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PGPRW-Locally Closed Continuous Maps in Topological Spaces

[1]Vivekananda Dembre and [2]Sandeep.N Patil,
[1]Assistant Professor, Department of Mathematics, Sanjay Ghodawat University, Kolhapur, India
[2]Assistant Professor, Department of Civil Engineering, Sanjay Ghodawat Polytechnic, Kolhapur, India

Abstract: In this paper,we introduce PGPRW-LC-continuous maps, PGPRW-LC*-continuous maps,PGPRW-LC**-continuous maps function, which are weaker than LC-continuous maps and obtain some of their properties.

Keywords:*pgprw-locally closed sets,Pgprw-locally closed continuous maps,pgprw-closed sets,pgprw-open sets.*

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I. INTRODUCTİON

Sheik john[1] introduced three new classes of sets denoted byW-LC set, W-LC*set &W-LC set** each of which contains LC(X,T) also various authors like park [2],Balachandran [3], and Veera kumar[4] have contributed to the development of generalization of locally closed sets and locally continuous maps maps in topological spaces.

II. PRELİMİNARİES

A subset A of t.s (X,T) is called a

(i) a pre generalized pre regular ωeakly closed set(briefly pgprω-closed set) if pCl(A)$\subseteq$U whenever A$\subseteq$U and[5] U is rgα open in (X,τ).

(ii) pre generalized pre regular ωeakly open set [6]in X if A^c is pgprω-closed in X.

(iii)PGPRW-LC set if A=U∩F[7]where U ispgprw-open and F is pgprw-closed in X.

(iv)PGPRW-LC*set if A=U∩F[7]where U ispgprw-open and F isclosed in X.

(v)PGPRW-LC** set if if A=U∩F[7]where U isopen and F is pgprw-closed in X.

(vi)W-LC set if A=U∩F[1]where U isw-open and F is w-closed in X.

(vii) W-LC* setif A=U∩F[1]where U isw-open and F isclosed in X.

(viii)W-LC set** if if A=U∩F[1]where U isopen and F is w-closed in X.

(ix) door space[9] if every subset of X is either open or closed in (X,T).

(x) Submaximal space[8] if every dense subset of X is open in X.

(xi) pgprw-submaximal space[10] if every dense subset of X is pgprw-open in X.

(xii)Regular openset [13] if A = int(cl(A)) and a regular closed set if A = cl(int(A)).

Defıntıon 2.1 A map f : (X,T) $\longrightarrow$(Y,T) is called

(i) LC-continuous maps[12] if f^{-1}(V) is a lc-set in (X,T) for each V$\epsilon\sigma$.

(ii) W-LC continuous maps if [1]f^{-1}(V) is a w-lc-set in (X,T) for each V$\epsilon\sigma$.

Theorem 2.2 [7]:

(i)Every locally closed set is a PGPRW-LC set but not conversely.

(ii) Every PGPRW-LC* set is PGPRW-LC set.

(iii) Every PGPRW-LC** set is PGPRW-LC set.

III. PGPRW-LOCALLY CLOSED CONTİNUOUS MAPS İN TOPOLOGİCAL SPACES

Definition 3.1:A function f : (X,T)$\longrightarrow$(Y,σ) is called PGPRW-LC continuous maps

(resp. PGPRW-LC* continuous maps, PGPRW-LC** -continuous maps) if f^{-1}(V)ϵPGPRW-LC(X,T)(resp.f^{-1}(V) ϵPGPRW-LC*(X,T),f^{-1}(V) ϵPGPRW-LC**(X,T) for each V$\epsilon\sigma$.

Example 3.2: Let X=Y={a,b,c}, T= {X,∅,{a}}and ,σ= {Y,∅,{a},{a,b}}then PGPRW-LC(X,T)= P(X) and the identity mapf : (X,T)$\longrightarrow$(Y,T) is PGPRW-LCcontinuous maps.

Theorem 3.3: Let f: (X,T)$\longrightarrow$ (Y,σ) be a function. Then we have the following.

(i) If f is locally continuous maps then it is PGPRW-LCcontinuous maps.
(ii) If f is PGPRW-LC* continuous maps then it is PGPRW-LCcontinuous maps.
(iii) If f is PGPRW-LC** continuous maps then it is PGPRW-LCcontinuous maps.

Proof: The proof follows from the definitions and theorem [2.2]However the converse of the above results are not true as seen by the following example.

Example 3.4: Let X=Y={a,b,c} T= {X,∅,{a},{a,b}}and σ = { Y,∅,{a},{b},{a,b}}.

f: (X,T)⟶ (Y,σ) be the identity map then f is PGPRW-LC continuous maps but not locally continuous maps, since for the the open set {b} in (Y,σ) f^{-1}(b) = b is not locally closed in (X,T).

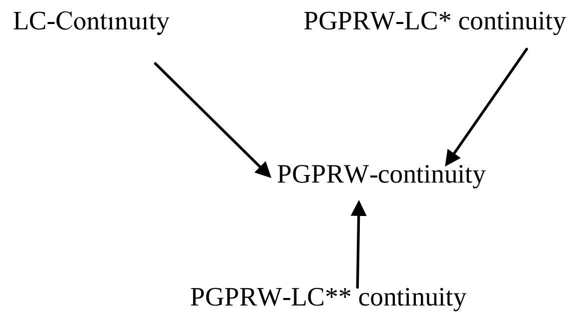
Example 3.5: Let X=Y={a,b,c} T= {X,∅,{a}}and σ = { Y,∅,{a},{b},{a,b}}.

f: (X,T)⟶(Y,σ) be the identity map then f is PGPRW-LC-continuous maps but not PGPRW-LC*continuous maps, since for the the open set {b} in (Y,σ) f^{-1}(b) = b is not locally closed in (X,T).

Example 3.6: Let X=Y={a,b,c} T= {X,∅,{a}}and σ = { Y,∅,{a},{b},{a,b}}.

f: (X,T)⟶(Y,σ) be the identity map then f is pgprw-continuous maps but not PGPRW-LC**continuous maps, since for the the open {a,b} of (Y,σ) f^{-1}(a,b) = {a,b}which is not a PGPRW-LC** set in (X,T).

Remark3.7: From the above information we have the following observation.



Theorem 3.8: Any map defined on a door space is pgprw-continuous maps (resp.PGPRW-LC* continuous maps,PGPRW**-continuous maps).

Proof: Let f : (X,T)⟶(Y,σ) be a function, where (X,T) be a door space and (Y,σ) be a door space and (Y,σ) be any space. Let Aϵ σ then by the assumption on (X,T),f^{-1}(A) is either open or closed. In the both the cases f^{-1}(A)ϵPGPRW-LC (X,T)(resp.f^{-1}(A)ϵPGPRW-LC* (X,T), f^{-1}(A)ϵPGPRW-LC** (X,T) and therefore f is pgprw-continuous maps (resp.PGPRW-LC* continuous maps,Pgprw**-continuous maps).

Theorem 3.9:A topological space (X,T) is pgprw-submaximal iff every function having (X,T) as domain is PGPRW-LC* continuous maps.

Proof: Let f: (X,T)⟶(Y,σ) be a function and (X,T) be pgprw-submaximal then w.k.t . A t.s (X,T); P(X) = PGPRW-LC*(X,T). If U is any open set of (Y,σ), f^{-1}(U)ϵP(X) = PGPRW-LC*(X,T) and so, f is PGPRW-LC* continuous maps.

Conversely, assume that every function having (X,T) as domain be PGPRW-LC* continuous maps. Consider the sierpinski space Y= {0,1} with σ = { Y,∅,{0}}. Let U be a subset of (X,T) and define f: (X,T)⟶(Y,σ) by f(X)=0 for every xϵu and f(x)=1 for every x does not belong to u by assumption f^{-1}{0}= U ϵPGPRW-LC*(X,T). Therefore we have P(X)= PGPRW-LC*(X,T)and so (X,T)is pgprw-submaximal and w.k.t A t.s (X,T) is pgprw-submaximal iff P(X)= PGPRW-LC*(X,T).

Theorem 3.10: If f : (X,T)⟶(Y, σ) is PGPRW-LC* continuous maps and a subset B is both open and closed in (X,T) then restiction f_B : (B, T_B)⟶(Y,σ) is PGPRW-LC* continuous maps.

Proof : Let G be an open set of (Y, σ)by hypothesis f^{-1}(G) is a PGPRW-LC* set in (X,T) then

f^{-1}(G) = U ∩ F, for some pgprw-open set U and closed set F of (X,T);

then f_B^{-1}(G)= B∩f^{-1}(G)= B ∩U ∩ F = (B ∩U) ∩ (B∩ F). SinceB∩ F is closed in (X,T)

and B∩ F⊆ B, B∩ Fis closed in (B, T_B) ,Since B ∩U is pgprw-open in (X,T),B ∩U⊆ B

and B is regular open in (X,T),B ∩U is pgprw-open in (B, T_B) .This shows that f_B^{-1}(G) ϵPGPRW-LC* (B, T_B)and hence f_B is PGPRW-LC* continuous maps.

Theorem 3.11: Let f : (X,T)⟶(Y, σ) be PGPRW-LCcontinuous maps and B an open set in Y containing f(X) .Then f: (X,T) (B,T_B) is PGPRW-LCcontinuous maps.

Proof: Let V be an open set in B . Then V is open in (Y, σ) sinceB is open in (Y, σ) . Therefore By hypothesis , f^{-1} (V) is a PGPRW-LC set in (X,T) that is f: (X,T)⟶(B,T_B) is pgprw-continuous maps.

Example 3.12: Composition of two PGPRW-LCcontinuous maps maps not be PGPRW-LCcontinuous maps as seen from the following example.

Example 3.13: Let X=Y=Z={a,b,c} and τ ={X,∅,{a},{b,c}} and σ = {Y, ∅,{a}}, μ = { Z,∅,{a},{b},{a,b},{a,c}} and a maps f: X Y, g: Y⟶Z be identity mapthen both f and g are PGPRW-LCcontinuous maps but their composition

gof: (X,T)⟶(Z,μ) is not PGPRW-LC-continuous maps, since for open set {a,b} of (Z, μ) $(gof)^{-1}(a,b)= f^{-1}(g^{-1}(a,b))= f^{-1}((a,b))$={a,b}, which is not a PGPRW-LC set in (X,T).

Theorem 3.14: If f : (X,T)⟶(Y, σ) is PGPRW-LC-continuous maps (resp. PGPRW-LC* continuous maps,PGPRW-LC** continuous maps) and g: (Y,T)⟶(Z,,μ) is continuous maps then gof: (X,T)⟶(Z,μ) is PGPRW-LCcontinuous maps (resp. PGPRW-LC*-continuous maps,PGPRW-LC**-continuous maps).

Proof: Let U be an open set in (Z,μ) since g is continuous maps $g^{-1}(U)$ is open in (Y, σ).since f is pgprw-continuous maps, $f^{-1}(g^{-1}(u))= (gof)^{-1}(u)$= is a PGPRW-LC set in (X,T) and hence gof :(X,T) (Z,μ) is PGPRW-LCcontinuous maps.

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