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AXIOMS IN RW TOPOLOGICAL SPACES

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ABSTRACT
In this paper, we study some separation axioms namely, rw-T_o-space,rw-T_1 -space and rw-T_2-space and their properties. We also obtain some of their characterizations.

KEYWORDS: RW-T_O-SPACE**,** RW-T_1 –SPACE, RW-T_2-SPACE.

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I. INTRODUCTION
In the year 2007,S.S.Benchalli and R.S.Wali introduced and studied rw-closed and rw-open sets respectively.In this paper we define and study the properties of a new topological axioms called rw-T_o-space**,** rw-T_1 –space,rw-T_2-space.

II. PRELIMINARIES
Throughout this paper space (X,τ) and (Y,σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c,P-Cl(A) and P-int(A) denote the Closure of A, Interior of A , Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

Definition 2.1:A subset A of a topological space (X, τ) is called
(i) A regular ωeakly closed set (briefly, rω-closed set) if Cl(A)⊆U whenever A⊆U and U is w-open in (X, τ).
(ii)A subset A of a topological space (X,τ) is called regular ωeakly open[2] (briefly rω-open) set in X if A^c is rω-closed in X.
(iii)A topological space X is called a τ_{rw} space if every rw -closed set in it is closed.

Defintion 3: A map f:(X, τ) -» (Y, σ) is called
(i) Rw-continuous map [1] if f^{-1}(v) is rw closed in (X,τ) for every closed V in (Y,σ).
(ii)Rw-irresolute map[1]if f^{-1}(v) is rw closed in (X,τ)for every rw-closed V in (Y,σ).
(iii)Rw-closed map[1] if f^{-1}(v) is rw closed in (X,τ) for every closed V in (Y,σ).
(iv)Rw-open map[1] if f^{-1}(v) is rw closed in (X,τ) for every closed V in (Y,σ).

III. RW-T_O-SPACE:

Definition 4.4.1: A topological space (X, τ) is called rw-T_o-space if for any pair of distinct points x,y of (X,τ) there exists an rw-open set G such that x∈G, y∉G or x∉G, y∈G.

Example 4.4.2: Let X = {a, b}, τ ={φ,{b}, X}. Then (X, τ) is rw-T_o-space, since for any pair of distinct points a, b of (X,τ) there exists an rw-T_o open set {b} such that a ∉{b},b∈{b}.

Remark 4.4.3: Every rw-space is rw-T_o-space.

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Theorem 4.4.4: Every subspace of a rw-T_o-space is rw-T_o-space.
Proof: Let (X,τ) be a rw-T_o-space and (Y,τ_y) be a subspace of (X,τ). Let $Y_1$and Y_2 be two distinct points of (Y,τ_y). Since (Y,τ_y) is subspace of (X,τ),Y_1 and Y_2 are also distinct points of (X,τ). As (X,τ) is rw-T_o-space , thereexists an rw-open set G such that $Y_1 \in G$, $Y_2 \notin G$. Then $Y \cap G$ is rw-openin (Y,τ_y) containing but Y_1 not Y_2. Hence (Y,τ_y) is rw-T_o-space.

Theorem 4.4.5: Let f: (X,τ) -» (Y, μ) be an injection, rw-irresolute map. If (Y,μ) is rw-T_o-space, then (X,τ) is rw-T_o-space.
Proof: Suppose (Y, μ) is rw-T_o-space.Let a and b be two distinct points in(X,τ).
As f is an injection f(a) and f(b) are distinct points in (Y,μ). Since(Y,μ) is rw-T_o-space, there exists an rw-open set G in (Y,μ) such that $f(a) \in G$ and $f(b) \notin G$. As f is rw-irresolute, $f^{-1}(G)$ is rw-open set in (X,τ) suchthat $a \in f^{-1}(G)$ and $b \notin f^{-1}(G)$. Hence (X,τ) is rw-T_o-space.

Theorem 4.4.6: If (X,τ) is rw-T_o-space, T_{Rw}-space and (Y,τ_y) is rw-closedsubspace of (X,τ), then (Y,τ_y) is rw-T_o -Space.
Proof: Let (X,τ) be rw-T_o-space, T_{Rw}-space and (Y,τ_y) is rw-closed subspaceof (X,τ). Let a and b be two distinct points of Y. Since Y is subspace of(X,τ), a and b are distinct points of (X,τ). As $(X,\tau)$is rw-T_o -space, thereexists an rw-open set G such that $a \in G$ and $b \notin G$. Again since (X,τ) isT_{Rw}-space, G is open in (X,τ).Then $Y \cap G$ is open. So $Y \cap G$ is rw-open such that $a \in Y \cap G$ and $b \notin Y \cap G$. Hence (Y,τ_y) is Rw-T_o –space.

Theorem 4.4.7: Let f: (X,τ) -» (Y, μ) be bijective rw-open map from a rw-T_0 Space(X,τ) onto a topological space (Y,τ_y). If (X,τ) is T_{rw}-space, then(Y, μ) is rw-T_0 Space.
Proof: Let a and b be two distinct points of (Y,τ_y). Since f is bijective, thereexist two distinct points e and d of (X,τ) such that f(c) = a and f(d) = b. As(X,τ) is rw-T_0 Space,there exists a rw-open set G such that $c \in G$ and $d \notin G$.Since (X,τ) is T_{rw}-space, G is open in (X,τ). Then f(G) is rw-open in(Y, μ), since f is rw-open, such that $a \in f(G)$ and $b \notin f(G)$.
Hence (Y,τ_y) is rw-T_0-space.

Definition 4.4.8:A topological space (X,τ) is said to be rw-T_1-space if forany pair of distinct points a and b of (X,τ) there exist rw-open sets G and Hsuch that $a \in G$, $b \notin G$ and $a \notin H$, $b \in H$.

Example 4.4.9: Let X = {a,b} and $\tau = \{\emptyset,\{a\}, X\}$. Then (X,τ) is atopological space. Here a and b are two distinct points of (X,τ), then thereexist rw-open sets {a},{b} such that $a \in \{a\}$, $b \notin \{a\}$ and $a \notin \{b\}$, $b \in \{b\}$.Therefore (X,τ) is rw-T_0 space.

Theorem 4.4.10: If (X,τ) is rw-T_1-space,then (X,τ) is rw-T_o-space.
Proof: Let (X,τ) be a rw-T_1-space. Let a and b be two distinct points of(X,τ). Since (X,τ) is rw-T_1-space, there exist rw-open sets G and H such that$a \in G$, $b \notin G$ and $a \notin H$, $b \in H$. Hence we have $a \in G$, $b \notin G$. Therefore(X,τ) is rw-T_o-space.

The converse of the above theorem need not be true as seen from the following example.

Example 4.4.11: Let X = {a,b} and $\tau = \{\varphi,\{b\},X\}$. Then (X,τ) is rw-T_o-space but not rw-T_1-space. For any two distinct points a, b of X and an rw-open set {b} such that $a \notin \{b\}$, $b \in \{b\}$ but then there is no rw-open set Gwith $a \in G$, $b \notin G$ for $a \neq b$.

Theorem 4.4.12: If f: (X,τ) -» (Y,τ_y) is a bijective rw-open map from a rw-T_1-space and T_{rw}-space (X,τ) on to a topological space (Y,τ_y), then$(Y,\tau_y)$is rw-T_1-space.
Proof: Let (X,τ) be a rw-T_1-space and T_{rw}-space. Let a and b be two distinctpoints of (Y,τ_y). Since f is bijective there exist distinct points c and d of(X,τ) such that f(c) = a and f(d) = b. Since (X,τ) is rw-T_1-space there exist rw-open sets G and H such that $c \in G$, $d \notin G$ and $c \notin H$, $d \in H$. Since(X,τ) is T_{rw}-space, G and H are open sets in (X,τ) also f is rw-open f(G)and f(H) are rw-open sets such that $a = f(c) \in f(G)$, $b = f(d) \notin f(G)$ and$a= f(c) \notin f(H)$, $b= f(d) \in f(H)$. Hence (Y,τ_y) is rw-T_1-space.

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Theorem 4.4.13: If (X,τ) is rw T_1 space and T_{rw}-space, Y is a subspace of(X,τ), then Y is rw T_1 space.
Proof: Let (X,τ) be a rw T_1 space and T_{rw}-space. Let Y be a subspace of (X,τ).Let a and b be two distract points of Y. Since Y⊆X,a and b are alsodistinct points of X. Since (X,τ) is rw-T_1-space, there exist rw-open sets Gand H such that a∈G, b∉G and a∉H, b∈H. Again since (X,τ) is T_{rw}-space,G and H are open sets in (X,τ), then Y∩G and Y∩H are open setsso rw-open sets of Y such that a∈Y∩G, b∉Y∩G and a∉Y∩H,b∈Y∩H. Hence Y is rw T_1 space.

Theorem 4.4.14: Iff: (X,τ) -»(Y,τ_y) is injective rw-irresolute map from atopological space (X,τ) into rw-T_1-space(Y,τ_y), then $(X,\tau)$is rw-T_1 - space.
Proof: Let a and b be two distinct points of (X,τ). Since f is injective, f(a)and f(b) are distinct points of (Y,τ_y). Since(Y,τ_y) is rw-T_1 space there exist rw-open sets G and H such that f(a)∈G, f(b) ∉G and f(a) ∉H, f(b)∈H.Since f is rw- irresolute, f^{-1}(G) and f^{-1}(H) are rw-open sets in (X,τ) such that a∈ f^{-1} (G), b∉f^{-1}(G) and a∉f^{1}(H), b∈f^{-1}(H). Hence (X,τ) is rw-T_1 space.

Definition 4.4.15: A topological space (X,τ). is said to be rw-T_2- space(or T_{rw}-Hausdorff space) if for every pair of distinct points x, y of X thereexist T_{rw}-open sets M and N such that x∈N, y∈M and N∩M = ∅.

Example 4.4.16: Let X = {a,b}, τ = {∅,{a},{b}, X}. Then (X,τ) istopological space. Then (X,τ) is rw-T_2-space. T_{rw}-open sets are ∅, {a}, {b},and X. Let a and b be a pair of distinct points of X, then there exist T_{rw} - open sets {a} and {b} such that a∈{a}, b∈{b} and {a}∩{b} = ∅. Hence (X,τ) is rw-T_2-space.

Theorem 4.4.17: Every rw-T_2- space is rwT_1space.
Proof: Let (X,τ) be a rw-T_2- space. Let x and y be two distinct points in X.Since (X,τ) is rw-T_2- space, there exist disjoint T_{rw}-open sets U and V such that x∈U, and y∈V. This implies, x∈U, y∉U and x∈V, y∉V. Hence (X,τ) is rw-T_2- space.

Theorem 4.4.18: If (X,τ) is rw-T_2-space, T_{rw}- space and (Y,τ_y) is subspace of (X,τ), then (Y,τ_y) is also rw-T_2-space.

Proof: Let (X,τ), be a rw-T_2 - space and let Y be a subset of X. Let x and y beany two distinct points in Y. Since Y⊆X,x and y are also distinct points ofX. Since $(X,\tau)$is rw-T_2 - space, there exist disjoint T_{rw}-open sets G and Hwhich are also disjoint open sets, since (X,τ) is T_{rw} - space. So G∩Y and H∩Y are open sets and so T_{rw}-open sets in (Y,τ_y). Also x∈G, x ∈Yimplies x∈G∩V and y∈H and y∈Y this implies y ∈Y∩H, since G∩H = ∅, we have (Y∩G)∩(Y∩H) = ∅. Thus G∩Y and H∩Y are disjointT_{rw}-open sets in Y such that x∈G∩Y, y∈H∩Y and (Y∩G)∩(Y∩ H)= ∅. Hence (Y,τ_y) is rw-T_2 - space.

Theorem 4.4.19: Let (X,τ), be a topological space. Then (X,τ),is rw-T_2-space if and only if the intersection of all T_{rw}-closed neighbourhood of eachpoint of X is singleton.
Proof: Suppose (X,τ), is rw-T_2-space. Let x and y be any two distinct pointsof X. Since X is rw-T_2-space, there exist open sets G and H such that x∈G,y∈H and G∩H = ∅.Since G∩H = ∅implies x∈G⊆X-H. SoX-HisT$_{rw}$-closed neighbourhood of x, which does not contain y. Thus y does notbelong to the intersection of all T_{rw}-closed neighbourhood of x. Since y isarbitrary, the intersection of all T_{rw}-closed neighbourhoods of x is thesingleton {x}.

Conversely, let (x) be the intersection of all T_{rw}-closedneighbourhoods of an arbitrary point x∈X. Let y be any point of Xdifferent from x. Since y does not belong to the intersection, there exists aT_{rw}-closed neighbourhood N of x such that y∉N. Since N isT_{rw}-neighbourhood of x, there exists an T_{rw}-open set G such x ∈G⊆X.Thus G and X - N are T_{rw}-open sets such that x⊆G, y∈X-N and G∩(X - N) = ∅. Hence (X,τ) is rw-T_2-space.

Theorem 4.4.20: Let f: (X,τ) -» (Y,τ_y) be a bijective rw-open map. If(X,τ) is rw-T_2- space and T_{rw} space, then $(Y,\tau_y)$is also rw-T_2- space.
Proof: Let (X,τ), is rw-T_2- space and T_{rw}- space. Let y_1 and y_2 be two distinctpoints of Y. Since f is bijective map, there exist distinct points x_1 and x_2 of Xsuch that $f(x_i) = y_j$ and $f(x_2) = y_2$. Since (X,τ) is rw-T_2- space,there exist rw-opensets G and H such that X_1∈G, X_2∈H and
G∩H =∅. Since (X,τ) isT_{rw}- space, G and H are open sets, then f(G) and f(H) are rw- open sets of(Y,τ_y) , since f is pprw-open, such that $y_1 = f(x_1)$ ∈ f(G), $y_2 = f(x_2)$ ∈ f(H) and

f(G)∩f(H) = ∅. Therefore we have f(G)∩f(H) = f(G∩H) =∅ . Hence(Y,τ_y) is rwT_2-space.

Theorem 4.4.21: Let (X,τ) be a topological space and let (Y,τ_y) be a Rw-T_2-space. Let f: (X,τ) —>(Y,τ_y) be an injective rw-irresolute map. Then (X,τ) is rw-T_2-space.

Proof: Let$x_1$and x_2 be any two distinct points of X. Since f is injective,$x_1 \neq x_2$ implies $f(x_1) \neq f(x_2)$.Let $y_1 = f(x_1)$, $y_2 = f(x_2)$ so that $x_1 = f^{-1}(y_1)$,$x_2 = f^{-1}(y_2)$.Then $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since (Y,τ_y) is Rw-T_2-space thereexist T_{rw}-open sets G and H such that $y_1 \in G$, $y_2 \in$ G and G∩H =∅ . As f isT_{rw}-irresolute $f^{-1}(G)$ and $f^{-1}(H)$ are T_{rw}-open sets of (X,τ).

Now $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f^{-1}(\emptyset) = \emptyset$ and $y_1 \in G$ implies $f^{-1}(y_1) \in f^{-1}(G)$ implies $x_1 \in f^{-1}(G)$, $y_2 \in$ H implies $f^{-1}(y_2) \in f^{-1}(H)$ implies $x_2 \in f^{-1}(H)$.Thus forevery pair of distinct points x_1, x_2 of X there exist disjoint T_{rw}-open sets$f^{-1}(G)$ and $f^{-1}(H)$ such that $x_1 \in f^{-1}(G)$, $x_2 \in f'^{-1}(H)$. Hence (X,τ) is rw-T_2-space.

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