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FUZZY PGPRW-CLOSED SETS AND FUZZY PGPRW-OPEN SETS İN FUZZY TOPOLOGİCAL SPACES

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Abtract: In this paper, we introduce new class of fuzzy sets called fuzzy pgprw-closed sets in fuzzy topological spaces and investigate certain basic properties of these fuzzy sets. Also we introduce fuzzy pgprw-open sets in fuzzy topological spaces and study some of their properties.

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INTRODUCTION

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. In the year 1965, L.A.Zadeh [1] introduced the concept of fuzzy subset as a generalization of that of an ordinary subset. The introduction of fuzzy subsets paved the way for rapid research work in many areas of mathematical science. In the year 1968, C.L.Chang [2] introduced the concept of fuzzy topological spaces as an application of fuzzy sets to topological spaces . Subsequently several researchers contributed to the development of the theory and applications of fuzzy topology. The theory of fuzzy topological spaces can be regarded as a generalization theory of topological spaces. An ordinary subset A or a set X can be characterized by a function called characteristic function

$\mu_A : X \longrightarrow [0,1]$ of A, defined by
$\mu_A(x) = 1$, if $x \in A$.
$= 0$, if $x \notin A$.

Thus an element $x \in X$ is in A if $\mu_A(x) = 1$ and is not in A if $\mu_A(x) = 0$. In general if X is a set and A is a subset of X then A has the following representation. $A = \{ (x, \mu_A(x)) : x \in X\}$, here $\mu_A(x)$ may be regarded as the degree of belongingness of x to A, which is either 0 or 1. Hence A is the class of objects with degree of belongingness either 0 or 1 only. Prof. L.A.Zadeh [1] introduced a class of objects with continuous grades of belongingness ranging between 0 and 1 ; he called such a class as fuzzy subset. A fuzzy subset A in X is characterized as a membership function
$\mu_A : X \longrightarrow [0,1]$, which associates with each point in x a real number $\mu_A(x)$ between 0 and 1 which represents the degree or grade membership of belongingness of x to A.

The purpose of this paper is to introduce a new class of fuzzy sets called fuzzy pgprw-closed sets in fuzzy topological spaces and investigate certain basic properties of these fuzzy sets. Among many other results it is observed that every fuzzy closed set is fuzzy pgprw-closed but not conversely.Also we introduce fuzzy pgprw-open sets in fuzzy topological spaces and study some of their properties.

1. Preliminaries

1.1Definition:[1]A fuzzy subset A in a set X is a function A : X → [0, 1]. A fuzzy subset in X is empty iff its membership function is identically 0 on X and is denoted by 0 or μ_ϕ.The set X can be considered as a fuzzy subset of X whose membership function is identically 1 on X and is denoted by μ_x or I_x. In fact every subset of X is a fuzzy subset of X but not conversely. Hence the concept of a fuzzy subset is a generalization of the concept of a subset.

1.2 Defnition :[1] If A and B are any two fuzzy subsets of a set X , thenA is said to be included in B or A is contained in B iff A(x) ≤ B(x) for all x in X. Equivalently, A ≤ B iff A(x) ≤ B(x) for all x in X.

1.3 Defnition: [1]Two fuzzy subsets A and B are said to be equal if A(x) = B(x) for every x in X. Equivalently A = B if A(x) = B(x) for every x in X.

1.4Defnition:[1] The complement of a fuzzy subset A in a set X, denoted by A′ or 1 − A, is the fuzzy subset of X defined by A′(x) = 1 − A(x) for all x in X. Note that(A′)′ = A.

1.5 Defnition:[1] The union of two fuzzy subsets A and B in X, denoted by A ∨ B, is a fuzzy subset in X defined by (A ∨ B)(x) = Max{μ_A(x), μ_B(X)} for all x in X.

1.6 Defnition:[1]The intersection of two fuzzy subsets A and B in X, denoted by A ∧ B, is a fuzzy subset in X defined by (A ∧ B)(x) = Min{A(x), B(x)} for all x in X.

1.7 Defnition:[1] A fuzzy set on X is 'Crisp' if it takes only the values 0 and 1 on X.

1.8 Defnition :[2]Let X be a set and be a family of fuzzy subsets of (X,τ) is called a fuzzy topology on X iff τ satisfies the following conditions.

(i) μ_ϕ ; $\mu_X \in \tau$: That is 0 and 1 ∈ τ

(ii) If $G_i \in \tau$ for i ∈ I then $\vee_{i \in I} G_i \in \tau$

(iii) If G,H ∈ τ then G ∧ H ∈ τ

The pair (X, τ) is called a fuzzy topological space (abbreviated as fts). The members of τ are called fuzzy open sets and a fuzzy set A in X is said to be closed iff 1 − A is an fuzzy open set in X.

1.9 Remark :[2] Every topological space is a fuzzy topological space but not conversely.

1.10 Defnition:[2]Let X be a fts and A be a fuzzy subset in X. Then ∧ {B : B is a closed fuzzy set in Xand B ≥ A} is called the closure of A and is denoted by A or cl(A).

1.11 Defnition:[2]Let A and B be two fuzzy sets in a fuzzy topological space (X,τ)and let A ≥ B. Then B is called an interior fuzzy set of A if there exists G ∈τ such that A ≥ G ≥ B, the least upper bound of all interior fuzzy sets of A is called the interior of A and is denoted by A^0.

1.12 Definition[3] A fuzzy set A in a fts X is said to be fuzzy semiopen if and only if there exists a fuzzy open set V in X such that $V \leq A \leq cl(V)$.

1.13 Definition[3] A fuzzy set A in a fts X is said to be fuzzy semi-closed if and only if there exists a fuzzy closed set V in X such that $int(V) \leq A \leq V$. It is seen that a fuzzy set A is fuzzy semiopen if and only if 1-A is a fuzzy semi-closed.

1.14 Theorem:[3] The following are equivalent:

(a) μ is a fuzzy semiclosed set,
(b) μ^{C} is a fuzzy semiopen set,
(c) $int(cl(\mu)) \leq \mu$.
(b) $int(cl(\mu)) \geq \mu^{c}$

1.15 Theorem [3] Any union of fuzzy semiopen sets is a fuzzy semiopen set and (b) any intersection of fuzzy semi closed sets is a fuzzy semi closed.

1.16 Remark[3]

(i) Every fuzzy open set is a fuzzy semiopen but not conversely.
(ii) Every fuzzy closed set is a fuzzy semi-closed set but not conversely.
(iii) The closure of a fuzzy open set is fuzzy semiopen set
(iv) The interior of a fuzzy closed set is fuzzy semi-closed set

1.17Definition:[3] A fuzzy set μ of a fts X is called a fuzzy regular open set of X if int(cl(μ).= μ.

1.18 Definition:[3] A fuzzy set μ of fts X is called a fuzzy regular closed set of X if $cl(int(\mu)) = \mu$.

1.19 Theorem:[3] A fuzzy set μ of a fts X is a fuzzy regular open if and only if μ^{C} fuzzy regular closed set.

1.20 Remark:[3]

(i) Every fuzzy regular open set is a fuzzy open set but not conversely.
(ii) Every fuzzy regular closed set is a fuzzy closed set but not conversely.

1.21 Theorem:[3]

(i) The closure of a fuzzy open set is a fuzzy regularclosed.
(ii) The interior of a fuzzy closed set is a fuzzy regular open set.

1.22 Definition:[4] A fuzzy set α in fts X is called fuzzy rwclosed
if $cl(\alpha) \leq \mu$ whenever $\alpha \leq \mu$ and μ is regular semi-open in X.

2. Fuzzy pgprw-closed sets and fuzzy pgprw-open sets in fts.

Definition2.1: Let (X, T) be a fuzzy topological space. A fuzzy set α of X is called fuzzy pre generalized pre regular w-closed (briefly, fuzzy pgprw-closed) if $p\text{-}cl(\alpha) \leq \sigma$ whenever $\alpha \leq \sigma$ and σ is fuzzy $rg\alpha -$ open in fts X.

We denote the class of all fuzzy pre generalized pre regular w-closed sets in fts X by FPGPRWC(X).

Theorem 2.2: Every fuzzy closed set is a fuzzy pgprw-closed set in a fts X.

Proof: Let α be a fuzzy closed set in a fts X. Let β be a fuzzy rg$\alpha -$ open set in X such that $\alpha \leq \beta$. Since α is fuzzy closed, p-cl(α)= α. Therefore p-cl(α) $\leq \beta$. Hence α is fuzzy pgprw-closed in fts X.

The converse of the above Theorem need not be true in general as seen from the following example.

Example 2.3: Let X= {a, b, c}. Define a fuzzy set α in X by

α (x) = 1 if x = a
0 othewise

β (x) = 1 if x = a,b
0 otherwise .

Let T= {1, 0, β}. Then (X, T) is a fuzzy topological space. Define a fuzzy set μ in X by

μ (x) = 1 if x = b
0 otherwise .

Then μ is a fuzzy pgprw-closed set but it is not a fuzzy closed set in fts X.

Corollary 2.4: By K.K.Azad we know that, every fuzzy regular closed set is a fuzzy closed set but not conversely. By Theorem 2.2 every fuzzy closed set is a fuzzy pgprw-closed set but not conversely and hence every fuzzy regular closed set is a fuzzy pgprw-closed set but not conversely.

Remark 2.5: Fuzzy pgprw closed sets and fuzzy semi-closed sets are independent.

Example: (i)Consider Let X= {a, b, c} and the functions α, β : X $\longrightarrow$ [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = a, b
0 otherwise

Consider T= {1, 0, α ,β}. Then (X, T) is a fuzzy topological space. Then the fuzzy set

μ(x) = 1 if x = b
0 otherwise

is a fuzzy pgprw-closed set but it is not a fuzzy semi-closed set in fts X.

Example: (ii) Let X= {a, b, c} and the functions α, β, γ: X $\longrightarrow$[0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = b
0 therwise

γ(x) = 1 if x = a,b
0 otherwise

Consider T= {1,0,α,β,γ}. Then (X, T) is a fuzzy topological space. Then the fuzzy set

α (x) = 1 if x = a
0 otherwise.

is a fuzzy semi-closed set but it is not a fuzy pgprw-closed set in fts X.

Remark 2.6: Fuzzy pgprw-closed sets and fuzzy rw-closed sets are independent.

Example (i):Let X= {a, b, c,d} and the functions $\alpha, \beta, \gamma, \delta$: X $\longrightarrow$ [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = b
0 therwise

γ(x) = 1 if x = a,b
0 otherwise

δ(x) = 1 if x = a,b,c
0 otherwise

Consider T= {1,0,α,β,γ,δ}. Then (X, T) is a fuzzy topological space. Then (X, T) is a fts then the fuzzy set

μ (x) = 1 if x = a ,d
0 otherwise.

is a fuzzy pgprw-closed set but it is not a fuzzy rw closed set in fts X.

Example: (ii) Let X= {a, b, c} and the functions α: X $\rightarrow$ [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

Consider T= {1,0,α}. Then (X, T) is a fuzzy topological space. Then the fuzzy set

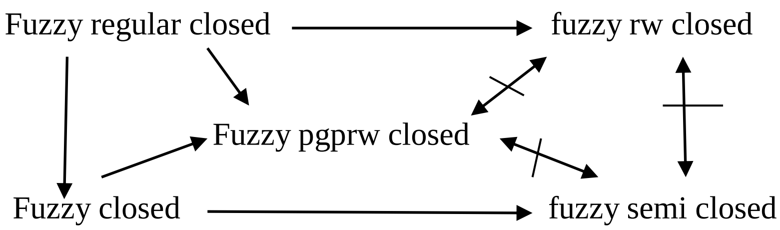
α (x) = 1 if x = a
0 otherwise.

is a fuzzy rw-closed set but it is not a fuzzy pgprw closed set in fts X.

Remark 2.7: From the above discussions and known results we have the following implications

In the following diagram,

A $\longrightarrow$ B we mean A implies B but not conversely and
A $\not\longrightarrow$ B means A and B are independent of each other.



Theorem 2.8: If α and β are fuzzy pgprw-closed sets in fts X, then $\alpha \vee \beta$ fuzzy pgprw-closed set in fts X.

Proof: Let σ be a fuzzy rg$\alpha -$ open set in fts X such that $\alpha \vee \beta \leq \sigma$. Now $\alpha \leq \sigma$ and $\beta \leq \sigma$ Since α and β are fuzzy pgprw-closed sets in fts X, p-cl(α) $\leq \sigma$ and p-cl(β) $\leq \beta$.
Therefore p-cl(α) $\vee$ p-cl(β) $\leq \sigma$ .But p-cl(α) $\vee$ p-cl(β)= p-cl($\alpha \vee \beta$). Thus p-cl($\alpha \vee \beta$)$\leq \sigma$. Hence $\alpha \vee \beta$ is a fuzzy pgprw-closed set in fts X.

Remark 2.9: If α and β are fuzzy pgprw-closed sets in fts X, then $\alpha \wedge \beta$ need not be a fuzzy pgprw-closed set in general as seen from the following example.

Example 2.10: Consider the fuzzy topological space (X, T) defined

Let X= {a, b, c,d},T= {1,0,α,β, γ}in this fts X, The fuzzy sets δ_1 δ_2 :X $\rightarrow$ [0, 1] are defined by

δ_1 (x)= 1 if x = {a,c,d}
0 otherwise.

δ_2 (x) = 1 if x = {a,b,c}
0 otherwise

and α, β, γ: X $\rightarrow$ [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = c,d
0 otherwise

γ(x) = 1 if x = a,c,d
0 otherwise

Then $\delta_1 and$ δ_2 are fuzzy pgprw closed sets in fts X.

Let μ = $\delta_1 \wedge \delta_2$, then

μ (x) = 1 if x = a ,c

0 otherwise.

μ = $\delta_1 \wedge \delta_2$, is not a fuzzy pgprw-closed set in fts X.

Theorem 2.11: If a fuzzy set α of fts X is both fuzzy regular open and fuzzy pgprw-closed, then α is a fuzzy pre-closed set in fts X.

Proof: Suppose a fuzzy set α of fts X is both fuzzy regular open and fuzzy pgprw-closed. As every fuzzy regular open set is a fuzzy rgα − open set and $\alpha \leq \alpha$ we have p-cl(α) $\leq \alpha$.
Also $\alpha \leq p$ −cl(α).Therefore p-cl(α) = α . That is α is fuzzy closed. Since α is fuzzy regular open, int(α)= α. Now cl(int(α))=cl(α) = α.Therefore α is a fuzzy p- closed set in fts X.

Theorem 2.12: If a fuzzy set α of a fts X is both fuzzy rgα − open and fuzzy pgprw closed, then α is a fuzzy closed set in fts X

Proof: Suppose a fuzzy set α of a fts X is both fuzy rgα − open and fuzzy pgprw-closed. Now , $\alpha \leq \alpha$, we have $\alpha \leq p$ −cl(α) .Therefore p-cl(α) = α and hence α is a fuzzy closed set in fts X.

Theorem2.13: If a fuzzy set α of a fts X is both fuzzy open and fuzzy generalized pre closed, then α is a fuzzy pgprw-closed set in fts X.

Proof: Suppose a fuzzy set α of a fts X is both fuzzy open and fuzzy generalized pre-closed. Now $\alpha \leq \alpha$, by hypothesis we have p-cl(α) $\leq \alpha$.Also $\alpha \leq p$ −cl(α) Therefore p-cl(α)= α. That is α is a fuzzy closed set and hence α is a fuzzy pgprw-closed set in fts X, as every fuzzy closed set is a fuzzy pgprw-closed set.

Remark2.14: If a fuzzy set γ is both fuzzy open and fuzzy pgprw-closed set in a fts X, then γ need not be a fuzzy generalized pre closed set in general as seen from the following example.

Example 2.15:

Let X= {a, b, c,d} and the functions α, β, γ: X → [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = c,d
0 therwise

γ(x) = 1 if x = a,c,d
0 otherwise

Consider T= {1,0,α,β,γ}. Then (X, T) is a fuzzy topological space.In this fts X, γ is both fuzzy open and pgprw closed but not fuzzy gp-closed.

Theorem 2.16: Let α be a fuzzy pgprw-closed set of a fts X and suppose $\alpha \leq \beta \leq p$ −cl(α). Then β is also a fuzzy pgprw-closed set in fts X.

Proof: Let $\alpha \leq \beta \leq p - \text{cl}(\alpha)$ and α be a fuzzy pgprw-closed set of fts X. Let σ be any fuzzy rgα open set such that $\beta \leq \sigma$ .Then $\alpha \leq \sigma$ and α is fuzzy pgprw-closed, we have p-cl(α)≤ σ.
But p-cl(β) $\leq p - \text{cl}(\alpha)$ and thus p-cl(β) $\leq \sigma$ Hence β is a fuzzy pgprw-closed set in fts X.

Theorem2.17: In a fuzzy topological space X if FRGα O(X) = {1, 0}, where FRGα O(X) is the family of all fuzzy rgα open sets then every fuzzy subset of X is fuzzy pgprw-closed.

Proof: Let X be a fuzzy topological space and FRGα (X) = {1, 0}. Let α be any fuzzy subset of X. Suppose α= 0. Then 0 is a fuzzy pgprw-closed set in fts X. Suppose $\alpha \neq 0$. Then 1 is the only fuzzy rg$\alpha -$ open set containing α and so p-cl(α) ≤1. Hence α is a fuzzy pgprw-closed set in fts X.

Theorem2.18: If α is a fuzzy pgprw-closed set of fts X and p-cl(α) ∧ (1-p-cl(α))=0,
then p-cl(α)- α does not contain any non-zero fuzzy rgα –open set in fts X.

Proof: Suppose α is a fuzzy pgprw-closed set of fts X and p-cl(α) ∧ (1-p-cl(α))=0,.We prove the result by contradiction. Let β be a fuzzy rgα -open set such that p-cl(α)- $\alpha \geq \beta$ and $\beta \neq 0$.
Now β ≤p-cl(α)- α i.e β ≤1-α which implies α ≤1- β, since β is fuzzy r$g\alpha$-open in fts X,then 1-β is fuzzy rgα-open in X; Since α is a fuzzy pgprw-closed set in fts X, by definition
p-cl(α)≤1- β So $\beta \leq \text{p} - \text{cl}(\alpha)$ Therefore, $\beta \leq$ p-cl(α) ∧ (1-p-cl(α))=0, by hypothesis. This shows that β =0 which is a contradiction. Hence p-cl(α)- α does not contain any non-zero fuzzy rg$\alpha - open$ set in fts X.

Corollary 2.19: If α is a fuzzy pgprw-closed set of fts X and p-cl(α) ∧ (1-p-cl(α))=0,
then p-cl(α)- α does not contain any non-zero fuzzy regular open set in fts X.

Proof: Follows form the Theorem 2.18 and the fact that every fuzzy regular
open set is a fuzzy rgα –open set in fts X.

Corollary 2.20: If α is a fuzzy pgprw-closed set of a fts X and p-cl(α) ∧ (1-p-cl(α))=0,
then p-cl(α)- α does not contain any non-zero fuzzy regular closed set in fts X.

Proof: Follows form the Theorem 2.18 and the fact that every fuzzy regular open set is a
fuzzy rgα –open set in fts X.

Theorem2.21: Let α be a fuzzy pgprw-closed set of fts X and p-cl(α) ∧ (1-p-cl(α))=0,Then α is a fuzzy closed set if and only if p-cl(α)- α is a fuzzy rgα − open set in fts X.

Proof: Suppose α is a fuzzy closed set in fts X. Then p-cl(α) = α and so p-cl(α) - $\alpha = 0$, which is a fuzzy rgα − open set in fts X. Conversely suppose p-cl(α)- α is a fuzzy rg$\alpha - open$ set in fts X. Since α is fuzzy pgprw-closed, by Theorem 2.18 p-cl(α)- α does not contain any non
zero fuzzy regular open set in fts X then That is p-cl(α) = α and hence α is a fuzzy closed set in fts X.

We introduce a fuzzy pgprw-open set in fuzzy topological space X as follows.

Definition2.22: A fuzzy set α of a fuzzy topological space X is called a fuzzy pre generalized pre regular w-open (briefly, fuzzy pgprw-open) set if its complement α^C is a fuzzy pgprw-closed set in fts X.

We denote the family of all fuzzy pgprw-open sets in fts X by FPGPRWO(X).

Theorem2.23: If a fuzzy set α of a fuzzy topological space X is fuzzy open, then it is fuzzy pgprw-open but not conversely.

Proof: Let α be a fuzzy open set of fts X. Then α^{C} is fuzzy closed. Now by Theorem 2.2, α^{C} is fuzzy pgprw-closed. Therefore α is a fuzzy pgprw-open set in fts X.

The converse of the above Theorem need not be true in general as seen from the following example.

Example2.24:

(ii) Let X= {a, b, c} and define the fuzzy set α in X by α: X $\longrightarrow$ [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

Let T= {1,0,α}Then (X, T) is a fuzzy topological space. Then the fuzzy set β in X by

β (x) = 1 if x = a,c
0 otherwise.

Then β is a fuzzy pgprw-open but it is not fuzzy open set in fts X.

Corollary2.25: By K.K.Azad, we know that, every fuzzy regular open set is a fuzzy open set but not conversely. By Theorem 2.23, every fuzzy open set is a fuzzy pgprw-open set but not conversely and hence every fuzzy regular open set is a fuzzy pgprw-open set but not conversely.

Theorem2.26: A fuzzy set α of a fuzzy topological space X is fuzzy pgprw-open if and only if $\delta \leq p-\text{int}(\alpha)$ whenever and $\delta \leq \alpha$ and δ is a fuzzy rg$\alpha - open$ set in fts X.

Proof: Suppose that $\delta \leq p - \text{int}(\alpha)$ whenever $\delta \leq \alpha$ and δ is a fuzzy rg$\alpha - open$ set in fts X. To prove that α is fuzzy pgprw-open in fts X. Let $\alpha^{C} \leq \beta$ and β is any fuzzy rg$\alpha - open$ set in fts X. Then $\beta^{c} \leq \alpha$, since β is fuzzy r$g\alpha$-open in fts X; then 1- β is also fuzzy rgα-open set in fts X .then β^{c} is also fuzzy r$g\alpha$-open in fts X By hypothesis $\beta^{c} \leq$p-int(α) Which implies [p-int(α)]$^{c} \leq \beta$ that is p-int(α^{c})$\leq \beta^{c}$ since p-cl(α^{c}) $=$ [p $-$ int(α)]c,Thus α^{C} is a fuzzy pgprw-closed and hence
α is fuzzy pgprw-open in X.

Conversely, suppose that α is fuzzy pgprw-open. Let $\beta \leq \alpha$ and β is any fuzzy rg$\alpha - open$ in fts X. Then $\alpha^{C} \leq \beta^{c}$ since β is fuzzy r$g\alpha$-open in fts X,then 1- β is fuzzy rgα-open in X; then β^{c} is also fuzzy r$g\alpha$-open in fts X. since α^{C} is fuzzy pgprw-closed,we have p-cl(α^{C}) $\leq \beta^{c}$ and so,
$\beta \leq$p-int(α), since p-cl(α^{c}) = [p $-$ int(α)]c.

Theorem2.27: If α and β are fuzzy pgprw-open sets in a fts X, then $\alpha \wedge \beta$ is also
a fuzzy pgprw-open set in fts X.

Proof: Let α and β be two fuzzy pgprw-open sets in a fts X. Then α^{C} and β^{C} are fuzzy pgprw-closed sets in fts X. By Theorem 2.8, $\alpha^{C} \vee \beta^{C}$ is also a fuzzy pgprw closed set in fts X.
That is $\alpha^{C} \vee \beta^{C} = \alpha^{C} \wedge \beta^{C}$ is a fuzzy pgprw-closed set in X. Therefore $\alpha \wedge \beta$ is also a fuzzy pgprw-open set in fts

Example2.28:

Consider the fuzzy topological space (X, T) defined in 2.6

Let X= {a, b, c,d},T= {1,0,α,β,γ, δ}in this fts X, The fuzzy sets δ_1 δ_2 :X ➔ [0, 1] are defined by

δ_1 (x)= 1 if x = a
0 otherwise.

δ_2 (x) = 1 if x = c
0 therwise

Then $\delta_1 and$ δ_2 are fuzzy pgprw open sets in fts X.

Let μ = $\delta_1 \vee \delta_2$, then

μ (x) = 1 if x = a ,c
0 otherwise.

μ = $\delta_1 \vee \delta_2$, Is not a fuzzy pgprw-closed set in fts X.

Theorem 2.29: If p-int(α)≤ $\beta \leq \alpha$ and α is a fuzzy pgprw-open set in a fts X, then β is also a fuzzy rw-open set in fts X.

Proof: Suppose p-int(α)≤ $\beta \leq \alpha$ and α is a fuzzy pgprw-open set in a fts X. To prove that β is a fuzzy pgprw-open set in fts X. Let σ be any fuzzy rgα-open set in fts X such that $\sigma \leq \beta$ .Now $\sigma \leq \beta \leq \alpha$ That is $\sigma \leq \alpha$.
Since α is fuzzy pgprw-open set of fts X, $\sigma \leq p - int(\alpha)$by Theorem 2.26. By hypothesis p-int(α)≤ β. Then int [p-int(α)] ≤ p-int (β) That is p-int(α) ≤ p-int(β).
Then $\sigma \leq$ p-int(β)Again by Theorem 2.26 β is a fuzzy pgprw-open set in fts X.

Theorem 2.30: If a fuzzy subset α of a fts X is fuzzy pgprw-closed and p-cl(α) ∧ (1-p-cl(α)) = 0, then p-cl(α)- α is a fuzzy pgprw-open set in fts X.

Proof: Let α be a fuzzy pgprw-closed set in a fts X and Let p-cl(α) ∧ (1-p-cl(α)) = 0,
Let β be any fuzzy rgα open set of fts X such that β ≤p-int(α), Then by Theorem 2.18 p-cl(α)- α does not contain any non-zero fuzzy rgα open set and so β = 0.
Therefore β ≤p-int(α)(cl(α)- α. By theorem 2.26 p-cl(α)- α is a fuzzy pgprw-open set in fts X.

Theorem2.31: Let α and β be two fuzzy subsets of a fts X. If β is a fuzzy pgprw-open set and α ≥p-int(β), then $\alpha \wedge \beta$ is a fuzzy pgprw-open set in fts X.

Proof: Let β be a fuzzy pgprw-open set of a fts X and α ≥p-int(α), That is p-int(B) ≤ $\alpha \wedge \beta$.
Also p-int(B) ≤ $\alpha \wedge \beta$ and β is a fuzzy pgprw-open set. By Theorem 2.29, $\alpha \wedge \beta$ is also a fuzzy pgprw-open set in fts X.

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