International Research Journal of Pure Algebra-8(2), 2018, 13-16
Available online through www.rjpa.info ISSN 2248–9037

D_{pgprw} (i, j)-σ_k-Continuous Maps in Bitopological spaces

VİVEKANANDA DEMBRE*

Assistant Professor,
Department of Mathematics,
Sanjay Ghodawat University, Kolhapur, India.

(Received On: 19-01-18; Revised & Accepted On: 17-02-18)

ABSTRACT

In this paper, a new class of maps called D_{pgprw} (i,j)-σ_k-continuous maps in bitopological spaces are introduced and investigated; during this process ,some of their properties are obtained.

Key words and phrases: *pgprw-closed sets,pgprw-open sets, pgprw-continuous maps.*

2000 Mathematics subject classification: *54A05.*

INTRODUCTİON

The Triple (X, τ_1, τ_2) where X is a set and τ_1 and τ_2 are topologies on X is called a bitopological space. H.Maki, P.Sundaram &K.Balachandran [1] introduced generalized maps & pasting lemma in Bitopological spaces.

2.PRELİMİNARİES: f: (X, τ_1,τ_2) → (Y,σ_1 σ_2)

(i) τ_j - σ_k-continuous maps [1] if $f^{-1}(V) \epsilon\ \tau_j$ –for every V ϵ τ_k.
(ii) C(i,j)- σ_k-continuous maps [2] if $f^{-1}(V)\epsilon C(i,j)$ for every σ_k – closed set in (Y,σ_1 σ_2).
(iii) D(i,j)- σ_k-continuous maps [1] if $f^{-1}(V)\epsilon D(i,j)$ for every σ_k – closed set in (Y,σ_1 σ_2).
(iv) W(i,j)- σ_k-continuous maps [3] if $f^{-1}(V)\epsilon W(i,j)$ for every σ_k – closed set in (Y,σ_1 σ_2).
(v) D_{rg}(i,j)- σ_k-continuous maps [4] if $f^{-1}(V)\epsilon D_{rg}(i,j)$ for every σ_k – closed set in (Y,σ_1 σ_2).
(vi) ω(i,j)- σ_k-continuous maps [5] if $f^{-1}(V)\epsilon\ \omega(i,j)$ for every σ_k – closed set in (Y,σ_1 σ_2).

2.1 Theorem: [6]

(i) If A is, τ_j- closed subset of a bitopological space (X, τ_1, τ_2), then the set A is (i,j) Pgprw-closed.
(ii) If A is a (i,j)- pgprw-closed subset of (X, τ_1, τ_2), then A is (i,j) gpr-closed.

2.2 Theorem: [6] If A and B be subsets of (X, τ_1,τ_2) then

(i) (i,j) pgprw-cl(X)=X and (i,j)-pgprw-cl($\emptyset$).
(ii) A $\subseteq$ (i,j)-pgprw-cl(A)
(iii) if B is any (i,j) pgprw-closed set containing A Then (i,j)-pgprw-cl(A) $\subseteq$ B.

3. D_{pgprw} (i,j)-σ_k-CONTİNUOUS MAPS MAPS İN BİTOPOLOGİCAL SPACES.

Definition 3.1: A map f: (X, τ_1, τ_2) → (Y,σ_1 σ_2) is called D_{pgprw} (i,j)-σ_k-continuous maps if the inverse image of every σ_k – closed set is an (i,j)-pgprw-closed set in (X,τ_1,τ_2).

Remark 3.2: If $\tau_1 = \tau_2 = \tau$ and $\sigma_1 = \sigma_2 = \sigma$ in definition 3.1 then the D_{pgprw} (i,j)-σ_k-continuous of maps coincides with pgprw-continuity of maps in topological spaces.

Theorem 3.3: If a map f: (X, τ_1, τ_2) → (Y,σ_1 σ_2) is τ_j- σ_k-continuous maps then it is D_{pgprw} (i,j)-σ_k-continuous maps.

Proof: Let V be a σ_k – closed set since f is τ_j- σ_k-continuous maps, $f^{-1}(V)$ is τ_j-closed by theorem 2.1[6] $f^{-1}(V)$ is (i,j)pgprw-closed in (X, τ_1, τ_2). Therefore f is D_{pgprw} (i,j)-σ_k-continuous maps.

*Corresponding Author: Vivekananda Dembre**
Assistant Professor, Department of Mathematics, Sanjay Ghodawat University, Kolhapur, India.

The converse of this theorem need not be true in general as seen from the following example,

Example 3.4: Let X = {a, b, c}, τ_1 = {X, ∅, {a}} and τ_2 = {X, ∅, {a,b }}, Y= {b, c}, σ_1 ={Y, ∅,{b}} and σ_2 = {Y, ∅, {c}}. Define a map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = c, f(b) = b, f(c) = c. Then f is D_{pgprw} (2,1)- σ_2-continuous maps but it is not τ_1- σ_2-continuous maps since for the closed set {b}, f^{-1}(b)={b}, which is not τ_1-closed.

Theorem 3.5: If a map f: (X, τ_1, τ_2) → (Y,σ_1 σ_2) is D_{pgprw}(i,j) σ_k-continuous maps then it is ω (i,j)-σ_k-continuous maps.

Proof: Let V be a σ_k – closed set since f is Dpgprw(i, j) σ_k-continuous maps, f^{-1}(V) is (i,j) pgprw-closed by theorem 2.1[6] then f^{-1}(V) is gpr −closed in (X, τ_1, τ_2).Therefore f is ω-(i,j)-σ_k-continuous maps.

The converse of this theorem need not be true in general as seen from the following example,

Example 3.6: Let X={a, b, c} τ_1={X, ∅,{a},{b},{a,b}} and τ_2 = {X, ∅,{a}} & Y= {a, b}, σ_1 = {Y, ∅} and σ_2 = {Y, ∅, {b}}. Define a map f: (X,τ_1, τ_2)→ (Y,σ_1 σ_2) by f(a) = f(b) = a and f(c) = b. Then this function f is gpr(1,2)−σ_2 − continuous maps, but it is not D_{pgprw}(1,2) σ_2 − continuous maps. since for the σ_2 closed set{a}, f^{-1}(a)={a,b}, which is not (1,2) pgprw-closed set.

Remark 3.7: D_{pgprw}(i,j) σ_k-continuous maps and D (i,j)-σ_k-continuous maps maps are independent.

Example 3.8: Let X={a, b, c} τ_1={X, ∅, {a},{b},{a, b}}and τ_2 = {X, ∅,{a}} & Y= {a, b}, σ_1 ={Y, ∅} and σ_2 = {Y, ∅, {a}}. Define a map f: (X,τ_1,τ_2) → (Y,σ_1 σ_2) by f(a) = f(c) = a and f(b) = b. Then this function f is D_{pgprw}(i,j) σ_2-continuous maps, but it is not (1,2) $\sigma 2$ − continuous maps. since for the σ_2 closed set{b}, f^{-1}(b) ={b}, which is not (1,2) g-closed set.

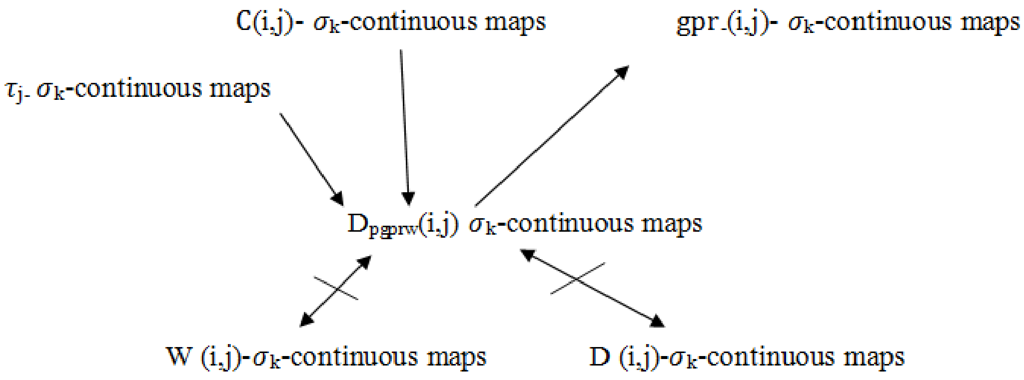
Example 3.9: Let X={a, b, c} τ_1={X, ∅, {a},{b},{a, b}} and τ_2 = {X, ∅, {a},{a, b}} & Y= {a, b}, σ_1=P(Y) and σ_2 = {Y, ∅, {b}}. Define a map f: (X, τ_1, τ_2) → (Y,σ_1, σ_2) by f(b) = b, f(c) = a and f(a) = a. Then this function f is D(1,2) σ_2-continuous maps, but it is not D(1,2) $\sigma 2$ − continuous maps. since for the σ_2 closed set{a}, f^{-1}(a)={a,c}, which is not (1,2) pgprw-closed set.

Remark 3.10: D_{pgprw}(i,j) σ_k-continuous maps and W(i,j)-σ_k-continuous maps maps are independent

Example 3.11: Let X={a,b,c} τ_1={X,∅,{a},{a,b}}and τ_2 = {X,∅,{a}}&Y= {a,b}, σ_1 =P(Y) and σ_2={Y, ∅, {a}}. Define a map f: (X,τ_1,τ_2) → (Y,σ_1 σ_2) by f(a)=a,f(b)=b & f(c) = a. Then this function f is D_{pgprw}(i,j) σ_2-continuous maps, but it is not W(1,2) $\sigma 2$ − continuous maps. since for the σ_2 closed set{b}, f^{-1}(b) ={b},which is not (1,2) wg-closed set.

Example 3.12: Let X={a, b, c} τ_1={X, ∅,{a}}and τ_2={X, ∅,{a},{a,b}}&Y={b,c}, σ_1={Y, ∅} and σ_2 = {Y, {∅}, {c}}. Define a map f: (X,τ_1,τ_2) → (Y,σ_1 σ_2) by f(a) = b, f(b) = c, f(c) = b. Then this function f is W(1,2)-σ_2-continuous maps, but it is not $_{Pgprw}$(1,2) $\sigma 2$ − continuous maps. since for the σ_2 closed set{b}, f^{-1}(b) ={a,c}, which is not (1,2) pgprw-closed set.

Remark 3.13: From the above discussions and known results we have the following implication form.



Theorem 3.14: The following statements are equivalent.

(i) A map f: (X,τ_1,τ_2) → (Y,σ_1 σ_2) is D_{pgprw}(i,j) σ_k-continuous maps.

(ii) The inverse image of σ_k –open set in Y is (i,j)-pgprw-open in Y.

Proof: (i) implies (ii) Let G be a σ_k –open in Y.Then G^c is σ_k –closed set in Y. Since f is D_{pgprw}(i,j) σ_k continuous maps, $f^{-1}(G^c)$ is (i,j)pgprw-closed in X That is $f^{-1}(G^c) = (f^{-1}(G^c))^C$ and so on $(f^{-1}(G^c)$ is (i,j) pgprw-open in (X, τ_1, τ_2).

(ii) implies (i) Let F be a σ_k –closed in Y. Then F^c is σ_k –open set in Y, By hypothesis $f^{-1}(F^c)$ is (i,j)pgprw-open in X. That is $f^{-1}(F^c) = (f^{-1}(F^c))^C$ and so $f^{-1}(F)$ is (i,j) pgprw-closed in (X,τ_1,τ_2). Therefore f is D_{pgprw}(i,j) σ_k-continuous maps.

Theorem 3.15: If a map f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1\ \sigma_2)$ is D_{pgprw}(i,j) σ_k-continuous maps, then f(i,j)-pgprw-cl(A) $\subseteq \sigma_k$ –p-cl(A) holds for every subset A of X.

Proof: Let A be any subset of X then f(A) $\subseteq \sigma_k$ –p-cl(A) and σ_k –p-cl(A) is σ_k- closed set in Y also $f^{-1}(f(A)) \subseteq f^{-1}(\sigma_k$ –p-cl(A)) that is A$\subseteq f^{-1}(\sigma_k$ –p-cl(A)) since f is D_{pgprw}(i,j) σ_k-continuous maps, $f^{-1}(\sigma_k$ –p-cl(A)) is a (i,j) pgprw-closed set in (X, τ_1, τ_2) by thm 2.2 (i,j) pgprw-cl(A) $\subseteq f^{-1}(\sigma_k$ –p-cl(A)),Therefore f(i,j)pgprw-cl(A) $\subseteq$ f(f^{-1} σ_k –p-cl(A)) $\subseteq \sigma_k$ –p-cl(A)) hence f(i,j)- pgprw-cl(A) $\subseteq$ (σ_k –p-cl(A)), for every subset A of (X,τ_1,τ_2).

Remark 3.16: Converse of the Theorem 3.15 is not true in general as seen from the following example. Let X={a,b,c} τ_1={X,∅,{a},{b},{a,b}} and τ_2 = {X,∅,{a},{b,c}}&Y= {a,b}, σ_1 =P(Y) and σ_2 = {Y, ,∅,{a}}.D_{pgprw} (1,2) = {X,∅,{b,c}} Define a map f: $(X, \tau_1, \tau_2) \rightarrow (Y,\sigma_1\ \sigma_2)$ by f(a)=f(c)=a & f(b)=b. Then f(1,2)-pgprw-cl(A) $\subseteq \sigma_2$ –p-cl(A) for every subset A of X but f is not D_{pgprw}(i,j) σ_2-continuous maps,Since for the closed set {b}, f^{-1}({b}) = b which is not a (1,2) pgprw-closed in (X, τ_1, τ_2).

Theorem 3.17: If a map f: $(X, \tau_1,\tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is D_{pgprw}(i,j) σ_k-continuous maps and g:$(Y,\sigma_1\ \sigma_2) \rightarrow (Z,\mu_1,\mu_2)$ is σ_k - μn continuous maps, then gof is D_{pgprw}(i,j) - μn-continuous maps.

Proof: Let f be μn closed set in $(Z,\mu 1,\mu 2)$ since g is σ_k - μn continuous maps, g^{-1}(F) is a σ_k-closed set in $(Y,\sigma_1\ \sigma_2)$ since f is D_{pgprw}(i,j) σ_k-continuous maps, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is a (i,j) pgprw-closed in (X,τ_1,τ_2) and hence gof is D_{pgprw}(i,j) - μn-continuous maps.

REFERENCES

1. H.Maki,P.Sundram&K.Balachandran,on g-continuous maps maps & pasting lemma in bitopological spaces, Bull Fukuoka univ, Ed, part-III, 40 (1991), 23-31.
2. T.Fukutake,P.sundaram and m.sheik john, on weakly closed sets,w-open sets and w-continuity in Bitopological spaces bull fukuoka univ.ed.part III, 51 (2002), 1-9.
3. T.Fukutake,P.Sundaram and N.Nagaveni, on weakly generalized continuous maps and T_{wg} spaces in bitopological spaces,bull,fukuoka univ.ed part III,51(2002),1-9.
4. I.Arockiarani, studies on generalization of g-closed sets and maps in topological spaces, ph.d thesis, Bharathiar university, coimbatore(1997).
5. Y.Gnanambal,studies on generalized pre-regular closed sets and generalization of locally closed sets, ph.d thesis bharathiar university,coimbatore, (1998).
6. R.S.Wali and Vivekananda Dembre,On Pgprw-locally closed sets in topological spaces, International Jounal of Mathematical Archive-7(3),2016,119-123.
7. R.S.Wali and Vivekananda Dembre; On Pre Generalızed Pre Regular Weakly Closed Sets in Topologıcal Spaces;Journal of Computer and Mathematical Sciences, Vol.6(2),113-125, February 2015.
8. R.S.Wali and Vivekananda Dembre, Minimal weakly open sets and maximal weakly closed sets in topological spaces; International Journal of Mathematical Archieve; Vol-4(9)-Sept-2014.
9. R.S.Wali and Vivekananda Dembre, Minimal weakly closed sets and Maximal weakly open sets in topological spaces; International Research Journal of Pure Algebra; Vol-4(9)-Sept-2014.
10. R.S.Wali and Vivekananda Dembre, on semi-minimal open and semi-maximal closed sets in topological spaces; Journal of Computer and Mathematical Science; Vol-5(9)-Oct.-2014 (International Journal).
11. R.S.Wali and Vivekananda Dembre, on pre genearalized pre regular open sets and pre regular weakly neighbourhoods in topological spaces; Annals of Pure and Applied Mathematics; Vol-10- 12 2015.
12. R.S.Wali and Vivekananda Dembre, on pre generalized pre regular weakly interior and pre generalized pre regular weakly closure in topological spaces, International Journal of Pure Algebra- 6(2), 2016, 255-259.
13. R.S.Wali and Vivekananda Dembre, on pre generalized pre regular weakly continuous maps in topological spaces, Bulletin of Mathematics and Statistics Research Vol.4.Issue.1.2016 (January-March).
14. R.S.Wali andVivekananda Dembre, on Pre-generalized pre regular weakly irresolute and strongly pgprw-continuous maps in topological spaces, Asian Journal of current Engineering and Maths 5;2 March-April (2016)44-46.
15. R.S.Wali and Vivekananda Dembre,(τ_1,τ_2) pgprw-closed sets and open sets in Bitopological spaces, International Journal of Applied Research 2016;2(5); 636-642.

16. R.S.Wali and Vivekananda Dembre,Fuzzy pgprw-continuous maps and fuzzy pgprw-irresolute in fuzzy topological spaces; International Journal of Statistics and Applied Mathematics 2016;1(1):01-04.
17. R.S.Wali and Vivekananda Dembre,On pgprw-closed maps and pgprw-open maps in Topological spaces; International Journal of Statistics and Applied Mathematics 2016;1(1); 01-04.
18. Vivekananda Dembre, Minimal weakly homeomorphism and Maximal weakly homeomorphism in topological spaces, Bulletin of the Marathons Mathematical Society, Vol. 16, No. 2, December 2015, Pages 1-7.
19. Vivekananda Dembre and Jeetendra Gurjar, On semi-maximal weakly open and semi-minimal weakly closed sets in topological spaces, International Research Journal of Pure Algebra-Vol-4(10), Oct – 2014.
20. Vivekananda Dembre and Jeetendra Gurjar, minimal weakly open map and maximal weakly open maps in topological spaces, International Research Journal of Pure Algebra-Vol.-4(10), Oct – 2014; 603-606.
21. Vivekananda Dembre, Manjunath Gowda and Jeetendra Gurjar, minimal weakly and maximal weakly continuous functions in topological spaces, InternationalResearch Journal of Pure Algebra-vol.-4(11), Nov– 2014.
22. Arun kumar Gali and Vivekananda Dembre, mınımal weakly generalızed closed sets and maxımal weakly generalızed open sets in topologıcal spaces, Journal of Computer and Mathematical sciences, Vol.6(6), 328-335, June 2015.
23. R.S.Wali and Vivekananda Dembre; Fuzzy Pgprw-Closed Sets and Fuzzy Pgprw-Open Sets in Fuzzy Topological SpacesVolume 3, No. 3, March 2016; Journal of Global Research in Mathematical Archives.
24. Vivekananda Dembre and Sandeep.N.Patil; On Contra Pre Generalized Pre Regular Weakly Continuous Functions in Topological Spaces; IJSART - Volume 3 Issue 12 – December 2017.
25. Vivekananda Dembre and Sandeep.N.Patil; On Pre Generalized Pre Regular Weakly Homeomorphism in Topological Spaces; Journal of Computer and Mathematical Sciences, Vol.9(1), 1-5 January 2018.
26. Vivekananda Dembre and Sandeep.N.Patil ;on pre generalized pre regular weakly topological spaces; Journal of Global Research in Mathematical Archives volume 5, No.1**,** January 2018.
27. Vivekananda Dembre and Sandeep.N.Patil; Fuzzy Pre Generalized Pre Regular Weakly Homeomorphism in Fuzzy Topological Spaces ;International Journal of Computer Applications Technology and Research Volume 7–Issue 02, 28-34, 2018, ISSN:-2319–8656.
28. Vivekananda Dembre and Sandeep.N.Patil; PGPRW-Locally Closed Continuous Maps in Topological Spaces; International Journal of Trend in Research and Development, Volume 5(1), January 2018.
29. Vivekananda Dembre and Sandeep.N.Patil; Rw-Separation Axioms in Topological Spaces**;** International Journal of Engineering Sciences & Research Technology; Volume 7(1): January, 2018.
30. Vivekananda Dembre and Sandeep.N.Patil ; Fuzzy pgprw-open maps and fuzzy pgprw-closed maps in fuzzy topological spaces; International Research Journal of Pure Algebra-8(1), 2018, 7-12.
31. Vivekananda Dembre and Sandeep.N.Patil; Pgprw-Submaximal spaces in topological spaces; International Journal of applied research 2018; Volume 4(2): 01-02.

Source of Support: Nil, Conflict of interest: None Declared