International Journal of Computer Applications Technology and Research
Volume 7–Issue 03, 109-113, 2018, ISSN:-2319–8656

New Axioms in Topological Spaces

Vivekananda Dembre
Assistant Professor
Department of Mathematics,
Sanjay Ghodawat University,
Kolhapur,India

Abstract: In this paper, we study some separation axioms namely, w-To-space, w-T1 -space and w-T2-space and their properties. We also obtain some of their characterizations.

Keywords: W-TO-Space, W-T1 –Space, W-T2-Space.

1. INTRODUCTION

In the year 2000,Sheik John introduced and studied w-closed and w-open sets respectively. In this paper we define and study the properties of a new topological axioms called w-To-space, w-T1 –space, w-T2-space.

II.PRELIMINARIES

Throughout this paper space (X,τ) and (Y,σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c, P-Cl(A) and P-int(A) denote the Closure of A, Interior of A , Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

Definition 2.1: A subset A of a topological space (X, τ) is called

Definition 2.1: A subset A of a topological space (X, τ) is called

(i)A ωeakly closed set (briefly, ω-closed set) if Cl(A)⊆U whenever A⊆U and U is open in (X, τ).

(ii)A subset A of a topological space (X,τ) is called ωeakly open(briefly ω-open) set in X if A^c is ω-closed in X.

(iii)A topological space X is called a τ_w space if every w - closed set in it is closed.

Defintion 3: A map f:(X, τ) -» (Y, σ) is called

(i) W-continuous map [1] if f^{-1} (v) is w closed in (X,τ) for every closed V in (Y,σ).

(ii)W-irresolute map[1]if f^{-1}(v) is w closed in (X,τ)for every w-closed V in (Y,σ).

(iii)W-closed map[1] if f^{-1}(v) is w closed in (X,τ) for every closed V in (Y,σ).

(iv)W-open map[1] if f^{-1}(v) is w closed in (X,τ) for every closed V in (Y,σ).

4. W-To-SPACE:

Definition 4.4.1: A topological space (X, τ) is called w-T_o-space if for any pair of distinct points x,y of (X,τ) there exists an w-open set G such that x∈G, y∉G or x∉G, y∈G.

Example 4.4.2: Let X = {a, b}, τ ={φ,{b}, X}. Then (X, τ) is w-T_o-space, since for any pair of distinct points a, b of (X,τ) there exists an w-T_o open set {b} such that a ∉{b},b∈{b}.

Remark 4.4.3: Every w-space is w-T_o-space.

Theorem 4.4.4: Every subspace of a w-T_o-space is w-T_o-space.

Proof: Let (X,τ) be a w-T_o-space and (Y,τ_y) be a subspace of (X,τ). Let $Y_1$and Y_2 be two distinct points of (Y,τ_y). Since (Y,τ_y) is subspace of (X,τ),Y_1 and Y_2 are also distinct points of (X,τ). As (X,τ) is w-T_o-space , there exists an w-open set G such that Y_1∈G, Y_2 ∉ G. Then Y∩G is w-open in (Y,τ_y) containing but Y_1 not Y_2. Hence (Y,τ_y) is w-T_o-space.

Theorem 4.4.5: Let f: (X,τ) -» (Y,μ) be an injection, w-irresolute map. If (Y,μ) is w-T_o-space, then (X,τ) is w-T_o-space.

Proof: Suppose (Y,μ) is w-T_o-space. Let a and b be two distinct points in (X,τ).

As f is an injection f(a) and f(b) are distinct points in (Y,μ). Since(Y,μ) is w-T_o-space, there exists an w-open set G in (Y,μ) such that f(a)$\in$G and f(b)$\notin$G. As f is w-irresolute, $f^{-1}(G)$ is w-open set in (X,τ) such that a$\in f^{-1}(G)$ and b$\notin f^{-1}(G)$. Hence (X,τ) is w-T_o-space.

Theorem 4.4.6: If (X,τ) is w-T_o-space, Tw-space and (Y,τ_y) is w-closed subspace of (X,τ), then (Y,τ_y) is w-T_o -Space.

Proof: Let (X,τ) be w-T_o-space, Tw-space and (Y,τ_y) is w-closed subspace of (X,τ). Let a and b be two distinct points of Y. Since Y is subspace of (X,τ), a and b are distinct points of (X,τ). As $(X,\tau)$is w-T_o -space, there exists an w-open set G such that a$\in$G and b$\notin$G. Again since (X,τ) is Tw-space, G is open in (X,τ).Then Y$\cap$G is open. So Y$\cap$G is w-open such that a$\in$Y$\cap$G and b$\notin$Y$\cap$G. Hence (Y,τ_y) is W-T_o –space.

Theorem 4.4.7: Let f: (X,τ) -» (Y,μ) be bijective w-open map from a w-T_0 Space (X,τ) onto a topological space (Y,τ_y). If (X,τ) is T_w-space, then (Y,μ) is w-T_0 Space.

Proof: Let a and b be two distinct points of (Y,τ_y). Since f is bijective, there exist two distinct points e and d of (X,τ) such that f(c) = a and f(d) = b. As (X,τ) is w-T_0 Space,there exists a w-open set G such that c$\in$ G and d$\notin$G. Since (X,τ) is T_w-space, G is open in (X,τ). Then f(G) is w-open in (Y,μ), since f is w-open, such that a$\in$f(G) and b$\notin$f(G). Hence (Y,τ_y) is w-T_0-space.

Definition 4.4.8: A topological space (X,τ) is said to be w-T_1-space if for any pair of distinct points a and b of (X,τ) there exist w-open sets G and H such that a$\in$G, b$\notin$G and a$\notin$H, b$\in$H.

Example 4.4.9: Let X = {a,b} and τ = {$\emptyset$,{a},X}. Then (X,τ) is a topological space. Here a and b are two distinct points of (X,τ), then there exist w-open sets {a},{b} such that a$\in${a}, b$\notin${a} and a$\notin${b}, b$\in${b}. Therefore (X,τ) is w-T_0 space.

Theorem 4.4.10: If (X,τ) is w-T_1-space,then (X,τ) is w-T_o-space.

Proof: Let (X,τ) be a w-T_1-space. Let a and b be two distinct points of (X,τ). Since (X,τ) is w-T_1-space, there exist w-open sets G and H such that a$\in$G, b$\notin$G and a$\notin$H, b$\in$H. Hence we have a$\in$G, b$\notin$G. Therefore (X,τ) is w-T_o-space.

The converse of the above theorem need not be true as seen from the following example.

Example 4.4.11: Let X = {a,b} and τ ={φ,{b},X}. Then (X,τ) is w-T_o-space but not w-T_1-space. For any two distinct points a, b of X and an w-open set {b} such that a$\notin${b}, b$\in${b} but then there is no w-open set G with a$\in$G, b$\notin$G for a$\neq$b.

Theorem 4.4.12: If f: (X,τ) -» (Y,τ_y) is a bijective w-open map from a w-T_1-space and T_w-space (X,τ) on to a topological space (Y,τ_y), then$(Y,\tau_y)$is w-T_1-space.

Proof: Let (X,τ) be a w-T_1-space and T_w-space. Let a and b be two distinct points of (Y,τ_y). Since f is bijective there exist distinct points c and d of (X,τ) such that f(c) = a and f(d) = b. Since (X,τ) is w-T_1-space there exist w-open sets G and H such that c$\in$G, d$\notin$G and c$\notin$H, d $\in$H. Since (X,τ) is T_w-space, G and H are open sets in (X,τ) also f is w-open f(G) and f(H) are w-open sets such that a = f (c)$\in$f(G), b = f(d)$\notin$f(G) and a= f(c)$\notin$ f(H), b= f(d)$\in$f(H). Hence (Y,τ_y) is w-T_1-space.

Theorem 4.4.13: If (X,τ) is wT_1 space and T_w-space, Y is a subspace of (X,τ), then Y is w-T_1 space.

Proof: Let (X,τ) be a w T_1 space and T_w-space. Let Y be a subspace of (X,τ). Let a and b be two distract points of Y. Since $Y \subseteq X$, a and b are also distinct points of X. Since (X,τ) is w-T_1-space, there exist w-open sets G and H such that $a \in G$, $b \notin G$ and $a \notin H$, $b \in H$. Again since (X,τ) is T_w-space,G and H are open sets in (X,τ), then $Y \cap G$ and $Y \cap H$ are open sets so w-open sets of Y such that $a \in Y \cap G$, $b \notin Y \cap G$ and $a \notin Y \cap H$, $b \in Y \cap H$. Hence Y is w T_1 space.

Theorem 4.4.14: Iff: (X,τ) -» (Y,τ_y) is injective w-irresolute map from a topological space (X,τ) into w-T_1-space(Y,τ_y), then $(X,\tau)$is w-T_1 - space.

Proof: Let a and b be two distinct points of (X,τ). Since f is injective, f(a) and f(b) are distinct points of (Y,τ_y). Since(Y,τ_y) is w-T_1 space there exist w-open sets G and H such that $f(a) \in G$, $f(b) \notin G$ and $f(a) \notin H$, $f(b) \in H$.Since f is w-irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are w-open sets in (X,τ) such that $a \in f^{-1}(G)$, $b \notin f^{-1}(G)$ and $a \notin f^{1}(H)$, $b \in f^{-1}(H)$. Hence (X,τ) is w-T_1 space.

Definition 4.4.15: A topological space (X,τ). is said to be w-T_2- space (or T_w-Hausdorff space) if for every pair of distinct points x, y of X there exist T_w-open sets M and N such that $x \in N$, $y \in M$ and $N \cap M = \emptyset$.

Example 4.4.16: Let X = {a,b}, $\tau = \{\emptyset, \{a\}, \{b\}, X\}$. Then (X,τ) is topological space. Then (X,τ) is w-T_2-space. T_w-open sets are $\emptyset$, {a}, {b},and X. Let a and b be a pair of distinct points of X, then there exist T_w - open sets {a} and {b} such that $a \in \{a\}$, $b \in \{b\}$ and $\{a\} \cap \{b\} = \emptyset$. Hence (X,τ) is w-T_2-space.

Theorem 4.4.17: Every w-T_2- space is w T_1space.

Proof: Let (X,τ) be a w-T_2- space. Let x and y be two distinct points in X. Since (X,τ) is w-T_2- space, there exist disjoint T_w-open sets U and V such that $x \in U$, and $y \in V$. This implies, $x \in U$, $y \notin U$ and $x \in V$, $y \notin V$. Hence (X,τ) is w-T_2-space.

Theorem 4.4.18: If (X,τ) is w-T_2-space, T_w- space and (Y,τ_y) is subspace of (X,τ), then (Y,τ_y) is also w-T_2-space.

Proof: Let (X,τ), be a w-T_2 - space and let Y be a subset of X. Let x and y be any two distinct points in Y. Since $Y \subseteq X$, x and y are also distinct points of X. Since $(X,\tau)$is w-T_2 - space, there exist disjoint T_w-open sets G and H which are also disjoint open sets, since (X,τ) is T_w - space. So $G \cap Y$ and $H \cap Y$ are open sets and so T_w- open sets in (Y,τ_y). Also $x \in G$, $x \in Y$ implies $x \in G \cap V$ and $y \in H$ and $y \in Y$ this implies $y \in Y \cap H$, since $G \cap H = \emptyset$, we have $(Y \cap G) \cap (Y \cap H) = \emptyset$. Thus $G \cap Y$ and $H \cap Y$ are disjoint T_w-open sets in Y such that $x \in G \cap Y$, $y \in H \cap Y$ and $(Y \cap G) \cap (Y \cap H) = \emptyset$. Hence (Y,τ_y) is w-T_2 - space.

Theorem 4.4.19: Let (X,τ), be a topological space. Then (X,τ),is w-T_2-space if and only if the intersection of all T_w-closed neighbourhood of each point of X is singleton.

Proof: Suppose (X,τ), is w-T_2-space. Let x and y be any two distinct points of X. Since X is w-T_2-space, there exist open sets G and H such that $x \in G$, $y \in H$ and $G \cap H = \emptyset$.Since $G \cap H = \emptyset$ implies $x \in G \subseteq X-H$. SoX-H is T_w-closed neighbourhood of x, which does not contain y. Thus y does not belong to the intersection of all T_w-closed neighbourhood of x. Since y is arbitrary, the intersection of all T_w-closed neighbourhoods of x is the singleton {x}.

Conversely, let (x) be the intersection of all T_w-closed neighbourhoods of an arbitrary point $x \in X$. Let y be any point of X different from x. Since y does not belong to the intersection, there exists a T_w-closed neighbourhood N of x such that $y \notin N$. Since N is T_w-neighbourhood of x, there exists an T_w-open set G such $x \in G \subseteq X$. Thus G and X - N are T_w-open sets such that $x \subseteq G$, $y \in X-N$ and $G \cap (X - N) = \emptyset$. Hence (X,τ) is w-T_2-space.

Theorem 4.4.20: Let f: (X,τ) -» (Y,τ_y) be a bijective w-open map. If (X,τ) is w-T_2- space and T_w space, then $(Y,\tau_y)$is also w-T_2- space.

Proof: Let (X,τ), is w-T_2- space and T_w- space. Let y_1 and y_2 be two distinct points of Y. Since f is bijective map, there exist distinct points x_1 and x_2 of X such that $f(x_i) = y_j$ and $f(x_2) = y_2$. Since (X,τ) is w-T_2- space,there exist w-open sets G and H such that $X_1 \in G$, $X_2 \in H$ and $G \cap H = \emptyset$. Since (X,τ) is T_w-space, G and H are open sets, then f(G) and f(H) are w- open sets of (Y,τ_y) , since f is ppw-open, such that $y_1 = f(x_1) \in f(G)$, $y_2 = f(x_2) \in f(H)$ and $f(G) \cap f(H) = \emptyset$. Therefore we have $f(G) \cap f(H) = f(G \cap H) = \emptyset$. Hence (Y,τ_y) is wT_2-space.

Theorem 4.4.21: Let (X,τ) be a topological space and let (Y,τ_y) be a W-T_2-space. Let f: (X,τ) —> (Y,τ_y) be an injective w-irresolute map. Then (X,τ) is w-T_2-space.

Proof: Let x_1 and x_2 be any two distinct points of X. Since f is injective, $x_1 \neq x_2$ implies $f(x_1) \neq f(x_2)$. Let $y_1 = f(x_1)$, $y_2 = f(x_2)$ so that $x_1 = f^{-1}(y_1)$, $x_2 = f^{-1}(y_2)$.Then $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since (Y,τ_y) is W-T_2-space there exist T_w-open sets G and H such that $y_1 \in G$, $y_2 \in$ G and $G \cap H = \emptyset$. As f is T_w-irresolute $f^{-1}(G)$ and $f^{-1}(H)$ are T_w-open sets of (X,τ).
Now $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f^{-1}(\emptyset) = \emptyset$ and $y_1 \in G$ implies $f^{-1}(y_1) \in f^{-1}(G)$ implies $x_1 \in f^{-1}(G)$, $y_2 \in H$ implies $f^{-1}(y_2) \in f^{-1}(H)$ implies $x_2 \in f^{-1}(H)$.Thus for every pair of distinct points x_1, x_2 of X there exist disjoint T_w-open sets $f^{-1}(G)$ and $f^{-1}(H)$ such that
$x_1 \in f^{-1}(G)$, $x_2 \in f^{-1}(H)$. Hence (X,τ) is w-T_2-space.

REFERENCES:

[1] M.Sheik john, a study on g-closed sets on continuous maps in topological and bitopological spaces ph.d Thesis Bharathiar University Coimbatote (2002).

[2]R.S.Wali and Vivekananda Dembre, " Minimal weakly open sets and maximal weakly closed sets in topological spaces"; International Journal of Mathematical Archieve; Vol-4(9)-Sept-2014.

[3] R.S.Wali and Vivekananda Dembre, "Minimal weakly closed sets and Maximal weakly open sets in topological spaces" ; International Research Journal of Pure Algebra; Vol-4(9)-Sept-2014.

[4] R.S.Wali and Vivekananda Dembre, " on semi-minimal open and semi-maximal closed sets in topological spaces ";

Journal of Computer and Mathematical Science;Vol-5(9)-0ct-2014 (International Journal).

[5] R.S.Wali and Vivekananda Dembre, "on pre generalized pre regular weakly closed sets in topological spaces "; Journal of Computer and Mathematical Science;Vol-6(2)-Feb-2015 (International Journal)

[6]R.S.Wali and Vivekananda Dembre, " on pre genearalized pre regular open sets and pre regular weakly neighbourhoods in topological spaces"; Annals of Pure and Applied Mathematics" ; Vol-10- 12 2015.

[7]R.S.Wali and Vivekananda Dembre, "on pre generalized pre regular weakly interior and pre generalized pre regular weakly closure in topological spaces", International Journal of Pure Algebra- 6(2),2016,255-259.

[8]R.S.Wali and Vivekananda Dembre , "on pre generalized pre regular weakly continuous maps in topological spaces", Bulletin of Mathematics and Statistics Research Vol.4.Issue.1.2016 (January-March).

[9]R.S.Wali andVivekananda Dembre,on Pre-generalized pre regular weakly irresolute and strongly pgprw-continuous maps in topological spaces, Asian Journal of current Engineering and Maths 5;2 March-April (2016)44-46.

[10]R.S.Wali and Vivekananda Dembre,On Pgprw-locally closed sets in topological spaces,
International Jounal of Mathematical Archive-7(3),2016,119-123.

[11]R.S.Wali and Vivekananda Dembre,(τ_1,τ_2) pgprw-closed sets and open sets in Bitopological spaces,International Journal of Applied Research 2016;2(5);636-642.

[12]R.S.Wali and Vivekananda Dembre,Fuzzy pgprw-continuous maps and fuzzy pgprw-irresolute in fuzzy topological spaces; International Journal of Statistics and Applied Mathematics 2016;1(1):01-04.

[13]R.S.Wali and Vivekananda Dembre,On pgprw-closed maps and pgprw-open maps in Topological spaces;International Journal of Statistics and Applied Mathematics 2016;1(1);01-04.

[14]Vivekananda Dembre, Minimal weakly homeomorphism and Maximal weakly homeomorphism in topological spaces, Bulletin of the Marathons Mathematical Society,Vol. 16, No. 2, December 2015, Pages 1-7.

[15]Vivekananda Dembre and Jeetendra Gurjar, On semi-maximal weakly open and semi-minimal weakly closed sets in topological spaces, International Research Journal of Pure Algebra-Vol-4(10), Oct – 2014.

[16]Vivekananda Dembre and Jeetendra Gurjar, minimal weakly open map and maximal weakly open maps in topological spaces, International Research Journal of Pure Algebra-Vol.-4(10), Oct – 2014; 603-606.

[17]Vivekananda Dembre ,Manjunath Gowda and Jeetendra Gurjar, minimal weakly and maximal weakly continuous functions in topological spaces, International Research Journal of Pure Algebra-vol.-4(11), Nov– 2014.

[18]Arun kumar Gali and Vivekananda Dembre, mınımal weakly generalızed closed sets and maxımal weakly generalızed open sets in topologıcal spaces, Journal of Computer and Mathematical sciences,Vol.6(6),328-335, June 2015.

[19]R.S.Wali and Vivekananda Dembre; Fuzzy Pgprw-Closed Sets and Fuzzy Pgprw-Open Sets in Fuzzy Topological SpacesVolume 3, No. 3, March 2016; Journal of Global Research in Mathematical Archives.

[20]Vivekananda Dembre and Sandeep.N.Patil; On Contra Pre Generalized Pre Regular Weakly Continuous Functions in Topological Spaces; IJSART - Volume 3 Issue 12 – DECEMBER 2017

[21]Vivekananda Dembre and Sandeep.N.Patil ; On Pre Generalized Pre Regular Weakly Homeomorphism in Topological Spaces; Journal of Computer and Mathematical Sciences, Vol.9(1), 1-5 January 2018.

[22]Vivekananda Dembre and Sandeep.N.Patil ;on pre generalized pre regular weakly topological spaces; Journal of Global Research in Mathematical Archives volume 5, No.1, January 2018.

[23]Vivekananda Dembre and Sandeep.N.Patil ; Fuzzy Pre Generalized Pre Regular Weakly Homeomorphism in Fuzzy Topological Spaces ;International Journal of Computer Applications Technology and Research Volume 7–Issue 02, 28-34, 2018, ISSN:-2319–8656.

[24]Vivekananda Dembre and Sandeep.N.Patil; PGPRW-Locally Closed Continuous Maps in Topological Spaces; International Journal of Trend in Research and Development, Volume 5(1), January 2018.

[25]Vivekananda Dembre and Sandeep.N.Patil ; Rw-Separation Axioms in Topological Spaces; International Journal of Engineering Sciences & Research Technology; Volume 7(1): January, 2018.

[26]Vivekananda Dembre and Sandeep.N.Patil ; Fuzzy pgprw-open maps and fuzzy pgprw-closed maps in fuzzy topological spaces; International Research Journal of Pure Algebra-8(1), 2018, 7-12.

[27]Vivekananda Dembre and Sandeep.N.Patil ; Pgprw-Submaximal spaces in topological spaces ; International Journal of applied research 2018; Volume 4(2): 01-02.