International Journal of Applied Research 2018; 4(2): 99-102

International Journal of Applied Research

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2018; 4(2): 99-102
www.allresearchjournal.com
Received: 20-12-2017
Accepted: 21-01-2018

**Vivekananda Dembre**
Assistant Professor,
Department of Mathematics,
Sanjay Ghodawat University,
Kolhapur, Maharashtra, India

**Pravin G Dhawale**
Assistant Professor,
Department of Electrical
Engineering Sanjay Ghodawat
University, Kolhapur,
Maharashtra, India

**Devendra Gowda**
Assistant Professor,
Department of Electrical
Engineering Sanjay Ghodawat
University, Kolhapur,
Maharashtra, India

**Correspondence**
**Vivekananda Dembre**
Assistant Professor,
Department of Mathematics,
Sanjay Ghodawat University,
Kolhapur, Maharashtra, India

# New semi generalized spaces in topological spaces

**Vivekananda Dembre, Pravin G Dhawale and Devendra Gowda**

**Abstract**
In this paper, we introduce and investigate topological spaces called Semi generalized-compactness spaces and Semi generalized-connectedness space and we get several characterizations and some of their properties. Also we investigate its relationship with other types of functions.

**Keywords:** Semi generalized-open set, semi generalized-closed sets, semi generalized-compact spaces, semi generalized-connectedness

## 1. Introduction
Many researchers have investigated the basic properties of compactness and connectedness. The productivity and fruitfulness of these notions of compactness and connectedness motivated mathematicians to generalize these notions. In the course of these attempts many stronger and weaker forms of compactness and connectedness have been introduced and investigated.
The aim of this paper is to introduce the concept of Semi generalized-compactness and Semi generalized-connectedness in topological spaces and is to give some characterizations of Semi generalized-compact spaces .

## 2. Preliminary Notes
Throughout this paper $(X, \tau)$, $(Y, \sigma)$ are topological spaces with no separation axioms assumed unless otherwise stated. Let $A \subseteq X$. The closure of A and the interior of A will be denoted by Cl(A) and Int(A) respectively.

**Definition 2.1:** A subset A of X is said to be b-open [1] if $A \subseteq Int(Cl(A)) \cup Cl(Int(A))$. The complement of b-open set is said to be b-closed. The family of all b-open sets (respectively b-closed sets) of $(X, \tau)$ is denoted by $bO(X, \tau)$ [respectively $bCL(X, \tau)$].

**Definition 2.2:** Let A be a subset of X. Then A is said to be Semi generalized-closed
if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in RSO(X, \tau)$. The complement of Semi generalized-closed set is called Semi generalized-open. The family of all Semi generalized-open [respectively Semi generalized-closed] sets of $(X, \tau)$ is denoted by $SGO(X, \tau)$ [respectively, $SGU\text{-}CL(X, \tau)$].

**Definition 2.3:** The Semi generalized-closure [35] of a set A, denoted by SG-Cl(A), is the intersection of all Semi generalized-closed sets containing A.

**Definition 2.4:** The Semi generalized-interior [35] of a set A, denoted by Semi generalized-Int(A), is the union of all Semi generalized-open sets contained in A.

**Remark 2.6:** Every closed set is Semi generalized-closed.

## 3. Compactness IN Semi Generalized Closed Sets
**Definition 3.1:** A collection $\{A_i : i \in \wedge\}$ of Semi generalized-open sets in a topological space X is called a Semi generalized-open cover of a subset B of X if $B \subset \{A_i : i \in \wedge\}$ holds.

**Definition 3.2:** A topological space X is Semi generalized-compact if every Semi generalized-open cover of X has a finite sub-cover.

**Definition 3.3:** A subset B of a topological space X is said to be Semi generalized-compact relative to X if, for every collection $\{A_i : i \in \wedge\}$ of Semi generalized-open subsets of X such that $B \subset U \{A_i : i \in \wedge\}$ there exists a finite subset $\wedge_0$ of $\wedge$ such that $B \subseteq U \{A_i : i \in \wedge_0\}$.

**Definition 3.4:** A subset B of a topological space X is said to be Semi generalized-compact if B is Semi generalized-compact as a subspace of X.

**Theorem 3.5:** Every Semi generalized-closed subset of a Semi generalized-compact space is Semi generalized-compact relative to X.

**Proof:** Let A be Semi generalized-closed subset of Semi generalized-compact space X. Then $A^C$ is Semi generalized-open in X. Let $M = \{G_\alpha : \alpha \in \wedge\}$ be a cover of A by semi generalized-open sets in X. Then $M* = M \cup A^c$ is a semi generalized-open cover of X. Since X is semi generalized-compact $M*$ is reducible to a finite subcover of X, say $X = G\alpha_1 \cup G\alpha_2 \cup \cdot \cdot \cdot \cup G\alpha_m \cup A^C$, $G\alpha_k \in M$. But A and $A^C$ are disjoint hence $A \subset G\alpha_1 \cup \cdot \cdot \cdot \cup G\alpha_m$, $G\alpha_k \in M$, which implies that any semi generalized-open cover M of A contains a finite sub-cover. Therefore A is semi generalized-compact relative to X. Thus every semi generalized-closed subset of a semi generalized-compact space x is semi generalized-compact.

**Definition 3.6:** A function $f : X \to Y$ is said to be semi generalized-continuous [5] if
$f^{-1}(V)$ is semi generalized-closed in X for every closed set V of Y .

**Definition 3.7:** A function $f : X \to Y$ is said to be semi generalized-irresolute [5] if $f^{-1}(V)$ is semi generalized-closed in X for every semi generalized-closed set V of Y .

**Theorem 3.8:** A semi generalized-continuous image of a semi generalized-compact space is compact

**Proof.** Let $f : X \to Y$ be a semi generalized-continuous map from a semi generalized-compact space X onto a topological space Y. Let $\{A_i : i \in \wedge\}$ be an open cover of Y . Then $\{f^{-1}(A_i) : i \in \wedge\}$ is a semi generalized-open cover of X. Since X is semi generalized-compact it has a finite sub-cover say $\{f^{-1}(A_1), f^{-1}(A_2), \cdot \cdot \cdot , f^{-1}(A_n)\}$. Since f is onto $\{A_1, \cdot \cdot \cdot, An\}$ is a cover of Y , which is finite. Therefore Y is compact.

**Theorem 3.9:** If a map $f : X \to Y$ is semi generalized-irresolute and a subset B of X is
semi generalized-compact relative to X, then the image f(B) is semi generalized-compact relative to Y .

**Proof.** Let $\{A_\alpha : \alpha \in \wedge\}$ be any collection of semi generalized-open subsets of Y such that $f(B) \subset \cup\{A_\alpha : \alpha \in \wedge\}$. Then $B \subset \cup\{f^{-1} (A_\alpha) : \alpha \in \wedge\}$ holds. Since by hypothesis B is semi generalized-compact relative to X there exists a finite subset $\wedge_0$ of $\wedge$ such that $B \subset U \{f^{-1} (A_\alpha) : \alpha \in \wedge_0\}$ Therefore we have $f(B) \subset \cup\{A_\alpha : \alpha \in \wedge_0\}$, which shows that f(B) is semi generalized compact relative to Y.

## 4 Connectedness In Semi Generalized Closed Sets

**Definition 4.1:** A topological space X is said to be semi generalized-connected if X cannot be expressed as a disjoint union of two non-empty semi generalized-open sets. A subset of X is semi generalized-connected if it is semi generalized-connected as a subspace.

**Example 4.2:** Let $X = \{a, b\}$ and let $\tau = \{X, \varphi, \{a\}\}$. Then it is semi generalized-connected.

**Remark 4.3:** Every semi generalized-connected space is connected but the converse need not be true in general, which follows from the following example.

**Example 4.4:** Let $X = \{a, b\}$ and let $\tau = \{X, \varphi\}$. Clearly $(X, \tau)$ is connected. The semi generalized-open sets of X are $\{X, \varphi, \{a\} , \{b\}\}$. Therefore $(X, \tau)$ is not a semi generalized-connected space, because $X = \{a\} \cup \{b\}$ where $\{a\}$ and $\{b\}$ are non-empty semi generalized-open sets.

**Theorem 4.5:** For a topological space X the following are equivalent.

(i) X is semi generalized-connected.

(ii) X and $\varphi$ are the only subsets of X which are both semi generalized-open and semi generalized-closed.

(iii) Each semi generalized-continuous map of X into a discrete space Y with at least two points is a constant map.

**Proof:** (i) ⇒ (ii) :

Let O be any semi generalized-open and semi generalized-closed subset of X. Then $O^C$ is both semi generalized-open and semi generalized-closed. Since X is disjoint union of the semi generalized-open sets O and $O^C$ implies from the hypothesis of (i) that either $O = \varphi$ or $O = X$.

(ii) ⇒ (i) :

Suppose that $X = A \cup B$ where A and B are disjoint non-empty semi generalized-open subsets of X. Then A is both semi generalized-open and semi generalized-closed. By assumption $A = \varphi$ or X. Therefore X is semi generalized-connected.

(ii) ⇒ (iii) :

Let $f : X \to Y$ be a semi generalized-continuous map. Then X is covered by semi generalized-open and semi generalized-closed covering $\{f^{-1}(Y) : y \in (Y)\}$. By assumption $f^{-1}(y) = \varphi$ or X for each $y \in Y$. If $f-1(y) = \varphi$ for all $y \in Y$ , then f fails to be a map. Then there exists only one point $y \in Y$ such that $f^{-1}(y) \neq \varphi$ and hence $f^{-1}(y) = X$. This shows that f is a constant map.

(iii) ⇒ (ii) :

Let O be both semi generalized-open and semi generalized-closed in X. Suppose $O \neq \varphi$.
Let $f : X \to Y$ be a semi generalized-continuous map defined by $f(O) = y$ and $f(O^C) = \{w\}$ for some distinct points y and w in Y.By assumption f is constant. Therefore we have $O = X$.

**Theorem 4.6:** If f : X → Y is a semi generalized-continuous and X is semi generalized-connected, then Y is connected.

**Proof:** Suppose that Y is not connected. Let Y = A∪B where A and B are disjoint non-empty open set in Y . Since f is semi generalized-continuous and onto, X = $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty semi generalized-open sets in X. This contradicts the fact that X is semi generalized-connected. Hence Y is connected.

**Theorem 4.7:** If f : X → Y is a semi generalized-irresolute surjection and X is semi generalized-connected, then Y is semi generalized-connected.

**Proof:** Suppose that Y is not semi generalized-connected. Let Y =A∪B where A and B are disjoint non-empty semi generalized-open set in Y. Since f is semi generalized-irresolute and onto, X = $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty semi generalized-open sets in X. This contradicts the fact that X is semi generalized-connected. Hence Y is connected.

**Theorem 4.8:** In a topological space (X, τ) with at least two points, if semi generalized-o(x, τ) = semi generalized-cl(x, τ) then X is not semi generalized-connected.

**Proof:** By hypothesis we have semi generalized-O(X, τ) = semi generalized-CL(X, τ) and by Remark 2.6 we have every closed set is semi generalized-closed, there exists some non-empty proper subset of X which is both semi generalized-open and semi generalized-closed in X. So by last Theorem 4.5 we have X is not semi generalized-connected.

**Definition 4.9:** A topological space X is said to be $T_{\text{SEMI GENERALIZED}}$-space if every semi generalized-closed subset of X is closed subset of X.

**Theorem 4.10:** Suppose that X is a $T_{\text{SEMI GENERALIZED}}$-space then X is connected if and only if it is semi generalized-connected.

**Proof:** Suppose that X is connected. Then X can not be expressed as disjoint union of two non-empty proper subsets of X. Suppose X is not a semi generalized-connected space. Let A and B be any two semi generalized-open subsets of X such that X = A∪B, where A∩B = φ and A⊂X, B⊂X. Since X is $T_{\text{SEMI GENERALIZED}}$-space and A,B are semi generalized-open, A,B are open subsets of X, which contradicts that X is connected. Therefore X is semi generalized-connected. Conversely, every open set is semi generalized-open. Therefore every semi generalized-connected space is connected.

**Theorem 4.11:** If the semi generalized-open sets C and D form a separation of X and if Y is semi generalized-connected subspace of X, then Y lies entirely within C or D.

**Proof:** Since C and D are both semi generalized-open in X the sets C∩Y and D∩Y are semi generalized-open in Y these two sets are disjoint and their union is Y. If they were both non-empty, they would constitute a separation of Y. Therefore, one of them is empty. Hence Y must lie entirely in C or in D.

**Theorem 4.12:** Let A be a semi generalized-connected subspace of X. If A⊂B⊂SG-Cl(A) then B is also semi generalized-connected.

**Proof:** Let A be semi generalized-connected and let A⊂B⊂SG-Cl(A). Suppose that B = C∪D is a separation of B by semi generalized-open sets. Then by Theorem 4.11 above A must lie entirely in C or in D. Suppose that A ⊂ C, then SG-Cl(A) ⊆ SG-Cl(C). Since SG-Cl(C) and D are disjoint, B cannot intersect D. This contradicts the fact that D is non-empty subset of B. So D = φ which implies B is semi generalized-connected.

## 5. References


1. Andrijevic D. On b-open sets, Math. Vesnik, 1996; 1(2):59-64.
2. Caldas M. Jafari S. On some applications of b-open sets in topological spaces, Kochi. J. Math. 2007; 2:11-19.
3. Ekici E, Caldas M. Slightly γ-continuous functions, BolSoc. Parana. Mat 2004; 2:63-74.
4. Ganster M, Steiner M. On some questions about b-open sets, Questions - Answers Gen. Topology, 2007; 1:45-52.
5. Ganster M, Steiner M. On bτ-closed sets, Appl. Gen. Topol. 2007; 2(8):243-247.
6. Nasef AA. Some properties of contra-γ-continuous functions, Chaos Solitons Fractals, 2005; 2(24):471-477.
7. Nasef AA. On b-locally closed sets and related topics, Chaos Solitons Fractals, 2001; 10(12):1909-1915.
8. Park JH. θ-b-continuous functions, Acta Math. Hungar. 2006; 4(110):347-359.
9. Wali RS, Vivekananda Dembre. Minimal weakly open sets and maximal weakly closed sets in topological spaces; International Journal of Mathematical Achieve. 2014; 4(9).
10. Wali RS, Vivekananda Dembre. Minimal weakly closed sets and Maximal weakly open sets in topological spaces International Research Journal of Pure Algebra. 2014; 4(9).
11. Wali RS. Vivekananda Dembre. On semi-minimal open and semi-maximal closed sets in topological spaces; Journal of Computer and Mathematical Science International Journal. 2014.
12. Wali RS, Vivekananda Dembre. On pre generalized pre regular weakly closed sets in topological spaces; Journal of Computer and Mathematical Science. (International Journal). 2015; 6(2).
13. Wali RS, Vivekananda Dembre. On pre genearalized pre regular open sets and pre regular weakly neighbourhoods in topological spaces; Annals of Pure and Applied Mathematics. 2015; 10- 12.
14. RS Wali, Vivekananda Dembre. On pre generalized pre regular weakly interior and pre generalized pre regular weakly closure in topological spaces, International Journal of Pure Algebra. 2016; 6(2):255-259.
15. RS Wali, Vivekananda Dembre. On pre generalized pre regular weakly continuous maps in topological spaces, Bulletin of Mathematics and Statistics Research 2016; 4.(1).
16. RS Wali, Vivekananda Dembre. On Pre-generalized pre regular weakly irresolute and strongly pgprw-

continuous maps in topological spaces, Asian Journal of current Engineering and Maths. 2016; 5(2):44-46.
17. RS Wali, Vivekananda Dembre. On Pgprw-locally closed sets in topological spaces, International Jounal of Mathematical Archive. 2016; 7(3):119-123.
18. RS Wali, Vivekananda Dembre. ($\tau_1$,$\tau_2$) pgprw-closed sets and open sets in Bitopological spaces, International Journal of Applied Research. 2016; 2(5):636-642.
19. RS Wali, Vivekananda Dembre. Fuzzy pgprw-continuous maps and fuzzy pgprw-irresolute in fuzzy topological spaces; International Journal of Statistics and Applied Mathematics. 2016; 1(1):01-04.
20. RS Wali, Vivekananda Dembre. On pgprw-closed maps and pgprw-open maps in Topological spaces; International Journal of Statistics and Applied Mathematics. 2016; 1(1):01-04.
21. Vivekananda Dembre. Minimal weakly homeomorphism and Maximal weakly homeomorphism in topological spaces, Bulletin of the Marathons Mathematical Society. 2015; 6(2):1-7.
22. Vivekananda Dembre, Jeetendra Gurjar. On semi-maximal weakly open and semi-minimal weakly closed sets in topological spaces, International Research Journal of Pure Algebra. 2014, 4(10).
23. Vivekananda Dembre, Jeetendra Gurjar. Minimal weakly open map and maximal weakly open maps in topological spaces, International Research Journal of Pure Algebra. 2014; 4(10):603-606.
24. Vivekananda Dembre, Manjunath Gowda, Jeetendra Gurjar Minimal. Weakly and maximal weakly continuous functions in topological spaces, International Research Journal of Pure Algebra. 2014, 4(11).
25. Arun kumar Gali. Vivekananda Dembre. mınımal weakly generalızed closed sets and maxımal weakly generalızed open sets in topologıcal spaces, Journal of Computer and Mathematical sciences. [I.F = 4.655]. 2015; 6(6):328-335.
26. RS Wali, Vivekananda Dembre. Fuzzy Pgprw-Closed Sets and Fuzzy Pgprw-Open Sets in Fuzzy Topological Spaces. Journal of Global Research in Mathematical Archives. 2016, 3(3).
27. Vivekananda Dembre, Sandeep N Patil. On Contra Pre Generalized Pre Regular Weakly Continuous Functions in Topological Spaces. 2017, 3(12).
28. Vivekananda Dembre, Sandeep N Patil. On Pre Generalized Pre Regular Weakly Homeomorphism in Topological Spaces; Journal of Computer and Mathematical Sciences. 2018; 9(1):1-5.
29. Vivekananda Dembre, Sandeep N Patil. On pre generalized pre regular weakly topological spaces. Journal of Global Research in Mathematical Archives 2018, 5(1).
30. Vivekananda Dembre and Sandeep N Patil. Fuzzy Pre Generalized Pre Regular Weakly Homeomorphism in Fuzzy Topological Spaces, International Journal of Computer Applications Technology and Research. ISSN 2319–8656. 2018; 7(2):28-34.
31. Vivekananda Dembre, Sandeep N Patil. PGPRW-Locally Closed Continuous Maps in Topological Spaces, International Journal of Trend in Research and Development. 2018, 5(1).
32. Vivekananda Dembre, Sandeep N Patil. RW-Separation Axioms in Topological Spaces**,** International. Journal. of Engineering Sciences & Research Technology. 2018, 7(1).
33. Vivekananda Dembre, Sandeep N Patil. Fuzzy pgprw-open maps and fuzzy pgprw-closed maps in fuzzy topological spaces; International Research. Journal of Pure Algebra. 2018; 8(1):7-12.
34. Vivekananda Dembre, Sandeep N Patil. Pgprw-Submaximal spaces in topological spaces; International Journal of applied research 2018; 4(2):01-02.
35. P Bhattacharya, BK Lahiri, Semi-generalized closed sets in topology, Indian J. Math. 1987; 29:376-382.