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ON THE GENERALIZED xyz-LINE CUT TRANSFORMATION GRAPHS

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ABSTRACT

Given a graph G with vertex set $V(G)$, edge set $E(G)$ and cutvertex set $W(G)$, let $\overline{G}$ be the complement, $L(G)$ the line graph and $C(G)$ the cutvertex graph of G. Let G^0 be the graph with $V(G^0) = V(G)$ and without edges, G^1 the complete graph with vertex set $V(G)$, $G^+ = G$ and $G^- = \overline{G}$. Let $lc(G)$ ($\overline{lc}(G)$) be the graph whose vertices can be put in one to one correspondence with the set of edges and cutvertices of G in such a way that two vertices of $lc(G)$ $(resp., \overline{lc}(G))$ are adjacent if and only if one corresponds to an edge of G and other to a cutvertex and they are incident (resp., nonincident). Given three variables $x, y, z \in \{0,1,+,-\}$, the *generalized* xyz*-line cut transformation graph* $R^{xyz}(G)$ of G is graph with vertex set $V(R^{xyz}(G)) = E(G) \cup W(G)$ and edge set $E(R^{xyz}(G)) = E(L(G))^x \cup E(C(G))^y \cup E(H)$, where $H = lc(G)$ if $z = +$, $H = \overline{lc}(G)$ if $z = -$, H is the graph with $V(H)$=$E(G) \cup W(G)$ and without edges if $z = 0$ and H is the complete bipartite graph with parts $E(G)$ and $W(G)$ if $z = 1$. The graph $R^{xyz}(G)$ generalizes the definition of the graph G^{xz} when $y = 0$ and $\{x, z\} \subseteq \{+,-\}$, which is given in [1]. In this paper, we investigate some basic properties such as order, size, degree of a vertex and connectedness of generalized xyz-line cut transformation graphs.

Keywords: Cutvertex; line graph; generalized xyz-line cut transformation graphs.

2010 mathematics subject classification: 05C12.

1 Introduction

By a graph G=(V, E), we mean a finite, undirected graph without loops or multiple edges. For any graph G, $V(G) = \{v_1, v_2, \ldots, v_n; n \geq 2\}$, $E(G) = \{e_1, e_2, \ldots, e_m; m \geq 1\}$, $W(G) = \{c_1, c_2, \ldots, c_r; r \geq 1\}$ and $U(G) = \{B_1, B_2, \ldots, B_s; s \geq 2\}$ denote the vertex set, edge set, cutvertex set and block set of G, respectively.

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The *degree* of a vertex v_i in G is the number of edges incident to v_i and it is denoted by $d_i = deg(v_i)$. A *cutvertex* of a connected graph G is the one whose removal increases the number of components. A *nonseparable graph* is connected, nontrivial and has no cutvertices. A *block* of a graph G is a maximal nonseparable subgraph. A block is called *endblock* of a graph if it contains exactly one cutvertex of G. The *line graph* $L(G)$ of G is the graph whose vertex set is $E(G)$ in which two vertices are adjacent if and only if they are adjacent in G. The *jump graph* $J(G)$ [2] of G is the graph whose vertex set is $E(G)$ in which two vertices are adjacent if and only if they are nonadjacent in G. If $B = \{u_1, u_2, \dots, u_p; p \geq 2\}$ is a block of G, then we say that vertex u_1 and block B are incident with each other, as u_2 and B are and so on. If a block is incident with cutvertices $c_1, c_2, \dots, c_r$, $r \geq 2$, we say that c_i and c_j are *coadjacent* where $i \neq j$ and $1 \leq i$, $j \leq r$. The *cutvertex graph* $C(G)$ [3] of a graph G is the graph whose vertex set corresponds to the cutvertices of G and in which two vertices of $C(G)$ are adjacent if the cutvertices of G to which they correspond lie on a common block. Let $lc(G)$ ($\overline{lc}(G)$) be the graph whose vertices can be put in one to one correspondence with the set of edges and cutvertices of G in such a way that two vertices of $lc(G)$ ($resp., \overline{lc}(G)$) are adjacent if and only if one corresponds to an edge of G and other to a cutvertex and they are incident (resp., nonincident). Here we call $lc(G)$ as *line-cut incident graph* and $\overline{lc}(G)$ as *partial complementary line-cut incident graph*. Let $D_G(c)$ the degree of the vertex c in $C(G)$. In this paper the considered graph must have at least one cutvertex. For graph theoretic terminology, we refer to [4,5].

2 Generalized $xyz-$ Line Cut Transformation Graphs

Let $G = (V, E)$ be a graph, and let α, β be two elements of $E(G) \cup W(G)$. The associativity of α and β is $+$ if they are adjacent or incident in G, otherwise is $-$. Let xz be a 2-permutation of the set $\{+, -\}$. We say that α and β correspond to the first term x of xz if both α and β are in $E(G)$ and α and β correspond to the second term z of xz if one of α and β is in $E(G)$ and the other is in $W(G)$. The *line-cut transformation graph* G^{xz} of G is defined on the vertex set $E(G) \cup W(G)$. Two vertices α and β of G^{xz} are joined by an edge if and only if these associativity in G is consistent with corresponding term of xz. Since there are four distinct 2-permutations of $\{+, -\}$, we obtain four line-cut transformations of G namely G^{++}, G^{+-}, G^{-+} and G^{--}. This concept is introduced in [1]. In the following definition we more generalize the construction of line-cut transformation graph G^{xz} of G. For this purpose we need the following notations.

For a graph $G = (V, E)$, let G^0 be the graph with $V(G^0) = V(G)$ and with no edges, G^1 the complete graph with $V(G^1) = V(G)$, $G^+ = G$ and $G^- = \overline{G}$. In this paper, we consider certain graph transformations depending on parameters $x, y, z \in \{0,1,+,-\}$. These operations induce functions $R^{xyz}: \mathcal{G} \to \mathcal{G}$ and $R^{xyz}(G)$ will be called the generalized xyz-line cut transformation of G which is defined as follows.

Definition: Given a graph G with edge set $E(G)$ and cutvertex set $W(G)$ and three variables $x, y, z \in \{0,1,+,-\}$, the *generalized xyz-line cut transformation graph* $R^{xyz}(G)$ of G is the graph with vertex set $V(R^{xyz}(G)) = E(G) \cup W(G)$ and edge set $E(R^{xyz}(G)) = E(L(G))^x \cup E(C(G))^y \cup E(H)$ where

1. $H = lc(G)$ if $z = +$.
2. $H = \overline{lc}(G)$ if $z = -$.
3. H is the graph with $V(H) = E(G) \cup W(G)$ and without edges if $z = 0$.
4. H is the complete bipartite graph with parts $E(G)$ and $W(G)$ if $z = 1$.

Thus we obtain 64 generalized xyz- line cut transformation graphs. Here note that $R^{+0+}(G) = G^{++}$, $R^{+0-}(G) = G^{+-}$, $R^{-0+}(G) = G^{-+}$, $R^{-0-}(G) = G^{--}$, $R^{00+}(G) = lc(G)$ and $R^{00-}(G) = \overline{lc}(G)$.

A graph G and all its 64 generalized xyz-line cut transformation graphs are shown in Figs. 1-4. The vertex e_i' of $R^{xyz}(G)$ corresponding to an edge e_i of G will be referred as *edge point*. The vertex c_i' of $R^{xyz}(G)$ corresponding to a cutvertex c_i of G will be referred as *cutvertex point*. In generalized xyz-line cut transformation graphs the edge points are denoted by dark circles and the cutvertex points are denoted by light circles.

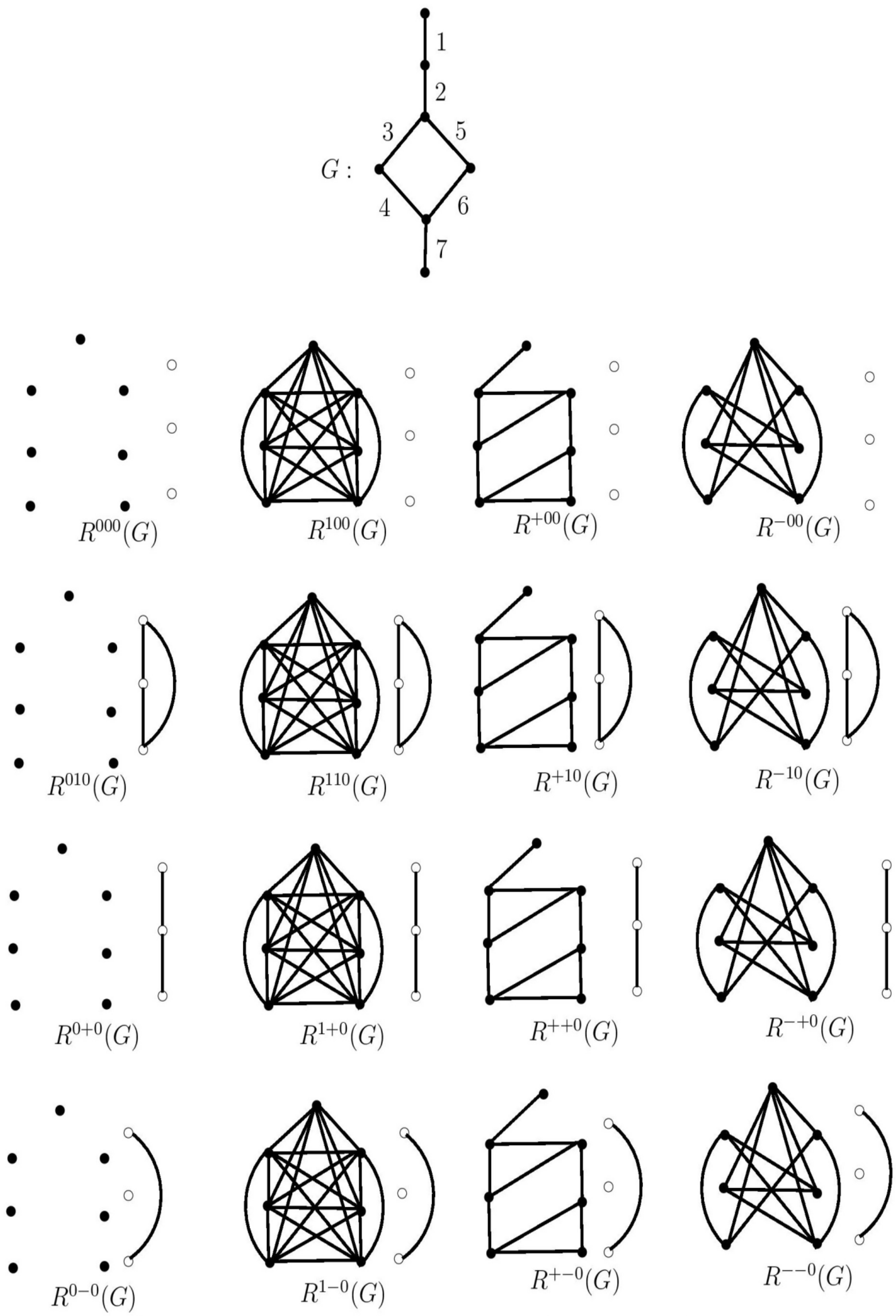

Fig. 1. Graph G and generalized xyz-line cut transformation graphs when $z = 0$

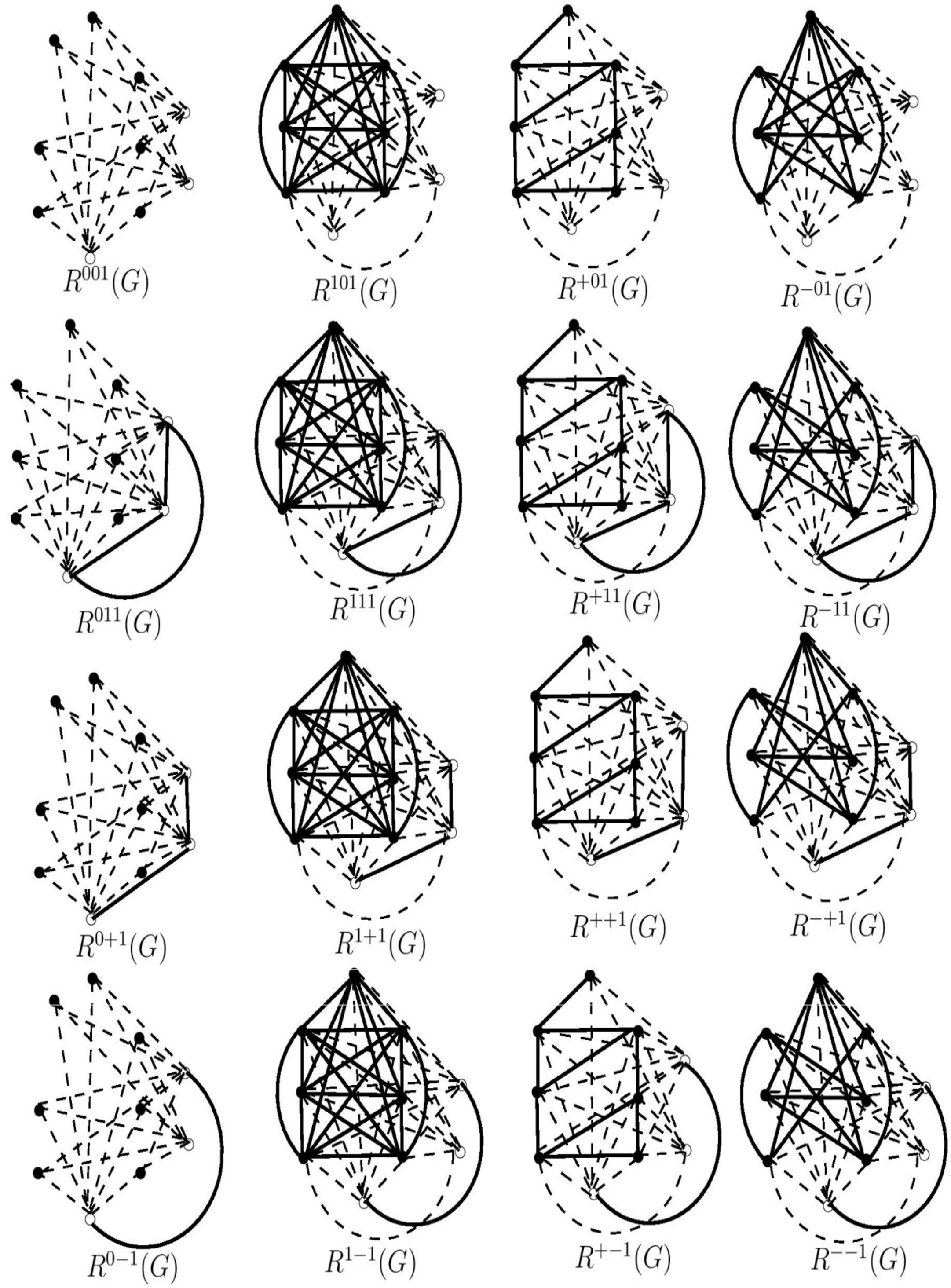

Fig. 2. Generalized xyz-line cut transformation graphs when $z = 1$

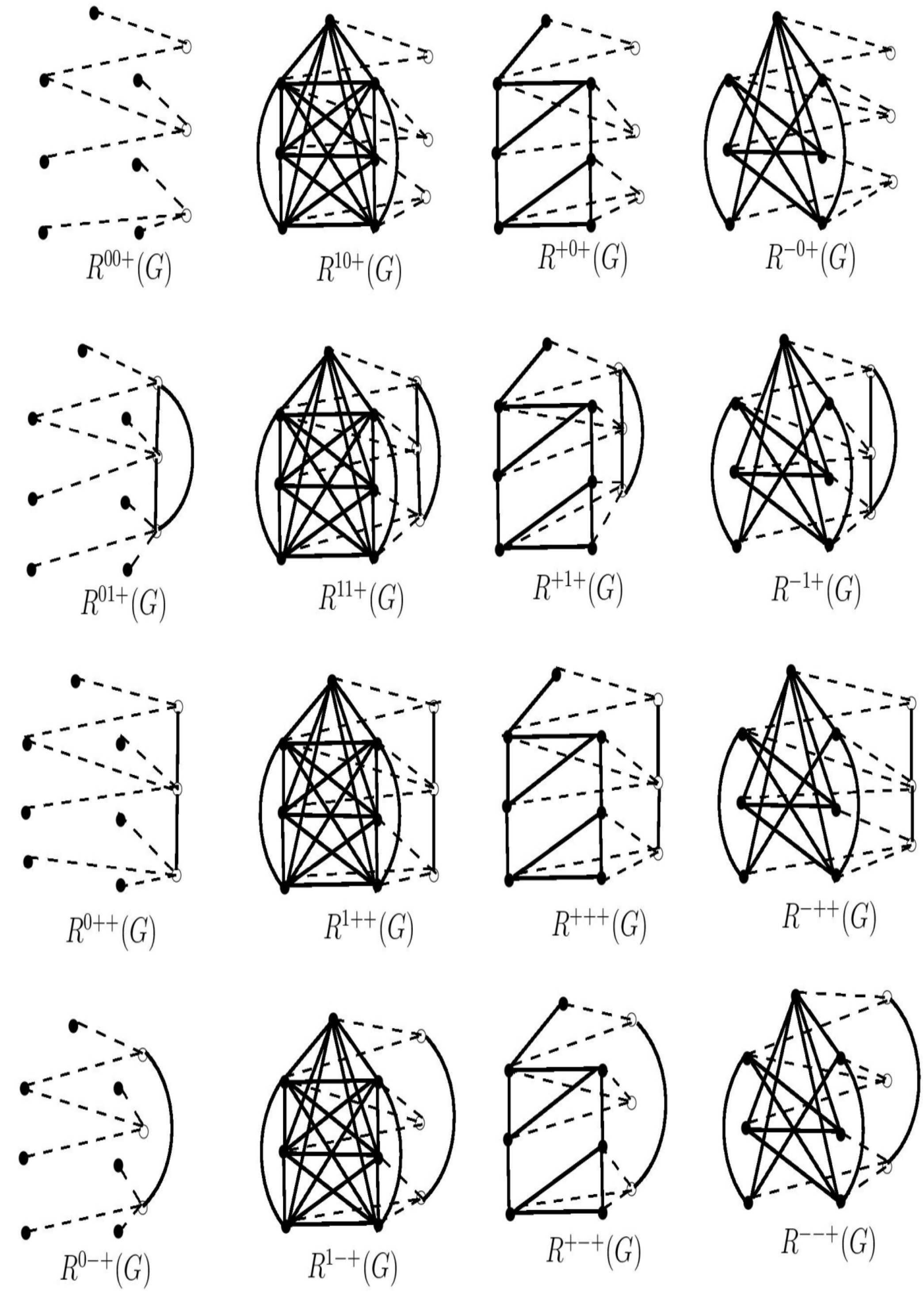

Fig. 3. Generalized xyz-line cut transformation graphs when $z = +$.

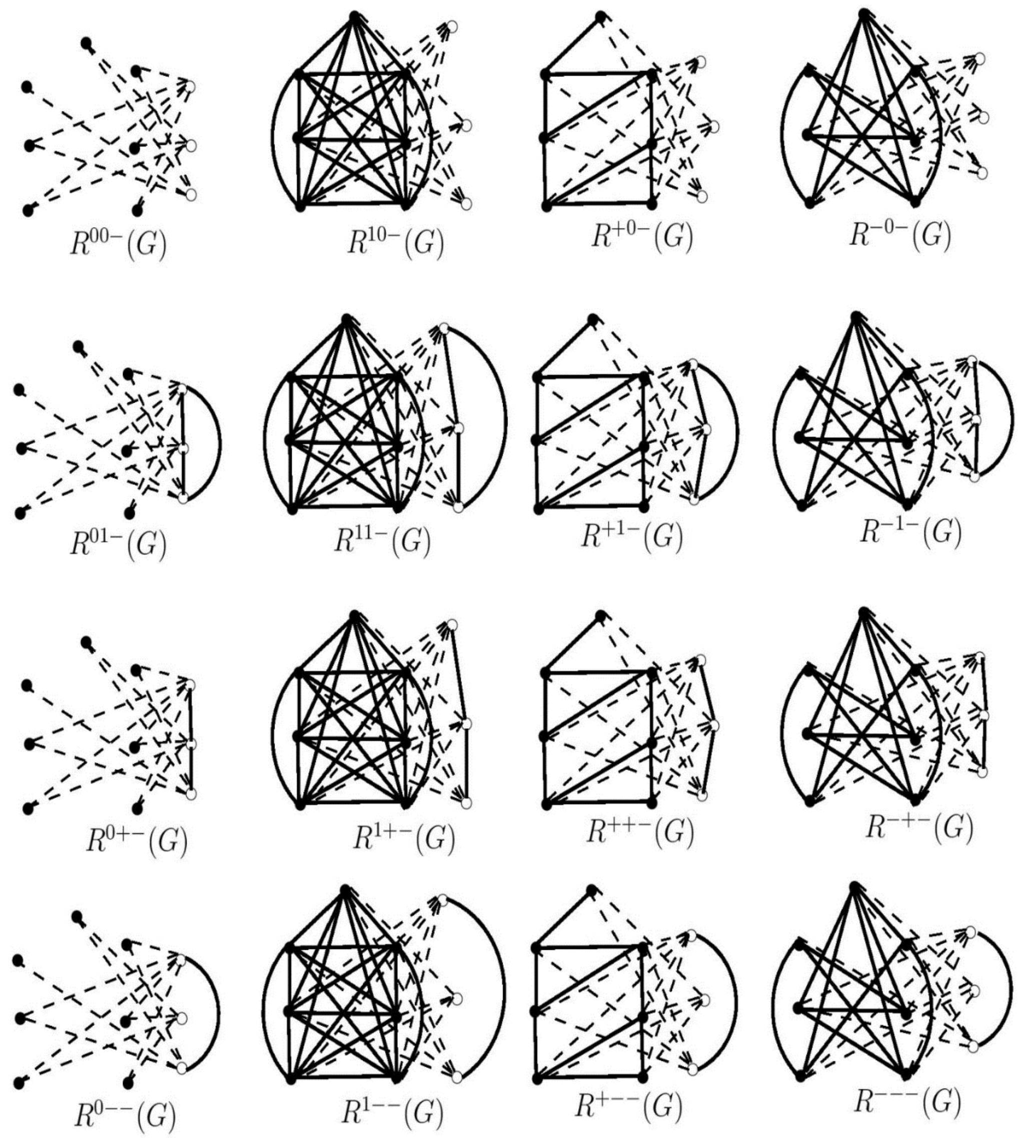

Fig. 4. Generalized xyz-line cut transformation graphs when $z = -$.

Remark 2.1

(i) $L(G)$ *is an induced subgraph of* $R^{+yz}(G)$.
(ii) $J(G)$ *is an induced subgraph of* $R^{-yz}(G)$.
(iii) K_m *is an induced subgraph of* $R^{1yz}(G)$.

Remark 2.2

(i) $C(G)$ *is an induced subgraph of* $R^{x+z}(G)$.
(ii) $\overline{C(G)}$ *is an induced subgraph of* $R^{x-z}(G)$.
(iii) K_r *is an induced subgraph of* $R^{x1z}(G)$.

Remark 2.3

(i) $lc(G)$ *is a spanning subgraph of* $R^{xy+}(G)$.
(ii) $\overline{lc}(G)$ *is a spanning subgraph of* $R^{xy-}(G)$.
(iii) $K_{m,r}$ *is a spanning subgraph of* $R^{xy1}(G)$.

Theorem 2.1: *[4] If G is connected, then $L(G)$ is connected.*

Theorem 2.2: *[6] Let G be a graph of size $m \geq 1$. Then $J(G)$ is connected if and only if G contains no edge that is adjacent to every other edges of G unless $G = K_4$ or C_4.*

Theorem 2.3: *([4], page 23) A graph G is connected if and only if for any partition of $V(G)$ into two subsets V_1 and V_2, there is an edge of G joining a vertex of V_1 with a vertex of V_2.*

3 Order, Size and Degree of Vertices of $R^{xyz}(G)$

Proposition 3.1: *Let G be a nontrivial connected (n,m)-graph with vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$, edge set $E(G) = \{e_1, e_2, \dots, e_m\}$, cutvertex set $W(G) = \{c_1, c_2, \dots, c_r; r \geq 1\}$ and block set $U(G) = \{B_1, B_2, \dots, B_s; s \geq 2\}$. Suppose that the vertex v_i of G has degree d_i, L_i is the degree of the vertex c_i in $lc(G)$ and $C(B_i)$ is the number of cutvertices of a connected graph G which are vertices of the block B_i. Then we have the following.*

$$E((L(G))^x) = \begin{cases} 0 & if \ \ x = 0 \\ \frac{m(m-1)}{2} & if \ \ x = 1 \\ -m + \frac{1}{2}\sum_{i=1}^{n} d_i^2 & if \ \ x = + \\ \frac{m(m+1)}{2} - \frac{1}{2}\sum_{i=1}^{n} d_i^2 & if \ \ x = - \end{cases}$$

$$E((C(G))^y) = \begin{cases} 0 & if \ \ y = 0 \\ \frac{r(r-1)}{2} & if \ \ y = 1 \\ \sum_{i=1}^{s} \frac{C(B_i)[C(B_i)-1]}{2} & if \ \ y = + \\ \frac{r(r-1)}{2} - \sum_{i=1}^{s} \frac{C(B_i)[C(B_i)-1]}{2} & if \ \ y = - \end{cases}$$

Theorem 3.2: *Let G be a nontrivial connected (n,m)-graph with vertex set $V(G)=\{v_1, v_2, \dots, v_n\}$, edge set $E(G)=\{e_1, e_2, \dots, e_m\}$, cutvertex set $W(G)=\{c_1, c_2, \dots, c_r\}$ and* block set *$U(G)=\{B_1, B_2, \dots, B_s\}$. Suppose that the vertex v_i of G has degree d_i, L_i is the degree of the vertex c_i in $lc(G)$ and $C(B_i)$ is the number of cutvertices of a connected graph G which are the vertices of the block B_i. Then*

(i) *The order of* $R^{xyz}(G) = |V(R^{xyz}(G))| = m + r = m + 1 + \sum_{i=1}^{s} (C(B_i) - 1)$.

(ii) *The size of* $R^{xyz}(G) = |E(R^{xyz}(G))| = \begin{cases} |E((L(G))^x)| + |E((C(G))^y)| & if \ \ z = 0. \\ |E((L(G))^x)| + |E((C(G))^y)| + mr & if \ \ z = 1. \\ |E((L(G))^x)| + |E((C(G))^y)| + \sum_{i=1}^{r} L_i & if \ \ z = +. \\ |E((L(G))^x)| + |E((C(G))^y)| + mr - \sum_{i=1}^{r} L_i & if \ \ z = -. \end{cases}$

Proof. (i) It is shown in [7] that if $C(B_i)$ is the number of cutvertices of a connected graph G which are vertices of the block B_i, then the number of cutvertices of G is given by $1 + \sum_{i=1}^{s} (C(B_i) - 1)$. On the other hand, by definition of $R^{xyz}(G)$, the number of vertices of $R^{xyz}(G)$ is the sum of the number of edges and cutvertices of G. Thus $|V(R^{xyz}(G))| = m + r$, where $r = 1 + \sum_{i=1}^{s} (C(B_i) - 1)$.

(ii) The proof of the theorem follows from the Definition of $R^{xyz}(G)$ and Proposition 3.1.

The proofs of the following results are straightforward.

Theorem 3.3: *Let G be an (n,m)-graph with r cutvertices. Then the degree of the edge point e' ($e = uv$ in G) and the cutvertex point c' (c in G) in $R^{xyz}(G)$, when $z = 0$, are*

1. $$d_{R^{xy0}(G)}(e') = \begin{cases} 0 & if\ x = 0\ and\ y \in \{0,1,+,-\} \\ m-1 & if\ x = 1\ and\ y \in \{0,1,+,-\} \\ d_G(u) + d_G(v) - 2 & if\ x = +\ and\ y \in \{0,1,+,-\} \\ m+1-d_G(u)-d_G(v) & if\ x = -\ and\ y \in \{0,1,+,-\} \end{cases}$$

2. $$d_{R^{xy0}(G)}(c') = \begin{cases} 0 & if\ y = 0\ and\ x \in \{0,1,+,-\} \\ r-1 & if\ y = 1\ and\ x \in \{0,1,+,-\} \\ D_G(c) & if\ y = +\ and\ x \in \{0,1,+,-\} \\ r-1-D_G(c) & if\ y = -\ and\ x \in \{0,1,+,-\}. \end{cases}$$

Corollary 3.4: *Let G be an (n,m)-graph with r cutvertices. Then the degree of the edge point e' and the cutvertex point c' in $R^{xyz}(G)$, when $z = 1$, are $d_{R^{xy1}(G)}(e') = d_{R^{xy0}(G)}(e') + r$ and $d_{R^{xy1}(G)}(c') = d_{R^{xy0}(G)}(c') + m$.*

Corollary 3.5: *Let G be an (n,m)-graph. Suppose that the degree of the vertex e_i in $lc(G)$ is r_i and L_i is the degree of the vertex c_i in $lc(G)$. Then the degree of the edge point e_i' ($e_i = uv$ in G) and the cutvertex point c_i' (c_i in G) in $R^{xyz}(G)$, when $z = +$, are $d_{R^{xy+}(G)}(e_i') = d_{R^{xy0}(G)}(e_i') + r_i$ and $d_{R^{xy+}(G)}(c_i') = d_{R^{xy0}(G)}(c_i') + L_i$.*

Corollary 3.6: *Let G be an (n,m)-graph. Suppose that the degree of the vertex e_i in $\overline{lc}(G)$ is p_i and L_i is the degree of the vertex c_i in $lc(G)$. Then the degree of the edge point e_i' ($e_i = uv$ in G) and the cutvertex point c_i' (c_i in G) in $R^{xyz}(G)$, when $z = -$, are $d_{R^{xy-}(G)}(e_i') = d_{R^{xy0}(G)}(e_i') + p_i$ and $d_{R^{xy-}(G)}(c_i') = d_{R^{xy0}(G)}(c_i') + m - L_i$.*

4 Connectedness of $R^{xyz}(G)$

The first theorem follows from the definition of $R^{xy0}(G)$.

Theorem 4.1: *For any graph G, $R^{xy0}(G)$ is disconnected.*

Theorem 4.2: *For any graph G, $R^{xy1}(G)$ is connected.*

Proof. The proof of the theorem follows from the fact that the complete bipartite graph $K_{m,r}$ is a connected spanning subgraph of $R^{xy1}(G)$ with partite sets $E(G)$ and $W(G)$.

When $z = +$, we have the following theorems.

Theorem 4.3: *For any connected graph G, $R^{00+}(G)$ is connected if and only if every edge is incident with at least one cutvertex in G and each cutvertex in a nonendblock is adjacent with at least one cutvertex in the same nonendblock.*

Proof. Suppose that every edge is incident with at least one cutvertex in G and each cutvertex in a nonendblock is adjacent with at least one cutvertex in the same block. Then each edge point is adjacent with at least one cutvertex point and each cutvertex point is adjacent with at least two edge points in $R^{00+}(G)$. Hence there exist a path from one vertex to any other vertex of $R^{00+}(G)$. Therefore $R^{00+}(G)$ is connected.

Conversely, if G contains an edge e is nonincident with cutvertex, then $R^{00+}(G) = R^{00+}(G-e) \cup K_1$ is disconnected, a contradiction. Let c be a cutvertex in a nonendblock. Consider the partition $\{V_1 = \{c'\} \cup \{a' : c$ and a are incident in G with $a \in E(G)\}, V_2 = V(R^{00+}(G)) \backslash V_1\}$ of $V(R^{00+}(G))$. It follows from Theorem 2.3 that there exist e' in V_1 and w' in V_2 such that $(e', w') \in E(R^{00+}(G))$, where clearly $e \in E(G)$ and $w \in W(G)$. Therefore, it follows from the definition of $R^{00+}(G)$ that e and w are incident and so c and w are adjacent in G.

Theorem 4.4: *For any graph G, $R^{1y+}(G)$ is connected.*

Proof. The proof of the theorem follows from the facts that K_m is subgraph of $R^{1y+}(G)$ with vertex set $E(G)$ and each cutvertex is incident with at least one edge in G.

Theorem 4.5: *For any graph G, $R^{01+}(G)$ is connected if and only if every edge is incident with at least one cutvertex in G.*

Proof. Suppose that every edge is incident with at least one cutvertex in G. Then K_r is a subgraph of $R^{01+}(G)$ with vertex set $W(G)$ and each edge point is adjacent with at least one cutvertex point in $R^{01+}(G)$. Therefore $R^{01+}(G)$ is connected.

Conversely, if G contains an edge e is nonincident with cutvertex, then $R^{01+}(G) = R^{01+}(G-e) \cup K_1$ is disconnected, a contradiction.

Theorem 4.6: *$R^{+1+}(G)$ is connected if and only if each component of G has at least one cutvertex.*

Proof. Suppose that each component of G has at least one cutvertex. Then K_r is an induced subgraph of $R^{+1+}(G)$ with vertex set $W(G)$ and line graph of each component of G are induced subgraph of $R^{+1+}(G)$ with vertex sets as respective edge set of components of G. By definition of $R^{+1+}(G)$, at least one edge point of line graph of each component of G in $R^{+1+}(G)$ is adjacent with one cutvertex point. Therefore $R^{+1+}(G)$ is connected.

Conversely, if one of the component say G_1 of G is block, then $R^{+1+}(G) = R^{+1+}(G - E(G_1)) \cup L(G_1)$ is disconnected, a contradiction.

Theorem 4.7: *For any graph G, $R^{-1+}(G)$ is connected.*

Proof. By definition of $R^{-1+}(G)$, K_r is an induced subgraph of $R^{-1+}(G)$ with vertex set $W(G)$ and each cutvertex point is adjacent to at least one edge point in $R^{-1+}(G)$. If G is disconnected, then result is obvious. Suppose G is connected. Now it is sufficient to show that every pair of edge point and cutvertex point are connected. We consider the following cases:

Case 1. If the edge is incident with a cutvertex in G, then result is obvious.

Case 2. If the edge e is nonincident with a cutvertex c in G, then there exists an edge e_1 which is nonincident with cutvertex c and is adjacent to e in G. Therefore e' and c' are connected through an edge point e' in $R^{-1+}(G)$.

Thus, every pair of vertices in $R^{-1+}(G)$ are connected. Hence $R^{-1+}(G)$ is connected.

Theorem 4.8: *[1] For any graph G, $R^{+0+}(G)$ is connected if and only if G is connected.*

Theorem 4.9: *For any connected graph G, $R^{0++}(G)$ is connected if and only if every edge is incident with at least one cutvertex in G.*

Proof. Suppose that every edge is incident with at least one cutvertex in G. Since $C(G)$ is a connected subgraph of $R^{0++}(G)$ with vertex set $W(G)$ (because G is connected) and each edge point is adjacent with at least one cutvertex point in $R^{0++}(G)$, it follows that $R^{0++}(G)$ is connected.

Conversely, let e be an edge in G and consider the partition $\{V_1 = \{e'\}, V_2 = V(R^{0++}(G))\backslash V_1\}$ of $V(R^{0++}(G))$. It follows from Theorem 2.3 that there exist w in V_2 such that $(e', w') \in E(R^{0++}(G))$, where clearly $w \in W(G)$ by definition of $R^{0++}(G)$. Therefore, it follows from the definition of $R^{0++}(G)$ that e and w are incident.

Theorem 4.10: *For any connected graph G, $R^{0-+}(G)$ is connected if and only if G satisfies the following conditions:*

(i) Each edge is incident with at least one cutvertex.
(ii) Each coadjacent cutvertex in a block B is adjacent to cutvertex or each coadjacent cutvertex in a block B is nonadjacent to cutvertex which is not in B.

Proof. Suppose that each edge is incident with at least one cutvertex and each coadjacent cutvertex in a block B is either adjacent to cutvertex or nonadjacent to cutvertex which is not in B. Then each edge point is adjacent with at least one cutvertex point in $R^{0-+}(G)$. Therefore it is sufficient to prove every pair of cutvertex points are connected. Let c'_1 and c'_2 be any two cutvertex points in $R^{0-+}(G)$. Then have the following three cases:

Case 1. If c_1 and c_2 are adjacent cutvertices by an edge e, then c'_1 and c'_2 are connected through an edge point e'.

Case 2. If c_1 and c_2 are nonadjacent cutvertices but noncoadjacent, then c'_1 and c'_2 are connected.

Case 3. If c_1 and c_2 are coadjacent cutvertices, then there exist a cutvertex c_3 which is nonadjacent c_1 and adjacent with c_2. Therefore c'_1 and c'_2 are connected in $R^{0-+}(G)$.

Therefore, every pair of vertices in R^{0-+} are connected. Hence $R^{0-+}(G)$ is connected.

Conversely, suppose $R^{0-+}(G)$ is connected. If G contains an edge e is nonincident with cutvetex, then $R^{0-+}(G) = R^{0-+}(G - e) \cup K_1$ is disconnected, a contradiction. If two coadjacent cutvertices are adjacent or nonadjacent with a cutvertex, then $R^{0-+}(G)$ contains two components, a contradiction.

Theorem 4.11: *$R^{+++}(G)$ is connected if and only if G is connected.*

Proof. It follows from Theorem 4.8 that $R^{+0+}(G)$ is connected and since $R^{+0+}(G)$ is an spanning subgraph of $R^{+++}(G)$ it follows that $R^{+++}(G)$ is connected.

Conversely, suppose $R^{+++}(G)$ is connected. If G is disconnected graph with at least two components G_1 and G_2, then $R^{+++}(G) = R^{+++}(G_1) \cup R^{+++}(G_2)$ is disconnected, a contradiction.

Theorem 4.12: *For a given graph G with at least one cutvertex, $R^{-y+}(G)$ is connected.*

Proof. Suppose that graph G with at least one cutvertex. We consider the following cases:

Case 1. If G contains no edge that is adjacent to every other edge of G, then by Theorem 2.2 and Remark 2.1 (ii), $J(G)$ is a connected induced subgraph of $R^{-y+}(G)$. Also each cutvertex point is adjacent to at least one edge point because every cutvertex is incident with at least one edge in G. Hence $R^{-y+}(G)$ is connected.

Case 2. If G contains an edge e that is adjacent to every other edge of G, then e is incident with at least one cutvertex c. And $R^{-y+}(G-e)$ is a connected induced subgraph of $R^{-y+}(G)$ and e', c', e'_1 is a path in $R^{-y+}(G)$ (see Fig. 5), where edge e_1 is incident with c, and each cutvertex point is adjacent to at least one edge in $R^{-y+}(G)$. Hence $R^{-y+}(G)$ is connected.

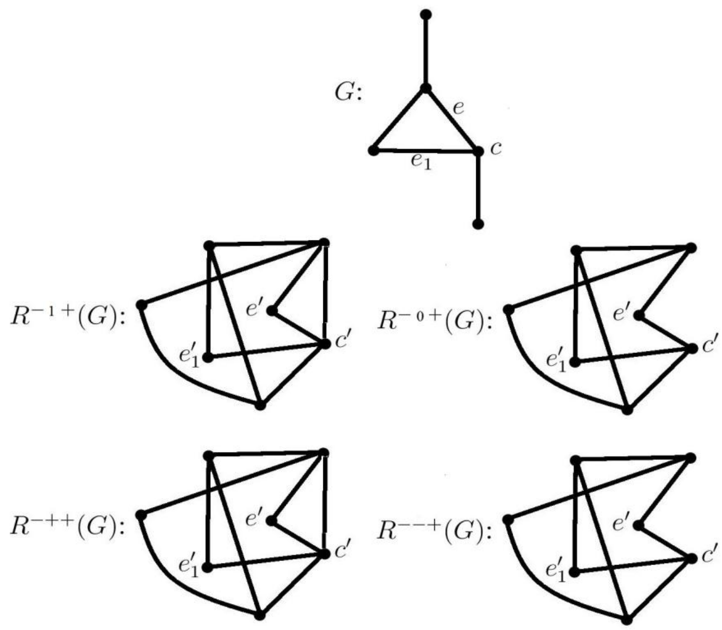

Fig. 5. Graph G and its $R^{-y+}(G)$

Theorem 4.13: *For any connected graph G, $R^{+-+}(G)$ is connected.*

Proof. Suppose that G is connected. Then by Theorem 2.1 and Remark 2.1 (i), $L(G)$ is a connected induced subgraph of $R^{+-+}(G)$ and also each cutvertex point is adjacent to at least one edge point because every cutvertex is incident with at least one edge in G. Hence $R^{+-+}(G)$ is connected.

Theorem 4.14: *For a disconnected graph G, $R^{+-+}(G)$ is connected if and only if every component of G contains at least one cutvertex.*

Proof. Suppose that every component of G contains at least one cutvertex. Then $\overline{C(G)}$ is a connected induced subgraph of $R^{+-+}(G)$. Since the line graph of each component of G is connected in $R^{+-+}(G)$ and also each cutvertex point is adjacent to at least one edge point because every cutvertex is incident with at least one edge in G, then $R^{+-+}(G)$ is connected.

Conversely, assume that there exists a component of G which is a block. Then $R^{+-+}(G)$ contains at least two components, a contradiction.

When $z = -$, we have the following theorems.

Theorem 4.15: *For any graph $G \neq K_{1,n}$, $R^{1y-}(G)$ is connected.*

Proof. Since K_m, with vertex set $E(G)$, is a subgraph of $R^{1y-}(G)$ and each cutvertex is nonincident with at least one edge in G (because $G \neq K_{1,n}$), it follows that $R^{1y-}(G)$ is connected.

Theorem 4.16: *$R^{01-}(G)$ is connected if and only if each edge is nonincident with at least one cutvertex in G.*
Proof. Suppose that every edge is nonincident with at least one cutvertex in G. Since K_r is a subgraph of $R^{01-}(G)$ and by hypothesis, each edge point is adjacent with at least one cutvertex point in $R^{01-}(G)$, then $R^{01-}(G)$ is connected.

Conversely, suppose $R^{01-}(G)$ is connected. Assume there is an edge e which is nonincident with every cutvertex in G. Then $R^{01-}(G) = R^{01-}(G-e) \cup K_1$ is disconnected, a contradiction.

Theorem 4.17: *For any graph $G \notin \{K_{1,n}, K_{1,s} \cup G_1\}$, where G_1 has no cutvertex, $R^{+1-}(G)$ is connected.*

Proof. Since K_r, with vertex set $W(G)$, is an induced subgraph of $R^{+1-}(G)$ and either edge is nonincident with at least one cutvertex or adjacent to an edge which is is nonincident with at least one cutvertex in G (because $G \notin \{K_{1,n}, K_{1,q} \cup G_1\}$), it follows that $R^{+1-}(G)$ is connected.

Theorem 4.18: *$R^{-1-}(G)$ is connected if and only if no edge which is adjacent to every other edges of G is incident with all cutvertices.*

Proof. Suppose that no edge which is adjacent to every other edges of G is incident with all cutvertices and since K_r is an induced subgraph of $R^{-1-}(G)$, it follows that $R^{-1-}(G)$ is connected.

Conversely, if G contains an edge e which is adjacent to every other edges of G is incident with all cutvertices, then $R^{-1-}(G) = R^{-1-}(G-e) \cup K_1$ is disconnected.

Theorem 4.19: *For any connected graph G with two cutvertices, $R^{00-}(G)$ is connected if and only if G contains an edge which is nonincident with both the cutvertices.*

Proof. Suppose that G contains an edge e which is nonincident with both cutvertices c_1 and c_2. Then c'_1 and c'_2 are connected through an edge point e' in $R^{00-}(G)$ and other edge points are adjacent with either c'_1 or c'_2. Therefore $R^{00-}(G)$ is connected.

Conversely, Suppose that u and v are the two cutvertices of G. Consider the partition $\{V_1 = \{u'\} \cup \{a' : u$ and a are incident in G with $a \in E(G)\}, V_2 = V(R^{00-}(G)) \backslash V_1\}$ of $V(R^{00-}(G))$. It follows from Theorem 2.3 that there exist w' in V_1 and k' in V_2 such that $(w', k') \in E(R^{00-}(G))$, where clearly $w \in E(G)$ and $k = v$ (by definition of $R^{00-}(G)$ and by choice of $\{V_1, V_2\}$). Therefore, it follows from the definition of $R^{00-}(G)$ that w is nonincident with both cutvertices.

Theorem 4.20: *For any connected graph G with at least three cutvertices, $R^{00-}(G)$ is connected.*

Proof. Since each edge is nonincident with at least one cutvertex in G. Then each edge-point is adjacent with at least one cutvertex point in $R^{00-}(G)$. Therefore it is sufficient to prove that every pair of cutvertex points are connected. Consider c'_1 and c'_2 any two cutvertex points in $R^{00-}(G)$. Then we have the following two cases.

Case 1. If there exist an edge e which is nonadjacent with c_1 and c_2, then c'_1 and c'_2 are connected through of the edge point e'.

Case 2. If there exists no an edge which is nonadjacent with c_1 and c_2, then there exist two edges e_1, e_2 and one cutvertex c_3 in which e_1 is nonadjacent with both c_2 and c_3 and e_2 is nonadjacent with both c_1 and c_3 such that $c'_1, e'_2, c'_3, e'_1, c'_2$ is a path in $R^{00-}(G)$.

Therefore, every pair of vertices in R^{00-} are connected. Hence $R^{00-}(G)$ is connected.

Theorem 4.21: *For any connected graph G, $R^{0+-}(G)$ is connected if and only if every edge is nonincident with at least one cutvertex in G.*

Proof. Suppose that every edge is nonincident with at least one cutvertex in G. Since G is connected. Then $C(G)$ is connected and by hypothesis each edge point is adjacent with at least one cutvertex point in $R^{0+-}(G)$. Therefore $R^{0+-}(G)$ is connected.

Conversely, suppose that $R^{0+-}(G)$ is connected. Assume there is an edge e which is nonincident with cutvertex in G. Then $R^{0+-}(G) = R^{0+-}(G-e) \cup K_1$ is disconnected, a contradiction.

Theorem 4.22: *[1] For a given graph G, with $m \geq 2$ and block set $U(G) = \{B_1, \dots, B_s; s \geq 2\}$, $R^{+0-}(G)$ is connected if and only if G satisfies following conditions:*

(i) $G \neq K_{1,n_1}$
(ii) $G \neq K_{1,n_2} \cup K_{1,n_3}$
(iii) $G \neq K_{1,n_1} \cup (\bigcup_{i=1}^{s} B_i)$.

Theorem 4.23: *For a given graph G, with $m \geq 2$ and block set $U(G) = \{B_1, \dots, B_s; s \geq 2\}$, $R^{++-}(G)$ is connected if and only if G satisfies following conditions:*

(i) $G \neq K_{1,n_1}$
(ii) $G \neq K_{1,n_2} \cup K_{1,n_3}$
(iii) $G \neq K_{1,n_1} \cup (\bigcup_{i=1}^{s} B_i)$.

Proof. It follows from Theorem 4.22, that $R^{+0-}(G)$ is a connected spanning subgraph of $R^{++-}(G)$ which implies that $R^{++-}(G)$ is connected.

Conversely, (i) If $G = K_{1,n_1}$, then $R^{++-}(G) = R^{+0-}(G) = K_{n_1} \cup K_1$ is disconnected, a contradiction.

(ii) If $G = K_{1,n_2} \cup K_{1,n_3}$, then $R^{++-}(G) = R^{+0-}(G) = K_{n_2+1} \cup K_{n_3+1}$ is disconnected, a contradiction.
(iii) If $G = K_{1,n_1} \cup (\bigcup_{i=1}^{s} B_i)$, then $R^{++-}(G) = R^{+0-}(G) = K_{n_1} \cup [(\bigcup_{i=1}^{s} L(B_i)) + K_1]$ is disconnected, a contradiction.

Theorem 4.24: *For any connected graph G with two cutvertices, $R^{0--}(G)$ is connected if and only if G contains an edge which is nonincident with both cutvertices.*

Proof. Since $R^{0--}(G) = R^{00-}(G)$ (because the cutvertices lie to the same block) the result follows from Theorem 4.19.

Theorem 4.25: *For any connected graph G with at least three cutvertices, $R^{0--}(G)$ is connected.*

Proof. Suppose that G is a connected graph with at least three cutvertices. Then R^{00-} is a connected spanning subgraph of $R^{0--}(G)$, Therefore the proof follows from the Theorem 4.20.

Theorem 4.26: *For a given graph G, with block set $U(G) = \{B_1, \dots, B_s; s \geq 2\}$, $R^{+--}(G)$ is connected if and only if G satisfies the following conditions:*

(i) $G \neq K_{1,n_1}$
(ii) $G \neq K_{1,n_1} \cup (\bigcup_{i=1}^{s} B_i)$.

Proof. Suppose a graph G satisfies conditions (i) and (ii). We prove the result by following cases.

Case 1. If G is connected, then we have the following subcases.
Subcase 1.1. If G is a block, then $R^{+--}(G) = L(G)$. Therefore by Theorem 2.1, $R^{+--}(G)$ is connected.
Subcase 1.2. If G has at least one cutvertex, then by Theorem 2.1 and Remark 2.1 (i), $L(G)$ is a connected

induced subgraph of $R^{+--}(G)$ and also by condition (i), each cutvertex point is adjacent to at least one edge point because every cutvertex is nonincident with at least one edge in G. Hence $R^{+--}(G)$ is connected.

Case 2. If G is disconnected with G_1, G_2,… ,G_t, $t \geq 2$ components. By condition (ii), $\overline{C(G)}$, $L(G_1)$, $L(G_2)$,…,$L(G_t)$ are connected induced subgraphs of $R^{+--}(G)$. Also at least one edge point of $L(G_i)$ is adjacent with at least one cutvertex point in $R^{+--}(G)$.

Conversely, (i) If $G = K_{1,n_1}$, then $R^{+--}(G) = K_{n_1} \cup K_1$ is disconnected, a contradiction.

(ii) If $G = K_{1,n_1} \cup (\bigcup_{i=1}^{s} B_i)$, then $R^{+--}(G) = K_{n_1} \cup [(\bigcup_{i=1}^{s} L(B_i)) + K_1]$ is disconnected, a contradiction.

Theorem 4.27 *For a given graph G with at least one cutvertex, $R^{-y-}(G)$ is connected if and only if G has no edge that is adjacent to all other edges.*

Proof. Suppose that G contains no edge that is adjacent to every other edge of G, then by Theorem 2.2 and Remark 2.1 (ii), $J(G)$ is connected induced subgraph of $R^{-y-}(G)$. Also each cutvertex point is adjacent to at least one edge point because every cutvertex is nonincident with at least one edge in G. Hence $R^{-y-}(G)$ is connected.

Conversely, assume that G contains an edge e that is adjacent to every other edge of G, then e is incident with one or two cutvertices. And $R^{-y-}(G) = R^{-y-}(G - e) \cup K_1$, a contradiction.

5 Conclusion

In this paper, we have introduced 64 generalized xyz-line cut transformation graphs and we studied order, size, degree of a vertex and connectedness of these new graphs. The study of diameter, traversability, planarity, chromatic number, domination number, spectra, energy and topological indices of 64 generalized xyz-line cut transformation graphs can be interesting. In [8,9], the authors gave the characterization of $R^{+0+}(G)$ and $R^{+++}(G)$, respectively. Characterization of remaining 62 generalized xyz-line cut transformation graphs can be quite challenging. (i.e., to prove that: A graph G is a generalized xyz- line cut transformation graph if and only if it is isomorphic to the generalized xyz-line cut transformation graph $R^{xyz}(H)$ of some graph H.)

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Competing Interests

Authors have declared that no competing interests exist.

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