



## NEW SPACES IN TOPOLOGICAL SPACES

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**Abstract :** The aim of this paper is to introduce and study two new classes of spaces, namely Pre generalized pre regular weakly normal and pre generalized pre regular weakly regular spaces and obtained their properties by utilizing pre generalized pre regular weakly closed sets.

**Keywords:** Pre generalized pre regular weakly closed set, Pre generalized pre regular weakly open sets, Pre generalized pre regular weakly regular space and pre generalized pre regular weakly normal space.

**Mathematics subject classification (2010):** 54A05.

### 1 INTRODUCTION

Maheshwari and Prasad[8] introduced the new class of spaces called s-normal spaces using semi-open sets. It was further studied by Noiri and Popa[10], Dorsett[6] and Arya[1]. Munshi[9], introduced g-regular and g-normal spaces using g-closed sets of Levine[7]. Later, Benchalli et al [3] and Shik John[12] studied the concept of  $g^*$ -pre-regular,  $g^*$ -pre normal and w-normal, w-regular spaces in topological spaces. Recently, Benchalli et al [2,11] introduced and studied the properties of regular weakly closed sets and regular weakly continuous functions.

### 2 PRELIMINARIES

Throughout this paper space  $(X, \tau)$  and  $(Y, \sigma)$  (or simply  $X$  and  $Y$ ) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset  $A$  of a space  $X$ ,  $Cl(A)$ ,  $Int(A)$ ,  $A^c$ , and  $\alpha-Cl(A)$ , denote the Closure of  $A$ , Interior of  $A$  and Compliment of  $A$  and  $\alpha$ -closure of  $A$  in  $X$  respectively.

**Definition 2.1:** A subset  $A$  of a topological space  $(X, \tau)$  is called

- (i) Generalized closed set (briefly g-closed) [7] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (ii) W-closed set [12] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open in  $X$ .

**Definition 2.2 :** A topological space  $X$  is said to be a

- (1)  $\alpha$ -regular [4], if for each  $\alpha$ -closed set  $F$  of  $X$  and each point  $x \notin F$ , there exists disjoint  $\alpha$ -open sets  $U$  and  $V$  such that  $F \subseteq V$  and  $x \in U$ .
- (2) w-regular [12], if for each closed set  $F$  of  $X$  and each point  $x \notin F$ , there exists disjoint w-open

sets  $U$  and  $V$  such that  $F \subseteq U$  and  $x \in V$ .

(3)  $g$ -regular [10], if for each  $g$ -closed set  $F$  of  $X$  and each point  $x \notin F$ , there exists disjoint open sets  $U$  and  $V$  such that  $F \subseteq U$  and  $x \in V$ .

**Definition 2.3.** A topological space  $X$  is said to be a

(1)  $\alpha$ -normal [4], if for any pair of disjoint  $\alpha$ -closed sets  $A$  and  $B$ , there exists disjoint  $\alpha$ -open sets  $U$  and  $V$  such that  $A \subseteq U$  and  $B \subseteq V$ .

(2)  $w$ -normal [12], if for any pair of disjoint  $w$ -closed sets  $A$  and  $B$ , there exists disjoint open sets  $U$  and  $V$  such that  $A \subseteq U$  and  $B \subseteq V$ .

(3)  $g$ -normal [10], if for any pair of disjoint  $g$ -closed sets  $A$  and  $B$ , there exists disjoint open sets  $U$  and  $V$  such that  $A \subseteq U$  and  $B \subseteq V$ .

**Definition 2.4:** [2] A topological space  $X$  is called  $T_{\text{regular weakly}}$ -space if every pre generalized pre regular weakly closed set is closed set.

**Definition 2.5:** A map  $f: (X, \tau) \rightarrow (Y, \tau)$  is said to be

(i) Pre generalized pre regular weakly continuous map [19] if  $f^{-1}(V)$  is a pre generalized pre regular weakly closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \tau)$ .

(ii) Pre generalized pre regular weakly irresolute map [20] if  $f^{-1}(V)$  is a pre generalized pre regular weakly closed set of  $(X, \tau)$  for every pre generalized pre regular weakly closed set  $V$  of  $(Y, \tau)$ .

### 3. PRE GENERALIZED PRE REGULAR WEAKLY REGULAR SPACE

In this section, we introduce a new class of space called pre generalized pre regular weakly regular space using Pre generalized pre regular weakly closed set and obtain some of their characterizations.

**Definition 3.1.** A topological space  $X$  is said to be pre generalized pre regular weakly regular space if for each pre generalized pre regular weakly closed set  $F$  and a point  $x \notin F$ , there exist disjoint open sets  $G$  and  $H$  such that  $F \subseteq G$  and  $x \in H$ .

We have the following interrelationship between pre generalized pre regular weakly regularity and regularity.

**Theorem 3.2.** Every pre generalized pre regular weakly regular space is regular.

**Proof:** Let  $X$  be a pre generalized pre regular weakly regular space. Let  $F$  be any closed set in  $X$  and a point  $x \notin F$ . By [2],  $F$  is pre generalized pre regular weakly topological space-closed and  $x \notin F$ . Since  $X$  is a pre generalized pre regular weakly regular space, there exists a pair of disjoint open sets  $G$  and  $H$  such that  $F \subseteq G$  and  $x \in H$ . Hence  $X$  is a regular space.

**Remark 3.3:** If  $X$  is a regular space and  $T_{\text{pre generalized pre regular weakly topological space}}$ , then  $X$  is pre generalized pre regular weakly regular space then we have the following characterization.

**Theorem 3.4.** The following statements are equivalent for a topological space  $X$ .

(i)  $X$  is a pre generalized pre regular weakly regular space

(ii) For each  $x \in X$  and each pre generalized pre regular weakly topological spaces open neighbourhood  $U$  of  $x$ , there exists an open neighbourhood  $N$  of  $x$  such that  $cl(N) \subseteq U$ .

**Proof:** (i) implies (ii): Suppose  $X$  is a pre generalized pre regular weakly regular space. Let  $U$  be any pre generalized pre regular weakly neighbourhood of  $x$ . Then there exists pre generalized pre regular weakly open set  $G$  such that  $x \in G \subseteq U$ . Now  $X - G$  is pre generalized pre regular weakly closed set and  $x \notin X - G$ . Since  $X$  is pre generalized pre regular weakly regular space, then there exist open sets  $M$  and  $N$  such that  $X - G \subseteq M$ ,  $x \in N$  and  $M \cap N = \emptyset$  and so  $N \subseteq X - M$ . Now  $cl(N) \subseteq cl(X - M) = X - M$  and  $X - M \subseteq M$ . This implies  $X - M \subseteq U$ . Therefore  $cl(N) \subseteq U$ .

(ii) implies (i): Let  $F$  be any pre generalized pre regular weakly topological space closed set in  $X$  and  $x \in X - F$  and  $X - F$  is a pre generalized pre regular weakly topological space open and so  $X - F$  is a pre generalized pre regular weakly topological space neighbourhood of  $x$ . By hypothesis, there exists an open neighbourhood  $N$  of  $x$  such that  $x \in N$  and  $cl(N) \subseteq X - F$ . This implies  $F \subseteq X - cl(N)$  is an open set containing  $F$  and  $N \cap (X - cl(N)) = \emptyset$ . Hence  $X$  is pre generalized pre regular weakly regular space.

We have another characterization of pre generalized pre regular weakly regularity in the following.

**Theorem 3.5:** A topological space  $X$  is pre generalized pre regular weakly regular if and only if for each pre generalized pre regular weakly topological space closed set  $F$  of  $X$  and each  $x \in X - F$  there exist open sets  $G$  and  $H$  of  $X$  such that  $x \in G$ ,  $F \subseteq H$  and  $cl(G) \cap cl(H) = \emptyset$ .

**Proof:** Suppose  $X$  is pre generalized pre regular weakly regular space. Let  $F$  be a pre generalized pre regular weakly topological space closed set in  $X$  with  $x \notin F$ . Then there exists open sets  $M$  and  $H$  of  $X$  such that  $x \in M$ ,  $F \subseteq H$  and  $M \cap H = \emptyset$ . This implies  $M \cap cl(H) = \emptyset$ . As  $X$  is pre generalized pre regular weakly regular, there exist open sets  $U$  and  $V$  such that  $x \in U$ ,  $cl(H) \subseteq V$  and  $U \cap V = \emptyset$ . so  $cl(U) \cap V = \emptyset$ . Let  $G = M \cap U$ , then  $G$  and  $H$  are open sets of  $X$  such that  $x \in G$ ,  $F \subseteq H$  and  $cl(G) \cap cl(H) = \emptyset$ .

Conversely, if for each pre generalized pre regular weakly closed set  $F$  of  $X$  and each  $x \in X - F$  there exist open sets  $G$  and  $H$  such that  $x \in G$ ,  $F \subseteq H$  and  $cl(G) \cap cl(H) = \emptyset$ . This implies  $x \in G$ ,  $F \subseteq H$  and  $G \cap H = \emptyset$ . Hence  $X$  is pre generalized pre regular weakly regular.

Now we prove that pre generalized pre regular weakly topological spaces- regularity is a hereditary property.

**Theorem 3.6.** Every subspace of a pre generalized pre regular weakly regular space is pre generalized pre regular weakly regular.

**Proof:** Let  $X$  be a pre generalized pre regular weakly regular space. Let  $Y$  be a subspace of  $X$ . Let  $x \in Y$  and  $F$  be a pre generalized pre regular weakly closed set in  $Y$  such that  $x \notin F$ . Then there is a closed set and so pre generalized pre regular weakly closed set  $A$  of  $X$  with  $F = Y \cap A$  and  $x \notin A$ . Therefore we have  $x \in X - A$ ,  $A$  is pre generalized pre regular weakly closed in  $X$  such that  $x \notin A$ . Since  $X$  is pre generalized pre regular weakly regular, then there exist open sets  $G$  and  $H$  such that  $x \in G$ ,  $A \subseteq H$  and  $G \cap H = \emptyset$ . Note that  $Y \cap G$  and  $Y \cap H$  are open sets in  $Y$ . Also  $x \in G$  and  $x \in Y$ , which implies  $x \in Y \cap G$  and  $A \subseteq H$  implies  $Y \cap G \subseteq Y \cap H$ ,  $F \subseteq Y \cap H$ . Also  $(Y \cap G) \cap (Y \cap H) = \emptyset$ . Hence  $Y$  is pre generalized pre regular weakly regular space.

We have yet another characterization of pre generalized pre regular weakly topological spaces-regularity in the following.

**Theorem 3.7 :** The following statements about a topological space  $X$  are equivalent:

- (i)  $X$  is pre generalized pre regular weakly regular
- (ii) For each  $x \in X$  and each pre generalized pre regular weakly topological space open set  $U$  in  $X$  such that  $x \in U$  there exists an open set  $V$  in  $X$  such that  $x \in V \subseteq cl(V) \subseteq U$ .

(iii) For each point  $x \in X$  and for each pre generalized pre regular weakly topological space closed set  $A$  with  $x \notin A$ , then there exists an open set  $V$  containing  $x$  such that  $cl(V) \cap A = \emptyset$ .

**Proof:** (i) implies (ii): Follows from Theorem 3.5.

(ii) implies (iii): Suppose (ii) holds. Let  $x \in X$  and  $A$  be an pre generalized pre regular weakly topological space closed set of  $X$  such that  $x \notin A$ . Then  $X - A$  is a pre generalized pre regular weakly topological space open set with  $x \in X - A$ . By hypothesis, there exists an open set  $V$  such that  $x \in V \subseteq cl(V) \subseteq X - A$ . That is  $x \in V$ ,  $V \subseteq cl(V)$  and  $cl(V) \subseteq X - A$ . So  $x \in V$  and  $cl(V) \cap A = \emptyset$ .

(iii) implies (i): Let  $x \in X$  and  $U$  be an pre generalized pre regular weakly topological space open set in  $X$  such that  $x \in U$ . Then  $X - U$  is an pre generalized pre regular weakly topological space closed set and  $x \notin X - U$ . Then by hypothesis, there exists an open set  $V$  containing  $x$  such that  $cl(V) \cap (X - U) = \emptyset$ . Therefore  $x \in V$ ,  $cl(V) \subseteq U$  so  $x \in V \subseteq cl(V) \subseteq U$ .

The invariance of pre generalized pre regular weakly topological space regularity is given in the following.

**Theorem 3.8:** Let  $f : X \rightarrow Y$  be a bijective, pre generalized pre regular weakly topological space irresolute and open map from a pre generalized pre regular weakly topological space regular space  $X$  into a topological space  $Y$ , then  $Y$  is pre generalized pre regular weakly topological spaces-regular.

**Proof:** Let  $y \in Y$  and  $F$  be a pre generalized pre regular weakly topological space closed set in  $Y$  with  $y \notin F$ . Since  $F$  is pre generalized pre regular weakly topological space irresolute,  $f^{-1}(F)$  is pre generalized pre regular weakly topological space closed set in  $X$ . Let  $f(x) = y$  so that  $x = f^{-1}(y)$  and  $x \notin f^{-1}(F)$ . Again  $X$  is pre generalized pre regular weakly-regular space, then there exist open sets  $U$  and  $V$  such that  $x \in U$  and  $f^{-1}(F) \subseteq G$ ,  $U \cap V = \emptyset$ . Since  $f$  is open and bijective, we have  $y \in f(U)$ ,  $F \subseteq f(V)$  and  $f(U) \cap f(V) = f(U \cap V) = f(\emptyset) = \emptyset$ . Hence  $Y$  is pre generalized pre regular weakly regular space.

**Theorem 3.9.** Let  $f : X \rightarrow Y$  be a bijective, pre generalized pre regular weakly closed and open map from a topological space  $X$  into a pre generalized pre regular weakly regular space  $Y$ . If  $X$  is  $T_{pre\ generalized\ pre\ regular\ weakly\ topological\ spaces}$ , then  $X$  is pre generalized pre regular weakly regular.

**Proof:** Let  $x \in X$  and  $F$  be an pre generalized pre regular weakly closed set in  $X$  with  $x \notin F$ . Since  $X$  is  $T_{pre\ generalized\ pre\ regular\ weakly\ topological\ spaces}$ ,  $F$  is closed in  $X$ . Then  $f(F)$  is pre generalized pre regular weakly closed set with  $f(x) \notin f(F)$  in  $Y$ , since  $f$  is pre generalized pre regular weakly closed. As  $Y$  is pre generalized pre regular weakly regular, then there exist open sets  $U$  and  $V$  such that  $x \in U$  and  $f(x) \in U$  and  $f(F) \subseteq V$ . Therefore  $x \in f^{-1}(U)$  and  $F \subseteq f^{-1}(V)$ . Hence  $X$  is pre generalized pre regular weakly regular space.

**Theorem 3.10.** If  $f : X \rightarrow Y$  is w-irresolute, continuous injection and  $Y$  is pre generalized pre regular weakly topological spaces-regular space, then  $X$  is pre generalized pre regular weakly topological spaces-regular.

**Proof:** Let  $F$  be any closed set in  $X$  with  $x \notin F$ . Since  $f$  is w-irresolute,  $f$  is pre generalized pre regular weakly topological space closed set in  $Y$  and  $f(x) \in f(F)$ . Since  $Y$  is pre generalized pre regular weakly regular, then there exists open sets  $U$  and  $V$  such that  $f(x) \in U$  and  $f(F) \subseteq V$ . Thus  $x \in f^{-1}(U)$ ,  $F \subseteq f^{-1}(V)$  and  $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ . Hence  $X$  is pre generalized pre regular weakly regular space.

#### 4 PRE GENERALIZED PRE REGULAR WEAKLY NORMAL SPACES

In this section, we introduce the concept of pre generalized pre regular weakly normal spaces and study some of their characterizations.

**Definition 4.1.** A topological space  $X$  is said to be pre generalized pre regular weakly normal if for each pair of disjoint pre generalized pre regular weakly topological spaces closed sets  $A$  and  $B$  in  $X$ , then there exists a pair of disjoint open sets  $U$  and  $V$  in  $X$  such that  $A \subseteq U$  and  $B \subseteq V$

We have the following interrelationship.

**Theorem 4.2.** Every pre generalized pre regular weakly normal space is normal.

**Proof:** Let  $X$  be a pre generalized pre regular weakly normal space. Let  $A$  and  $B$  be a pair of disjoint closed sets in  $X$ . From [2],  $A$  and  $B$  are pre generalized pre regular weakly topological spaces closed sets in  $X$ . Since  $X$  is pre generalized pre regular weakly normal, then there exists a pair of disjoint open sets  $G$  and  $H$  in  $X$  such that  $A \subseteq G$  and  $B \subseteq H$ . Hence  $X$  is normal.

**Remark 4.3.** The converse need not be true in general as seen from the following example.

**Example 4.4.** Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c, d\}\}$ . Then the space  $X$  is normal but not pre generalized pre regular weakly normal, since the pair of disjoint pre generalized pre regular weakly topological spaces closed sets namely,  $A = \{a, d\}$  and  $B = \{b, c\}$  for which there do not exist disjoint open sets  $G$  and  $H$  such that  $A \subseteq G$  and  $B \subseteq H$ .

**Remark 4.5.** If  $X$  is normal and  $T_{\text{pre generalized pre regular weakly topological spaces}}$ , then  $X$  is pre generalized pre regular weakly -normal.

Hereditary property of pre generalized pre regular weakly normality is given in the following.

**Theorem 4.6.** A pre generalized pre regular weakly closed subspace of a pre generalized pre regular weakly normal space is pre generalized pre regular weakly normal. We have the following characterization.

**Theorem 4.7.** The following statements for a topological space  $X$  are equivalent:

- (i)  $X$  is pre generalized pre regular weakly topological spaces is normal
- (ii) For each pre generalized pre regular weakly closed set  $A$  and each pre generalized pre regular weakly topological space open set  $U$  such that  $A \subseteq U$ , there exists an open set  $V$  such that  $A \subseteq V \subseteq \text{cl}(V) \subseteq U$
- (iii) For any pre generalized pre regular weakly closed sets  $A, B$ , there exists an open set  $V$  such that  $A \subseteq V$  and  $\text{cl}(V) \cap B = \emptyset$ .
- (iv) For each pair  $A, B$  of disjoint pre generalized pre regular weakly closed sets then there exist open sets  $U$  and  $V$  such that  $A \subseteq U, B \subseteq V$  and  $\text{cl}(U) \cap \text{cl}(V) = \emptyset$ .

**Proof:** (i) implies (ii): Let  $A$  be a pre generalized pre regular weakly closed set and  $U$  be a pre generalized pre regular weakly open set such that  $A \subseteq U$ . Then  $A$  and  $X - U$  are disjoint pre generalized pre regular weakly closed sets in  $X$ . Since  $X$  is pre generalized pre regular weakly normal, then there exists a pair of disjoint open sets  $V$  and  $W$  in  $X$  such that  $A \subseteq V$  and  $X - U \subseteq W$ . Now  $X - W \subseteq X - (X - U)$ , so  $X - W \subseteq U$  also  $V \cap W = \emptyset$ . implies  $V \subseteq X - W$ , so  $\text{cl}(V) \subseteq \text{cl}(X - W)$  which implies  $\text{cl}(V) \subseteq X - W$ . Therefore  $\text{cl}(V) \subseteq X - W \subseteq U$ . So  $\text{cl}(V) \subseteq U$ . Hence  $A \subseteq V \subseteq \text{cl}(V) \subseteq U$ .

(ii) implies (iii): Let  $A$  and  $B$  be a pair of disjoint pre generalized pre regular weakly closed sets in  $X$ . Now  $A \cap B = \emptyset$ , so  $A \subseteq X - B$ , where  $A$  is pre generalized pre regular weakly closed and  $X - B$  is pre generalized pre regular weakly open. By (ii), there exists an open set  $V$  such that  $A \subseteq V \subseteq \text{cl}(V) \subseteq X - B$ . Hence  $\text{cl}(V) \cap B = \emptyset$ .

regular weakly open . Then by (ii) there exists an open set  $V$  such that  $A \subseteq V \subseteq \text{cl}(V) \subseteq X - B$ . Now  $\text{cl}(V) \subseteq X - B$  implies  $\text{cl}(V) \cap B = \emptyset$ . Thus  $A \subseteq V$  and  $\text{cl}(V) \cap B = \emptyset$ .

(iii) implies (iv): Let  $A$  and  $B$  be a pair of disjoint pre generalized pre regular weakly closed sets in  $X$ . Then from (iii) there exists an open set  $U$  such that  $A \subseteq U$  and  $\text{cl}(U) \cap B = \emptyset$ . Since  $\text{cl}(V)$  is closed, so pre generalized pre regular weakly closed set. Therefore  $\text{cl}(V)$  and  $B$  are disjoint pre generalized pre regular weakly closed sets in  $X$ . By hypothesis, then there exists an open set  $V$ , such that  $B \subseteq V$  and  $\text{cl}(U) \cap \text{cl}(V) = \emptyset$ .

(iv) implies (i): Let  $A$  and  $B$  be a pair of disjoint pre generalized pre regular weakly closed sets in  $X$ . Then from (iv) then there exist an open sets  $U$  and  $V$  in  $X$  such that  $A \subseteq U$ ,  $B \subseteq V$  and  $\text{cl}(U) \cap \text{cl}(V) = \emptyset$ . So  $A \subseteq U$ ,  $B \subseteq V$  and  $U \cap V = \emptyset$ . Hence  $X$  is pre generalized pre regular weakly normal.

**Theorem 4.8.** Let  $X$  be a topological space. Then  $X$  is pre generalized pre regular weakly normal if and only if for any pair  $A, B$  of disjoint pre generalized pre regular weakly closed set then there exist open sets  $U$  and  $V$  of  $X$  such that  $A \subseteq U, B \subseteq V$  and  $\text{cl}(U) \cap \text{cl}(V) = \emptyset$ .

**Theorem 4.9.** Let  $X$  be a topological space. Then the following are equivalent:

- (i)  $X$  is normal
- (ii) For any disjoint closed sets  $A$  and  $B$ , then there exist disjoint pre generalized pre regular weakly topological spaces- open sets  $U$  and  $V$  such that  $A \subseteq U, B \subseteq V$ .
- (iii) For any closed set  $A$  and any open set  $V$  such that  $A \subseteq V$ , there exists an pre generalized pre regular weakly open set  $U$  of  $X$  such that  $A \subseteq U \subseteq \alpha\text{-cl}(U) \subseteq V$ .

**Proof:** (i) implies (ii): Suppose  $X$  is normal. Since every open set is pre generalized pre regular weakly open [2], (ii) follows.

(ii) implies (iii): Suppose (ii) holds. Let  $A$  be a closed set and  $V$  be an open set containing  $A$ . Then  $A$  and  $X - V$  are disjoint closed sets. By (ii), then there exist disjoint pre generalized pre regular weakly open sets  $U$  and  $W$  such that  $A \subseteq U$  and  $X - V \subseteq W$ , since  $X - V$  is closed, so pre generalized pre regular weakly is closed. From [2], we have  $X - V \subseteq \alpha\text{-int}(W)$  and  $U \cap \alpha\text{-int}(W) = \emptyset$ . and so we have  $\alpha\text{-cl}(U) \cap \alpha\text{-int}(W) = \emptyset$ . Hence  $A \subseteq U \subseteq \alpha\text{-cl}(U) \subseteq X - \alpha\text{-int}(W) \subseteq V$ . Thus  $A \subseteq U \subseteq \alpha\text{-cl}(U) \subseteq V$ .

(iii) implies (i): Let  $A$  and  $B$  be a pair of disjoint closed sets of  $X$ . Then  $A \subseteq X - B$  and  $X - B$  is open. There exists a pre generalized pre regular weakly open set  $G$  of  $X$  such that  $A \subseteq G \subseteq \alpha\text{-cl}(G) \subseteq X - B$ . Since  $A$  is closed, it is  $w$ -closed, we have  $A \subseteq \alpha\text{-int}(G)$ . Take  $U = \text{int}(\text{cl}(\text{int}(\alpha\text{-int}(G))))$  and  $V = \text{int}(\text{cl}(\text{int}(X - \alpha\text{-cl}(G))))$ . Then  $U$  and  $V$  are disjoint open sets of  $X$  such that  $A \subseteq U$  and  $B \subseteq V$ . Hence  $X$  is normal.

We have the following characterization of pre generalized pre regular weakly topological spaces- normality and pre generalized pre regular weakly topological spaces- normality.

**Theorem 4.10.** Let  $X$  be a topological space. Then the following are equivalent:

- (i)  $X$  is  $\alpha$ -normal.
- (ii) For any disjoint closed sets  $A$  and  $B$ , there exist disjoint pre generalized pre regular weakly topological space- open sets  $U$  and  $V$  such that  $A \subseteq U, B \subseteq V$  and  $U \cap V = \emptyset$ .

**Proof:** (i) implies (ii): Suppose  $X$  is  $\alpha$ -normal. Let  $A$  and  $B$  be a pair of disjoint closed sets of  $X$ . Since  $X$  is  $\alpha$ -normal, there exist disjoint  $\alpha$ -open sets  $U$  and  $V$  such that  $A \subseteq U$  and  $B \subseteq V$  and  $U \cap V = \emptyset$ .

(ii) implies (i): Let  $A$  and  $B$  be a pair of disjoint closed sets of  $X$ . The by hypothesis there exist disjoint pre generalized pre regular weakly open sets  $U$  and  $V$  such that  $A \subseteq U$  and  $B \subseteq V$  and  $U \cap V = \emptyset$ . Since from [2],  $A \subseteq \alpha\text{-int}U$  and  $B \subseteq \alpha\text{-int}(V)$  and  $\alpha\text{-int}U \cap \alpha\text{-int}V = \emptyset$ . Hence  $X$  is  $\alpha$ -normal.

**Theorem 4.11.** Let  $X$  be a  $\alpha$ - normal, then the following hold good:

- (i) For each closed set  $A$  and every pre generalized pre regular weakly open set  $B$  such that  $A \subseteq B$  there exists a  $\alpha$  open set  $U$  such that  $A \subseteq U \subseteq \alpha\text{-cl}(U) \subseteq B$ .
- (ii) For every pre generalized pre regular weakly closed set  $A$  and every open set  $B$  containing  $A$ , there exist a  $\alpha$ -open set  $U$  such that  $A \subseteq U \subseteq \alpha\text{-cl}(U) \subseteq B$ .

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