPRW-LC* set is PGPRW-LC set
- (iv) Every PGPRW-LC** set is PGPRW-LC set

Proof: The proof are obivious from the definition and the relation between the sets however the converses of the above results are not true as seen from the following examples.

Example 3.12: Let X={a,b,c,d} and T ={X, ∅,{a},{b},{a,b},{a,b,c}}
- (i) The set {c} is PGPRW-LC* set but not a locally closed set in (X,T).
- (ii) The set {b,c} is PGPRW-LC** set but not a locally closed set in (X,T).
- (iii) The set {a,d} is PGPRW-LCset but not a PGPRW-LC* set in (X,T).
- (iv) The set {b,c} is PGPRW-LCset but not a PGPRW-LC** set in (X,T).

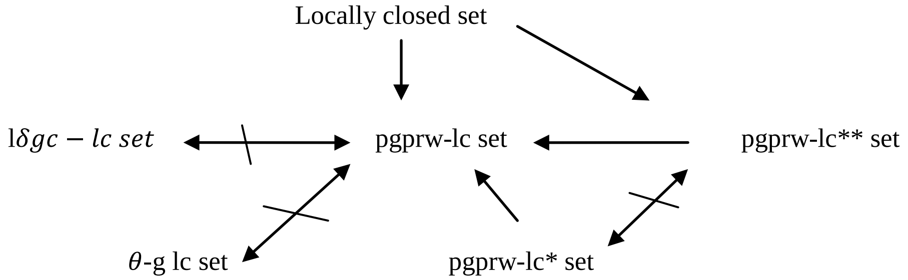
Remark 3.13: PGPRW-LC* sets and PGPRW-LC** sets are independent of each other as seen from the examples.

Example: Let X={a,b,c} and T ={X, ∅,{a},{a,b}} then the set {b} is PGPRW-LC** but not PGPRW-LC* set in (X,T).

Example: Let X={a,b,c} and T ={X, ∅,{a},{a,b}} then the set {a,c} is PGPRW-LC* but not $\mathrm{PGPRW-LC}**$ set in (X,T).

Remark 3.14: From the above discussion and known results we have the following implication.

In the diagram



Theorem 3.15: If pgprwo(X,T) then

(i) PGPRW-LC(X,T) = LC (X,T).

Proof: (i) For any space (X,T) ,W.k.t LC(X,T)⊆ PGPRW-LC(X,T).Since PGPRWO(X,T)=T, that is every pgprw-open set is open and every pgprw-closed set is closed in (X,T), PGPRW-LC(X,T) ⊆ LC (X,T); hence PGPRW-LC(X,T) = LC (X,T).

Theorem 3.16: If PGPRWO(X,T) = T, then PGPRW-LC*(X,T)= PGPRW-LC** (X,T)= PGPRW-LC(X,T).

Proof: for any space,(X,T)

LC (X,T) ⊆ PGPRW-LC*(X,T) ⊆ PGPRW-LC(X,T) and

LC (X,T) ⊆ PGPRW-LC**(X,T) ⊆ PGPRW-LC(X,T) since PGPRWO(X,T)=T.

PGPRW-LC(X,T)=LC(X,T) by theorem 3.15,it follows that

LC(X,T)=PGPRW-LC*(X,T) = PGPRW-LC**(X,T)= PGPRW-LC(X,T).

Remark 3.17: The converse of the theorem 3.16 need not be true in general as seen from the following example.

Theorem 3.18: Let X={a,b,c} with the topology T= {X, ∅,{a},{b},{a,c}} then PGPRW-LC*(X,T)=PGPRW-LC**(X,T)=PGPRW-LC(X,T)=P(X).

However PGPRWO(X,T)={X, ∅,{a},{b},{c},{a,b},{b,c},{a,c}}≠T.

Theorem 3.19: If GO(X,T)=T,then GLC(X,T)⊆ PGPRW-LC(X,T)

Proof: For any space (X,T) w.k.t LC(X,T) ⊆ GLC(X,T) and LC (X,T)⊆PGPRW-LC(X,T).....(i)

GO(X,T)=T, that is every g-open set is open and every g-closed set is closed in (X,T) and so

GLC(X,T) ⊆LC(X,T) that is GLC(X,T) =LC(X,T)......(ii)

From (i) and (ii) we have GLC(X,T) ⊆ PGPRW-LC(X,T).

Theorem 3.20: If PGPRW-LC(X,T) ⊆ LC(X,T). For any space (X,T), then PGPRW-LC(X,T) = PGPRW-LC*(X,T)

Proof: Let PGPRWC(X,T)⊆LC(X,T),For any space (X,T),w.k.t PGPRW-LC*(X,T) ⊆ PGPRW-LC(X,T)...(i) Let Aϵpgprwc (X,T), then A= U∩F, where U is pgprw-open and F is a pgprw-closed ın (X,T).Now, FϵPGPRW-LC(X,T) by hypothesis F is locally closed set in (X,T), then F= G∩E, where G is an open set and E is a closed set in (X,T).
Now, A= U∩F=U ∩ (G∩E)= (U∩G) ∩ E, where U∩G is pgprw-open as the intersection of pgprw-open sets is pgprw-open and E is a closed set in (X,T). It follows that A is PGPRW-LC*(X,T). That is Aϵ PGPRW-LC* (X,T) and so, PGPRWC(X,T) ⊆ PGPRW-LC*(X,T) ......(ii).

From (i) and (ii) we have PGPRW-LC(X,T) = PGPRW-LC*(X,T).

Example 3.21: The converse of the theorem 3.20 need not be true in general as seen from the following example.

Example 3.22: Consider X= {a,b,c,d} and T= {X, ∅,{a,b},{c,d}},then PGPRW-LC(X,T) = PGPRW-LC*(X,T) = P(X). But PGPRWC(X,T)=P(X) and LC(X,T)= { X, ∅,{a,b},{c,d}} That is pgprwc(X,T) ⊈ LC(X,T).

Theorem 3.23: For a subset A of (X,T) if Aϵ PGPRW-LC(X,T) then A = U ∩ (pgprw-cl(A)) for some open set V.

Proof : Let, Aϵ PGPRW-LC(X,T) then there exist a pgprw-open U and a pgprw-closed set F s.t. A= U∩F.Since A⊆ F, Pgprw-cl(A)= pgprw-cl(F) =F. Now U ∩ (pgprw-cl(A)) ⊆U∩F= A, that is U∩ (pgprw-cl(A))= A.

Conversely A⊆ U and A ⊆ Pgprw-cl(A) implies A ⊆ U ∩ (pgprw-cl(A)) and therefore A= U ∩ (Pgprw-cl(A)) for some pgprw-open set U.

Remark 3.24: The converse of the theorem 3.23 neeed not be true in general as seen from the following example.

Example 3.25: Consider X = {a,b,c} with the topology T= {X, ∅,{a},{b,c}} then

pgprwc(X,T)={X, ∅,{a},{b,c}} then PGPRW-LC(X,T)= {X, ∅,{a},{b,c}}

Take A={b}, pgprw-cl(A) = {b,c} now, A = X ∩ (pgprw-cl(A)) for some pgprw-open set X. but {b}∉PGPRW-LC (X,T).

Theorem 3.26: For a subset A of (X,T), the following are equivalent.
- (i) Aϵ PGPRW-LC*(X,T).
- (ii) A= U ∩ (p-cl(A) for some pgprw-open set U.
- (iii) pcl(A)-A is pgprw-closed.
- (iv) A ∪(p-cl(A)^c is pgprw-open.

Proof :
- (i) implies (ii) Let Aϵ PGPRW-LC*(X,T) then there exists a pgprw-open set U and a closed set F s.t A = U∩F. Since A ⊆ F, p-cl(A) ⊆ p-cl(F)= F. Now U∩p-cl(A) ⊆ U∩F = A that is U∩ p-cl(A)= A. Conversely A ⊆ U, and A ⊆ p-cl(A) implies A ⊆ U∩ p-cl(A) and therefore A = U∩p-cl(A) for some pgprw-open set U.
- (ii) implies (i) since U is a pgprw-open set and pcl(A) is a closed set, A= U ∩ (p-cl(A) ϵ PGPRW-LC*(X,T).
- (iii) implies (iv) let F = p-cl(A)-A, then F is pgprw-closed by the assumption and X-F=X-[p-cl(A)-A] = X ∩ [P-cl(A)-A]^c = A ∪ (X- p-cl(A)) = A ∪ (p-cl(A))^c. But X- F is pgprw-open. This shows that A ∪ (p-cl(A)) ^{c} is pgprw-open.
- (iv) implies (iii) Let U = A ∪ (p-cl(A)^c then U is pgprw-open ,this implies X-U is pgprw-closed and X-U = X – (A∪P-cl(A)^c) = p-cl(A) ∩ (X-A)= p-cl(A)-A is pgprw-closed.
- (v) impies (ii) Let U = A ∪ (p-cl(A)^c then U is pgprw-open .hence we prove that A =U∩ (p − cl(A)cfor some pgprw-open set U.

Now A = U∩ (p − cl(A)
= [A∪(p-cl(A)^c] ∩ p-cl(A)
= A∩ [p-cl(A)] ∪ p-cl(A)^c∩ p-cl (A).
= A ∪ ∅ = A. Therefore A = U∩ (p − cl(A) for some pgprw-open set U.

(ii) implies (iv) Let A = U ∩ (p-cl(A) for some pgprw-open set then we p.t A ∪ (p-cl(A)^c is pgprw-open. Now A ∪ (p-cl(A)^c = (U∩ (p − cl(A)) ∪ [p-cl(A))^c = U∩ (p − cl(A)) ∪ [p-cl(A))^c =U ∩ X = U, which is pgprw-open.Thus A= (p-cl(A))^c is pgprw-open.

Theorem 3.27: For a subset A of (X,T) if AϵPGPRW-LC**(X,T),then there exists an open set U s.t A= U ∩ pgprw-cl(A).

Proof: Let AϵPGPRW-LC**(X,T), then there exist an open set U and a pgprw-closed set s.t
A = U ∩ F Since A ⊆ U and A⊆pgprw-cl(A) we have A ⊆ pgprw-cl(A).

Conversely, Since A ⊆ F and pgprw-cl(A) ⊆ pgprw-cl(F) = F, as F is pgprw-closed. Thus
U ∩ pgprw-cl(A) ⊆ U ∩ F= A. That is U ∩ pgprw-cl(A) ⊆ A; hence A= U ∩ pgprw-cl(A). For some open set U.

Theorem 3.28: The converse of the theorem 3.27 need not be true in general as seen from the following example.

Example: Let X= {a,b,c,d} with the topology T= { X, ∅,{a},{b},{a,b},{a,b,c}} take A= {a,d}.

Then pgprw-cl(A) = pgprw-cl{a,d}={a,d}; also A= X ∩ pgprw-cl(A)= {a,b,c,d}∩{a,d}={a,d} for some open set X but {a,d}ϵ PGPRW-LC**(X,T).

Theorem 3.29: For A and B in (X,T) the following are true.
(i) if AϵPGPRW-LC*(X,T) and BϵPGPRW-LC*(X,T),then A∩BϵPGPRW-LC*(X,T).
(ii) if AϵPGPRW-LC**(X,T) and B is open,then A∩BϵPGPRW-LC**(X,T).
(iii) if AϵPGPRW-LC(X,T) and B is pgprw-open,then A∩BϵPGPRW-LC(X,T).
(iv) if AϵPGPRW-LC*(X,T) and B is pgprw-open,then A∩BϵPGPRW-LC*(X,T).
(v) if AϵPGPRW-LC*(X,T) and B is closed,then A∩BϵPGPRW-LC*(X,T).

Proof:
(i) Let A,BϵPGPRW-LC*(X,T),it follows from theorem 3.26 that there exist pgprw-open sets P and Q s.t A = P∩ P-cl(A) and B = Q ∩ P-cl(B). Therefore A∩B = P∩ P-cl(A) ∩ Q ∩ P-cl(B) =P ∩ Q ∩[P-cl(A) ∩ P-cl(B)] where P ∩ Q is pgprw-open and P-cl(A) ∩ P-cl(B) is closed. This shows that A∩BϵPGPRW-LC*(X,T).
(ii) Let AϵPGPRW-LC**(X,T) and B is open. Then there exist an open set P and pgprw-closed set F s.t A=P ∩ F. Now, A∩B=P∩F∩B = (P∩B) ∩ F, Where (P∩B) is open and F is pgprw-closed .This implies A∩BϵPGPRW-LC**(X,T).
(iii) Let AϵPGPRW-LC(X,T) and B is pgprw-open then there exists a pgprw-open set P and Q pgprw-closed set F s.t A = P∩ F. Now, A∩B = P∩ F ∩ B = (P∩B) ∩ F, Where (P∩B) is pgprw-open and F is pgprw-closed. This shows that A∩BϵPGPRW-LC(X,T).
(iv) Let AϵPGPRW-LC*(X,T) and B is pgprw-open then there exists a pgprw-open set p and Q pgprw-closed set F s.t A = P∩ F. Now, A∩B = (P∩F) ∩ B = (P∩B) ∩ F, Where (P∩B) is pgprw-open and F is closed .This implies that A∩BϵPGPRW-LC*(X,T).
(v) AϵPGPRW-LC*(X,T) and B is closed. Then there exist an pgprw-open set P and a closed set F s.t A = P ∩ F. Now, A∩B = (P∩F) ∩ B = P∩(F ∩ B), Where (F∩B) is closed and p is pgprw-open. This implies A∩BϵPGPRW-LC*(X,T).

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