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Pre Generalized Pre Regular Weakly Axioms in Topological Spaces

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Abstract: In this paper, we study some separation axioms namely, pgprw-T_o-space, pgprw-T_1 -space and pgprw-T_2-space and their properties. We also obtain some of their characterizations.

Keywords: PGPRW-T_O-SPACE**,** PGPRW-T_1 –SPACE, PGPRW-T_2-SPACE.

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I. INTRODUCTION

In the year 2015,R.S.Wali and Vivekananda Dembre introduced and studied pgprw-closed and pgprw-open sets respectively. In this paper we define and study the properties of a new topological axioms called pgprw-T_o-space**,** pgprw-T_1 –space,pgprw-T_2-space.

II. PRELIMINARIES

Throughout this paper space (X,τ) and (Y,σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c,P-Cl(A) and P-int(A) denote the Closure of A, Interior of A , Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

Definition 2.1:A subset A of a topological space (X, τ) is called

(i) A pre generalized pre regular ωeakly closed set(briefly pgprω-closed set)if pCl(A)⊆U whenever A⊆U and U is rgα-open in (X, τ).

(ii)A subset A of a topological space (X,τ) is called pre generalized pre regular ωeakly open (briefly pgprω-open) set in X if A^c is pgprω-closed in X.

(iii)A topological space X is called a τ_{pgprw} space if every pgprw -closed set in it is pre-closed.

Defintion 3: A map f:(X, τ) -» (Y, σ) is called

(i) Pgprw-continuous map[6] if f^{-1} (v) is pgprw closed in (X,τ) for every closed V in (Y,σ).

(ii)Pgprw-irresolute map[7]if f^{-1}(v) is pgprw closed in (X,τ)for every pgprw-closed V in (Y,σ).

(iii)Pgprw-closed map[8] if f^{-1}(v) is pgprw closed in (X,τ) for every closed V in (Y,σ).

(iv)Pgprw-open map[8] if f^{-1}(v) is pgprw closed in (X,τ) for every closed V in (Y,σ).

(v) P-irresolute map[11] if f^{-1}(v) is p-closed in (X,τ) for every p-closed V in (Y, σ).

IV. PRE GENERALIZED PRE REGULAR WEAKLY SPACE

Definition 4.4.1: A topological space (X, τ) is called pgprw-T_o-space if for anypair of distinct points x,y of (X,τ) there exists an pgprw-open set G such thatx∈G, y∉G or x∉G, y∈G.

Example 4.4.2: Let X = {a, b}, τ ={φ,{b}, X}. Then (X, τ) is pgprw-T_o-space,since for any pair of distinct points a, b of (X,τ) there exists an pgprw-T_o open set{b} such that a ∉{b},b∈{b}.

Remark 4.4.3: Every pgprw-space is pgprw-T_o-space.

Theorem 4.4.4: Every subspace of a pgprw-T_o-space ispgprw-T_o-space.

Proof: Let (X,τ) be a pgprw-T_o-space and (Y,τ_y) be a subspace of (X,τ). Let $Y_1$and Y_2 be two distinct points of (Y,τ_y). Since (Y,τ_y) is subspace of (X,τ),Y_1 and Y_2 are also distinct points of (X,τ). As (X,τ) is pgprw-T_o-space , thereexists an pgprw-open set G such that Y_1∈G, Y_2∉G. Then Y∩G is pgprw-openin (Y,τ_y) containing but Y_1 not Y_2. Hence (Y,τ_y) is pgprw-T_o-space.

Theorem 4.4.5: Let f: (X,τ) -» (Y, μ) be an injection, pgprw-irresolute map. If(Y,μ) is pgprw-T_o-space, then (X,τ) is pgprw-T_o-space.

Proof: Suppose (Y, μ) is pgprw-T_o-space.Let a and b be two distinct points in(X,τ).As f is an injection f(a) and f(b) are distinct points in (Y,μ). Since(Y,μ) ispgprw-T_o-space, there exists an pgprw-open set G in (Y,μ) such that f(a)∈G and f(b)∉G. As f is pgprw-irresolute, f^{-1}(G) is pgprw-open set in (X,τ) suchthat a∈f^{-1}(G) and b∉f^{-1}(G). Hence (X,τ) is pgprw-T_o-space.

Theorem 4.4.6: If (X,τ) is pgprw-T_o-space, T_{Pgprw}-space and (Y,τ_y) is pgprw-closed subspace of (X,τ), then (Y,τ_y) is pgprw-T_o -Space.

Proof: Let (X,τ) bepgprw-T_o-space, T_{Pgprw}-space and (Y,τ_y) is pgprw-closed subspace of (X,τ). Let a and b be two distinct points of Y. Since Y is subspace of(X,τ), a and b are distinct points of (X,τ). As (X,τ)is pgprw-T_o -space, thereexists an pgprw-open set G such that a∈G and b∉G. Again since (X,τ) isT_{Pgprw}-space, G is open in (X,τ).Then Y∩Gis open. So Y∩G is pgprw-open such that a∈Y∩G and b∉Y∩G. Hence (Y,τ_y) is Pgprw-T_o–space.

Theorem 4.4.7: Let f: (X,τ) -» (Y, μ) be bijective pgprw-open map from apgprw-T_0Space(X,τ) onto a topological space (Y,τ_y). If (X,τ) is T_{pgprw}-space, then(Y, μ) is pgprw-T_0 Space.

Proof: Let a and b be two distinct points of (Y,τ_y). Since f is bijective, thereexist two distinct points e and d of (X,τ) such that f(c) = a and f(d) = b. As(X,τ) ispgprw-T_0 Space,there exists a pgprw-open set G such that c∈ G and d∉G.Since (X,τ)

is T_{pgprw}-space, G is open in (X,τ). Then f(G) is pgprw-open in(Y, μ), since f is pgprw-open, such that a∈f(G) and b∉f(G). Hence (Y,τ_y) ispgprw-T_0-space.

Definition 4.4.8:A topological space (X,τ) is said to be pgprw-T_1-space if forany pair of distinct points a and b of (X,τ) there exist pgprw-open sets G and Hsuch that a∈G, b∉G and a∉H, b∈H.

Example 4.4.9: Let X = {a,b} and τ = {∅,{a}, X}. Then (X,τ) is atopological space. Here a and b are two distinct points of (X,τ), then thereexist pgprw-open sets {a},{b} such that a∈{a}, b∉{a} and a∉{b}, b∈{b}.Therefore (X,τ) is pgprw-T_0space.

Theorem 4.4.10: If (X,τ) is pgprw-T_1-space,then (X,τ) is pgprw-T_o-space.

Proof: Let (X,τ) be apgprw-T_1-space. Let a and b be two distinct points of(X,τ). Since (X,τ) ispgprw-T_1-space, there exist pgprw-open sets G and H such thata∈G, b∉G and a∉H, b∈H. Hence we have a∈G, b∉G. Therefore(X,τ) is pgprw-T_o-space.The converse of the above theorem need not be true as seen from thefollowing example.

Example 4.4.11: Let X = {a,b} and τ ={φ,{b},X}. Then (X,τ) is pgprw-T_o-space but not pgprw-T_1-space. For any two distinct points a, b of X and anpgprw-open set {b} such that a∉{b}, b∈{b} but then there is no pgprw-open set Gwith a∈G, b∉G for a≠b.

Theorem 4.4.12: If f:(X,τ) -» (Y,τ_y) is a bijective pgprw-open map from a pgprw-T_1-space and T_{pgprw}-space (X,τ) on to a topological space (Y,τ_y), then(Y,τ_y) is pgprw-T_1-space.

Proof: Let (X,τ) be a pgprw-T_1-space and T_{pgprw}-space. Let a and b be two distinct points of (Y,τ_y). Since f is bijective there exist distinct points c and d of(X,τ)such that f(c) = a and f(d) = b. Since (X,τ) is pgprw-T_1-space there existpgprw-open sets G and H such that c∈G, d∉G and c∉H, d ∈H. Since(X,τ) is T_{pgprw}-space, G and H are open sets in (X,τ) also f is pgprw-open f(G)and f(H) are pgprw-open sets such that a = f (c)∈f(G), b = f(d)∉f(G) and a= f(c)∉ f(H), b= f(d)∈f(H). Hence (Y,τ_y) is pgprw-T_1-space.

Theorem 4.4.13: If (X,τ) is pgprw T_1 space and T_{pgprw}-space, Y is a subspace of (X,τ), then Y is pgprw T_1 space.

Proof: Let (X,τ) be a pgprw T_1 space and T_{pgprw}-space. Let Y be a subspace of (X,τ).Let a and b be two distract points of Y. Since Y⊆X,a and b are alsodistinct points of X. Since $(X,\tau)$is pgprw-T_1-space, there exist pgprw-open sets Gand H such that a∈G, b∉G and a∉H, b∈H. Again since (X,τ) is T_{pgprw}-space,G and H are open sets in (X,τ), then Y∩G and Y∩H are open sets so pgprw-open sets of Y such that a∈Y∩G, b∉Y∩G and a∉Y∩H,b∈Y∩H. Hence Y is pgprw T_1 space.

Theorem 4.4.14: Iff: (X,τ) -»(Y,τ_y) is injective pgprw-irresolute map from a topological space (X,τ)into pgprw-T_1-space(Y,τ_y), then $(X,\tau)$is pgprw-T_1 - space.

Proof: Let a and b be two distinct points of (X,τ). Since f is injective, f(a)and f(b) are distinct points of (Y,τ_y). Since(Y,τ_y) is pgprw-T_1 space there exist pgprw-open sets G and H such that f(a)∈G, f(b) ∉G and f(a) ∉H, f(b)∈H.Since f is pgprw-irresolute, f^{-1}(G) and f^{-1}(H) are pgprw-open sets in (X,τ) such that a∈f $^{-1}$(G), b∉f $^{-1}$(G) and a∉f ^{1}(H), b∈f $^{-1}$(H). Hence (X,τ) is pgprw-T_1space.

Definition 4.4.15: A topological space (X,τ). is said to be pgprw-T_2- space(or T_{pgprw}-Hausdorff space) if for every pair of distinct points x, y of X thereexist T_{pgprw}-open sets M and N such that x∈N, y∈M and N∩M = ∅.

Example 4.4.16: Let X = {a,b}, τ = {∅,{a},{b}, X}. Then (X,τ) istopological space. Then (X,τ) is pgprw-T_2-space. T_{pgprw}-open sets are ∅, {a}, {b},and X. Let a and b be a pair of distinct points of X, then there exist T_{pgprw} - open sets {a} and {b} such that a∈{a}, b∈{b} and {a}∩{b} = ∅. Hence (X,τ) is pgprw-T_2-space.

Theorem 4.4.17: Every pgprw-T_2- space is pgprwT_1space.

Proof: Let (X,τ) be a pgprw-T_2- space. Let x and y be two distinct points in X.Since (X,τ) is pgprw-T_2- space, there exist disjoint T_{pgprw}-open sets U and V such that x∈U, and y∈V. This implies, x∈U, y∉U and x∈V, y∉V. Hence (X,τ) is pgprw-T_2- space.

Theorem 4.4.18: If (X,τ) is pgprw-T_2-space, T_{pgprw}- space and(Y,τ_y) is subspace of (X,τ), then (Y,τ_y) is also pgprw-T_2-space.

Proof: Let (X,τ), be a pgprw-T_2 - space and let Y be a subset of X. Let x and y beany two distinct points in Y. Since Y⊆X,x and y are also distinct points ofX. Since $(X,\tau)$is pgprw-T_2 - space, there exist disjoint T_{pgprw}-open sets G and Hwhich are also disjoint open sets, since (X,τ) is T_{pgprw} - space. So G∩Y andH∩Y are open sets and so T_{pgprw}- open sets in (Y,τ_y). Also x∈G, x ∈Yimplies x∈G∩V and y∈H and y∈Y this implies y∈Y∩H, since G∩H = ∅,we have (Y∩G)∩(Y∩H) = ∅. Thus G∩Y and H∩Y are disjointT_{pgprw}-open sets in Y such that x∈G∩Y, y∈H∩Y and (Y∩G)∩(Y∩ H)= ∅. Hence (Y,τ_y) is pgprw-T_2 - space.

Theorem 4.4.19: Let (X,τ), be a topological space. Then (X,τ),is pgprw-T_2-space if and only if the intersection of all T_{pgprw}-closed neighbourhood of each point of X is singleton.

Proof: Suppose (X,τ), is pgprw-T_2.space. Let x and y be any two distinct pointsof X. Since X is pgprw-T_2-space, there exist open sets G and H such that x∈G,y∈H and G∩H = ∅.Since G∩H =∅.implies x∈G⊆X-H. SoX-HisT_{pgprw}-closed neighbourhood of x, which does not contain y. Thus y does notbelong to the intersection of all T_{pgprw}-closed neighbourhood of x. Since y is arbitrary, the intersection of all T_{pgprw}-closed neighbourhoods of x is the singleton {x}.

Conversely, let (x) be the intersection of all T_{pgprw}-closedneighbourhoods of an arbitrary point x∈X. Let y be any point of Xdifferent from x. Since y does not belong to the intersection, there exists aT_{pgprw}-closed neighbourhood N of x such that y∉N. Since N isT_{pgprw}-neighbourhood of x, there exists an T_{pgprw}-open set G such x ∈G⊆X.Thus G and X - N are T_{pgprw}-open sets such that x⊆G, y∈X-N andG∩(X - N) = ∅. Hence (X,τ) is pgprw-T_2-space.

Theorem 4.4.20: Let f: (X,τ), -» (Y,τ_y) be a bijective pgprw-open map. If(X,τ) is pgprw-T_2- space and T_{pgprw} space, then $(Y,\tau_y)$is also pgprw-T_2- space.

Proof: Let (X,τ), is pgprw-T_2- space and T_{pgprw}- space. Let y_1 and y_2 be two distinct points of Y. Since f is bijective map, there exist distinct points x_1 and x_2 of Xsuch that $f(x_i) = y_j$ and $f(x_2) = y_2$. Since (X,τ) is pgprw-T_2- space, there exist pgprw-opensets G and H such that X_1∈G, X_2∈H and G∩H =∅. Since (X,τ) isT_{pgprw}- space, G and H are open sets, then f(G) and f(H) are pgprw- open sets of(Y,τ_y) , since f is pprw-open, such that y_1 = $f(x_1)$ ∈ f(G), y_2 = $f(x_2)$ ∈ f(H) andf(G)∩f(H) = ∅. Therefore we have f(G)∩f(H) = f(G∩H) =∅. Hence(Y,τ_y) is pgprwT_2-space.

Theorem 4.4.21: Let (X,τ) be a topological space and let (Y,τ_y) be aPgprw-T_2-space. Let f: (X,τ)—>(Y,τ_y) be an injective pgprw-irresolute map. Then(X,τ) is pgprw-T_2-space.

Proof: Let $\mathbf{X_1}$and X_2 be any two distinct points of X. Since f is injective,$x_1 \neq x_2$ implies $f(x_1) \neq f(x_2)$.Let $y_1 = f(x_1)$, $y_2 = f(x_2)$ so that $\mathbf{x_1} = f^{-1}(y_1)$,$x_2 = f^{-1}(y_2)$.Then $y_1,y_2 \in Y$ such that $y_1 \neq y_2$. Since (Y,τ_y) is Pgprw-T_2-space thereexist T_{pgprw}-open sets G and H such that $y_1 \in G$, $y_2 \in$ G and $G \cap H = \emptyset$. As f isT_{pgprw}-irresolute $f^{-1}(G)$ and f^{-1} (H) are T_{pgprw}-open sets of (X,τ). Now

$f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f^{-1}(\emptyset) = \emptyset$ and $y_1 \in G$ implies $f^{-1}(y_1) \in f^{-1}(G)$ implies$X_1 \in f^{-1}(G)$, $y_2 \in$ H implies $f^{-1}(y_2) \in f^{-1}(H)$ implies $x_2 \in f^{-1}(H)$.Thus forevery pair of distinct points x_1, x_2 of X there exist disjoint T_{pgprw}-open sets$f^{-1}(G)$ and $f^{-1}(H)$ such that $X_1 \in f^{-1}(G)$, $x_2 \in f^{-1}(H)$. Hence (X,τ) is pgprw-T_2-space.

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