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On Pre Generalized Pre Regular Weakly Homeomorphism in Topological Spaces

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ABSTRACT

In this paper we introduce and investigate new class of maps called pgprw-homeomorphism and several characterization and some of their properties. Also we investigate it's relationship with other types of functions.

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Keywords: Pgprw-homeomorphism,Pgprw-closed sets, Pgprw-open sets.

I. INTRODUCTION

The notion homeomorphism plays a very important role in topology. By definition a homeomorphism between two topological spaces X and Y is a bijective map $f: X \rightarrow Y$ when both f and f^{-1} are continuous map. Wali and Vivekananda Dembre[1] introduced Pgprw – closed set in topological spaces. Wali and Vivekananda Dembre[2] introduced pgprw–continuous map, in topological spaces. In this paper we introduce the concept of pgprw – homeomorphism and study the relationship between homeomorphism, pgprw – homeomorphism,gpr homeomorphism, gp homeomorphism & gspr homeomorphism.

II. PRELIMINARIES

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c, P-Cl(A) and P-int(A) denote the Closure of A, Interior of A, Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

Definition 2.1: A subset A of a topological space (X, τ) is called

(i) A pre generalized pre regular ωeakly closed set(briefly pgprω-closed set)[1] if $pCl(A) \subseteq U$ whenever $A \subseteq U$ and U is rgα open in (X, τ).
(ii) Pre generalized pre-regular ωeakly open(briefly pgprω-open)[2] set in X if A^c is pgprω-closed in X.
(iii) Regular open set if A = int(clA))[4] and a regular closed set if A = cl(int(A)).
(iv) Generalized pre regular closed set(briefly gpr-closed)[5] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
(v) Generalized semi pre regular closed (briefly gspr-closed) set[12] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
(vi) Generalized pre closed (briefly gp-closed) set[7] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Defintion 2.2: A map $f:(X, \tau) \rightarrow (Y, \sigma)$) is called

(i) pgprw-continuous map[3] if the inverse image of every closed in Y is pgprw closed set in X.
(ii) regular-continuous map[10] if the inverse image of every closed in Y is regular closed set in X.
(iii) gpr-continuous map[8] if the inverse image of every closed in Y is gpr closed set in X.
(iv) gspr-continuous map[6] if the inverse image of every closed in Y is gspr closed set in X.
(v) gp-continuous map[11] if the inverse image of every closed in Y is gp closed set in X.

Defintion2.3: A map $f:(X, \tau) \rightarrow (Y, \sigma)$ is called

(i) gpr homomorphism[5] if f & f^{-1} are gpr continuous map.
(ii) gspr homomorphism[6] if f &f^{-1} are gspr continuous map.
(iii) gp homomorphism[11] if f &f^{-1} are gp continuous map.
(iv) pgprw-closed map[9] if f(F) is pgprw-closed in (Y, σ) for every closed set of (X, τ) & pgprw-open map if f(F) is pgprw-open in (Y, σ) for every open set of (X, τ).

Theorem 2.4:

(i) Every pgprw-closed set is gspr-closed.
(ii) Every pgprw-closed set is gp-closed.

III. PGPRW-HOMEOMORPHISM IN TOPOLOGICAL SPACES

Definition 3.1 : A bijection $f : (X, \tau) \rightarrow (Y, \sigma)$ is called pre generalized pre regular weakly homeomorphism if f and f^{-1} are pgprw-continuous map. We denote the family of all pgprw-homeomorphisms of a topological space (X, τ) onto itself by pgprw-h(X, τ).

Example 3.2 : Consider X=Y={a,b,c,d} with topologies τ={X,∅,{a},{b},{a,b},{a,b,c}} and σ = {Y, ∅,{a},{b},{a,b},{a,b,c}}. Let f: X → Y be a map defined by f (a) = c, f (b) = a f(c) = b and f(d)=d. Then f is bijective, pgprw-continuous map and f^{-1} is pgprw –continuous map Hence f is pgprw–homeomorphism.

Theorem 3.3: Every homeomorphism is pgprw-homeomorphism.

Proof: Let f: (X, τ) → (Y, σ) be a homeomorphism. Then f and f^{-1} are continuous map and f is bijection. Since every continuous map is pgprw-continuous map,f and f^{-1} are pgprw-continuous map.Hence f is pgprw-homeomorphism.

Remark 3.4: The Converse of the above theorem need not be true as seen from the following example.

Example 3.5 : Consider X=Y={a,b,c,d} with topologies τ={X,∅,{a},{b},{a,b},{a,b,c}} and σ={Y,∅,{a},{b},{a,b},{a,b,c}}. Let f: X → Y be a map defined by f(a)=c,f(b)=a,f(c)=b,f(d)=d. Then f is pgprw-homeomorphism. But it is not homeomorphism since the inverse image of the closed set {c,d} in X is {b,d} is not closed in Y.

Theorem 3.6: Every regular homeomorphism is pgprw-homeomorphism.

Proof: The proof follows from the theorem 3.3

Remark 3.7: The Converse of the above theorem need not be true as seen from the following example.

Example 3.8 : Consider X=Y={a,b,c,d} with topologies τ={X,∅,{a},{b},{a,b},{a,b,c}} and σ={Y,∅,{a},{b},{a,b},{a,b,c}}. Let f: X → Y be a map defined by f(a)=c,f(b)=a,f(c)=b,f(d)=d. Then f is pgprw-homeomorphism. But it is not regular homeomorphism since the inverse image of the closed set {c,d} in X is {b,d} is not regular closed in Y.

Theorem 3.9: Every pgprw-homeomorphism is gpr homeomorphism.

Proof: Let f: (X, τ) → (Y, σ) be a pgprw homeomorphism.Then f and f^{-1} are pgprw-continuous map and f is bijection. Since every pgprw-continuous map is gpr-continuous map, f and f^{-1} are gpr-continuous map. Hence f is gpr-homeomorphism.

Remark 3.10: The Converse of the above theorem need not be true as seen from the following example.

Example 3.11: Consider X=Y={a,b,c} with topologies τ={X,∅,{a},{b,c}} and σ = {Y, ∅,{a}}. Let f: X → Y be a map defined by f(a)=b,f(b)=a,f(c)=c. Then f is gpr-homeomorphism. But it is not pgprw homeomorphism since the inverse image of the closed set {b,c} in X is {a,c} is not pgprw closed in Y.

Theorem 3.12: Every pgprw-homeomorphism is gspr-homeomorphism.

Proof: The proof follows from the definition and fact that every pgprw-closed set is gspr-closed [Theorem 2.4].

Remark 3.13: The Converse of the above theorem need not be true as seen from the following example.

Example 3.14 : Consider X=Y={a,b,c} with topologies τ={X,∅,{a},{b,c}} and σ={Y, ∅,{a}}. Let f: X → Y be the defined by f(a)=b,f(b)=a,f(c)=c. Then f is gspr-homeomorphism.But it is not pgprw-homeomorphism since the inverse image of the closed set {b,c} in X is {a,c} is not pgprw-closed in Y.

Theorem 3.15: Every pgprw-homeomorphism is gp-homeomorphism.

Proof: The proof follows from the definition and fact that every pgprw-closed set is gp-closed [Theorem 2.4].

Remark 3.16: The Converse of the above theorem need not be true as seen from the following example.

Example 3.17 : Consider X=Y={a,b,c} with topologies τ={X,∅,{a},{b,c}} and σ = {Y, ∅,{a}}.Let f: X → Y be the defined by f(a)=b,f(b)=a,f(c)=c. Then f is gp-homeomorphism.But it is not pgprw-homeomorphism since the inverse image of the closed set {b,c} in X is {a,c} is not pgprw-closed in Y.

Theorem 3.18: Let f: (X, τ) → (Y, σ) be a bijective pgprw-continuous map. Then the following statements are equivalent.

(i) f is a pgprw-open map.
(ii) f is pgprw-homeomorphism.
(iii) f is a pgprw-closed map.

Proof: Suppose (i) holds. Let V be open in(X, τ) .Then by (i), f (V) is pgprw-open in (Y, σ). But
$f(V) = (f^{-1})^{-1}(V)$ and so $(f^{-1})^{-1}(V)$ is pgprw-open in(Y, σ). This shows that f^{-1} is pgprw-continuous map and it proves (ii).

Suppose (ii) holds. Let F be a closed set in (X, τ). By (ii), f^{-1} is pgprw-continuous map and so $(f^{-1})^{-1}(V)(F) = f(F)$ is pgprw-closed in (Y, σ).This proves (iii).

Suppose (iii) holds. Let V be open in(X, τ) .Then V^c is closed in (X, τ) By (iii), $f(V^c)$ is pgprw-closed in (Y, σ). But $f(V^c) = (f(V))^c$. This implies that $(f(V))^c$ is pgprw-closed in (Y, σ) and so
f (V) is pgprw-open in (Y, σ). This proves (i).

Remark 3.19: The Composition of two pgprw-homeomorphism need not be a pgprw-homeomorphism in general as seen from the following example.

Example 3.20 : Consider X=Y=Z= {a,b,c} with topologies
τ = {X, ϕ,{a},{b},{a,b}} and σ ={Y, ϕ,{a}}& μ= { Z = ϕ,{a},{a,b},{a,c}}.
Define f: (X, τ) → (Y, σ) and g:(Y, σ) → (Z, μ) & gof: X → Z are identity maps both f & g are pgprw homeomorphism but gof not pgprw-homeomorphism.Since closed set v={b} in Z,
f^{-1}(v)={b},which is not pgprw-closed set in X.

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