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Vivekananda Dembre
 Assistant Professor,
 Department of Mathematics,
 Sanjay Ghodawat University,
 Kolhapur, Maharashtra, India

Patil Sandeep N
 Assistant Professor,
 Department of Civil
 Engineering, Sanjay Ghodawat
 Polytechnic, Kolhapur,
 Maharashtra, India

Pgprw- Submaximal spaces in topological spaces

Vivekananda Dembre and Patil Sandeep N

Abstract

In this paper, we define pgprw-submaximal spaces and obtain some of their properties.

Keywords: Pgprw-submaximal spaces, Pgprw closed set and Pgprw-open set

1. Introduction

In the year 1995, J.Dontchev^[3,4], defined on door spaces and submaximal spaces. and in this paper we define pgprw-submaximal spaces and obtain some of their properties.

2. Preliminaries: A subset A of $t.s (X,T)$ is called a

- (i) pre generalized pre regular weakly closed set (briefly pgprw-closed set) if $pCl(A) \subseteq U$ whenever $A \subseteq U$ and U is rg α open in (X, τ) .
- (ii) pre generalized pre regular weakly open set^[2] in X if A^c is pgprw-closed in X .
- (iii) Submaximal spaces^[3] if every dense subset of X is open in X .
- (iv) door space^[4] if every subset of X is either open or closed in X .
- (v) g-Submaximal space^[5] if every dense subset of X is open in X .
- (vi) w-Submaximal space^[6] if every dense subset of X is g-open in X .
- (vii) rg-submaximal space^[7] if every dense subset of X is rg-open in X .

2.1 Theorem^[8]: For a subset A of (X,T) if $A \in PGPRW-LC^{**}(X,T)$, then there exists an open set U s.t $A = U \cap pgprw-cl(A)$.

2.2 Theorem:^[8] For a subset A of (X,T) , the following are equivalent.

- (i) $A \in PGPRW-LC^*(X,T)$.
- (ii) $A = U \cap (p-cl(A))$ for some pgprw-open set U .
- (iii) $pCl(A) - A$ is pgprw-closed.
- (iv) $A \cup (p-cl(A))^c$ is pgprw-open.

3. Pgprw- submaximal spaces in topological spaces.

Definition 3.1: A topological space (X,T) is called pgprw-submaximal if every dense subset is pgprw-open.

Theorem 3.2: A $t.s (X,T)$ is pgprw-submaximal iff $p(X) = PGPRW-LC^*(X,T)$

Proof: Let (X,T) be pgprw-submaximal. $A \in P(X)$ and $V = A \cup (X - p-cl(A))^c$ then
 $Cl(V) = cl (A \cup (X - p-cl(A))^c)$
 $= cl (A) \cup (X - p-cl(A))$
 $= X$.

That is $cl(V) = X$. It follows that V is dense in (X,T) by assumption, V is pgprw-open by thm 2.1^[8], $A \in PGPRW-LC^*(X,T)$ Therefore $p(X) = PGPRW-LC^*(X,T)$.

Conversely, Let A be dense in (X,T) and $p(X) = PGPRW-LC^*(X,T)$ then $A = A \cup (X - p-cl(A))$ Since $A \in PGPRW-LC^*(X,T)$. $A = A \cup (X - p-cl(A))$ is pgprw-open by theorem 2.2; hence (X,T) is pgprw-submaximal.

Theorem 3.3: Every submaximal space is pgprw-submaximal but not conversely.

Correspondence

Vivekananda Dembre
 Assistant Professor,
 Department of Mathematics,
 Sanjay Ghodawat University,
 Kolhapur, Maharashtra, India

Proof: Let (X, T) be a submaximal space and A be a dense subset of (X, T) then A is open but every open set is pgprw-open and so A is pgprw-open. Therefore (X, T) is a pgprw-submaximal space.

The Converse of the above theorem need not be true in general as seen from the following example.

Example: Let $X = \{a, b, c\}$ with the topology $T = \{X, \emptyset, \{a\}, \{b, c\}\}$ however the set $A = \{a, b\}$ is dense in (X, T) . But it is not open in (X, T) therefore (X, T) is not submaximal.

Remark 3.4: Pgprw-submaximal and g-submaximal spaces are independent of each other.

Example: Let $X = \{a, b, c\}$ $T = \{X, \emptyset, \{a\}, \{a, b\}\}$, in this space (X, T) every dense subset is pgprw-open and hence (X, T) is pgprw-submaximal. However the set $A = \{a, c\}$ is dense in (X, T) but it is not g open in (X, T) therefore (X, T) is not g-submaximal.

Example: Let $X = \{a, b, c\}$ $T = \{X, \emptyset, \{a\}, \{b, c\}\}$, in this space (X, T) every dense subset is g-open and hence (X, T) is g-submaximal. However the set $A = \{a, b\}$ is dense in (X, T) but it is not pgprw-open in (X, T) therefore (X, T) is not pgprw-submaximal.

Remark 3.5: Pgprw-submaximal and w-submaximal spaces are independent of each other.

Example: Let $X = \{a, b, c\}$, $T = \{X, \emptyset, \{a\}\}$, in this space (X, T) every dense subset is pgprw-open and hence (X, T) is pgprw-submaximal. However the set $A = \{a, c\}$ is dense in (X, T) but it is not w-open in (X, T) therefore (X, T) is not w-submaximal.

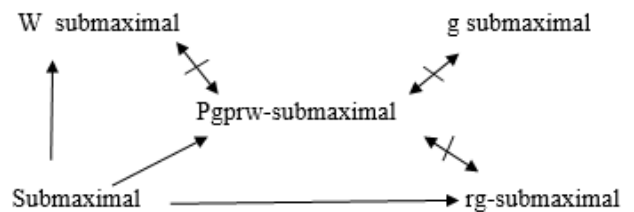
Example: Let $X = \{a, b, c\}$, $T = \{X, \emptyset, \{a\}, \{b, c\}\}$, in this space (X, T) every dense subset is w-open and hence (X, T) is w-submaximal. However the set $A = \{a, b\}$ is dense in (X, T) but it is not pgprw-open in (X, T) therefore (X, T) is not pgprw-submaximal.

Remark 3.6: Pgprw-submaximal and rg-submaximal spaces are independent of each other

Example: Let $X = \{a, b, c, d\}$ $T = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}\}$, in this space (X, T) every dense subset is pgprw-open and hence (X, T) is pgprw-submaximal. However the set $A = \{a, b, d\}$ is dense in (X, T) but it is not rg-open in (X, T) . Therefore (X, T) is not rg-submaximal.

Example: Let $X = \{a, b, c\}$ $T = \{X, \emptyset, \{a\}, \{b, c\}\}$, in this space (X, T) every dense subset is rg-open and hence (X, T) is rg-submaximal. However the set $A = \{a, b\}$ is dense in (X, T) but it is not pgprw-open in (X, T) . Therefore (X, T) is not pgprw-submaximal.

Remark 3.7: From the above discussions and known results we have the following results.



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