BLDE ASSOCIATION'S

S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

ADVANCE LEARNERS

For the Academic Year : 2019-20

BLDE ASSOCIATION'S

S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

SEMINAR REPORT

2019-20 (ODD SEMESTER)

BLDE Association's

S.B.Arts And K.C.P Science College, Vijayapur

DEPARTMENT OF MATHEMATICS

NOTICE:

The UG Department of Mathematics is conducting seminar for B.Sc students for the academic year 2019-20(Odd Semester)

IQAC, Co-ordinator S.B.Arts & K.C.P.Science College, Vijayapur.	Principal, S.B. Arts and KCP Science College VIJAYAPUR	**Head of Department** H. O. D. Department of Mathematics. S. B. Arts & K. C. P. Science College, BIJAPUR.

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S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

Seminar Report-2019-20

Odd Semester

Sl. No	Semester	Name of the Student	Topics	Date
1	I	Kiran Watharkar	Theory of Equations	19-09-2019
2	III	Apoorva Mirajkar	Homogenious Differential Equation	20-09-2019
3	V	Pallavi Akkavadi	Kinematics	25-09-2019

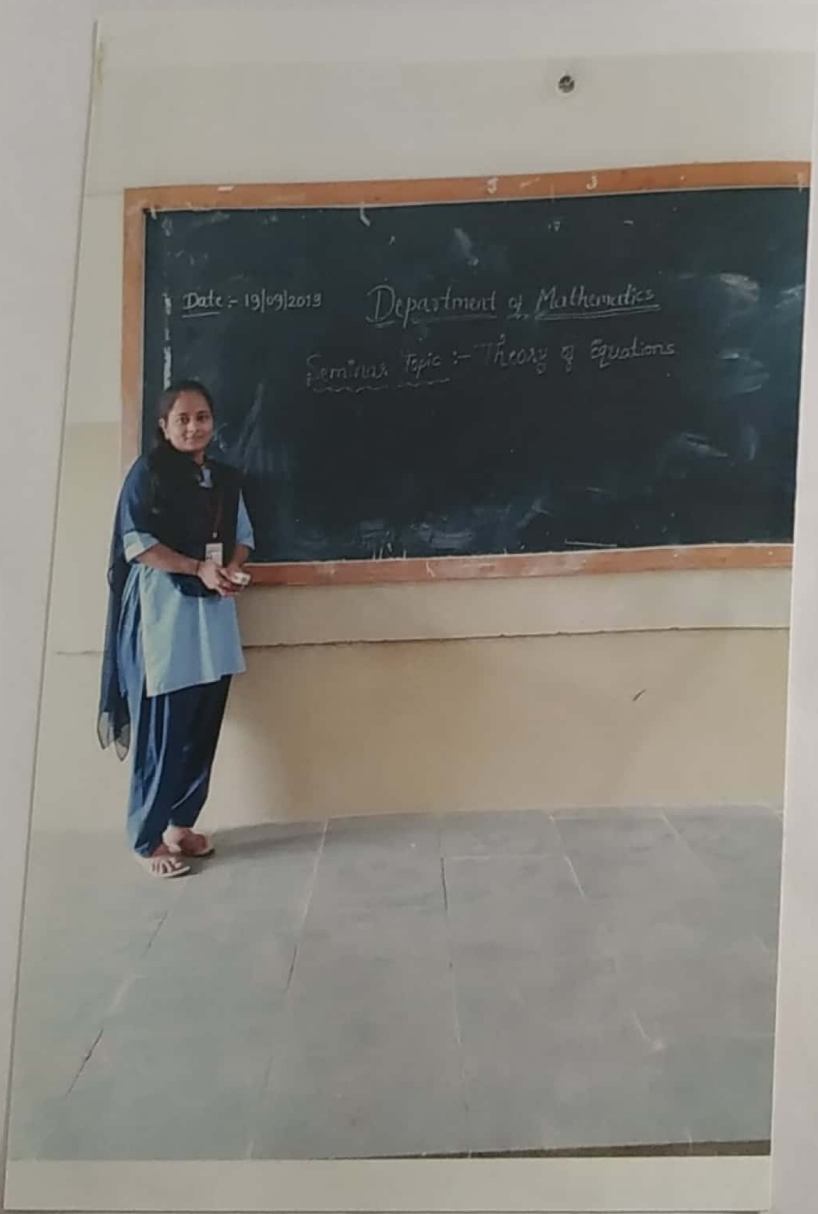
Name : Kiran. G. Watharkar

lass : Bsc Ist sem

Roll NO: 41

niversity no : S1925015

ubject : Mathematics.



Theory of Equation.

Factor theorem:

If the reminder $f(x) = 0$ then $x-r$ is a factor of $f(x)$

Proof: If $x-r$ is a factor of $f(x)$
then the reminder and dividing
$f(x)$ by $x-r$ must be equal to zero

i.e $R = f(x) = 0$

given $f(r) = x-r \cdot g(x)$

$f(r) = 0 \cdot g(x)$

$f(r) = 0$

Conversly:

If the reminder $f(x)$ is zero when $f(x)$ is divided by $x-r$ then we can write $f(x)$ is of the form.

$f(x) = (x-r) \cdot g(x) + 0$

$= (x-r) \cdot g(x)$

$\therefore x-r$ is a factor of $f(x)$.

undamental Theorem of Algebra:-

tatement: Every polynomial equation of $f(x)$ n^{th} degree has exactly 'n' number of roots.

roof: By the Fundamental theorem of Algebra.

$f(x) = a_0x^n + a_1x^{n-1} + \cdots$

has atleast 1 root r_1 then $(x-r_1)$ is a factor of $f(x)$ so that

$f(x) = (x-r_1)q_1(x)$

where $q_1(x)$ is the quotient obtained on dividing $f(x)$ by $(x-r_1)$

where $q_1(x)$ is degree $(n-1)$

i.e $q_1(x) = (x-r_2).q_2(x) \Rightarrow q_1(x) = a_0x^{n-1} + a_1x^{n-2} + \cdots$

If $(n-1)>0$ then $q_1(x)$ has at least 1 root say r_2

then $(x-r_2)$ is a factor of $q_1(x)$

$\therefore q_1(x) = (x-r_2) \cdot q_2(x)$

where $q_2(x)$ is the quotient obtained on dividing $q_1(x)$ by $(x-r_2)$.

Here $q_2(x)$ is a polynomial equation of degree $(n-2)$

i.e $q_2(x) = (x-r_3).q_3(x) \Rightarrow q_2(x) = a_0x^{n-2} + a_1x^{n-1} + \cdots$

This process can be continued until we obtain

$q_n(x) = a_0$

$\therefore f(x) = a_0(x-r_1)(x-r_2)\cdots(x-r_n)$

No value of 'x' other than $r_1, r_2 \cdots r_n$ makes $f(x)=0$

$\therefore r_1, r_2, r_3 \cdots r_n$ are zeros or roots of the equation

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n$

Hence the polynomial $f(x)=0$ of degree 'n' has exactly 'n' roots.

elation between Roots & Co-efficients:-

et us consider the polynomial function of n^{th} degree is

$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n = 0$

et $r_1, r_2, r_3 \dots r_n$ be the roots of n^{th} degree polynomial
and
$(x-r_1)(x-r_2)\dots\dots(x-r_n)$ be the factors of polynomial
unction $f(x)$

By Fundamental theorem of Algebra:-
very polynomial equation of n^{th} degree has exactly
number of roots.

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n = a_0[(x-r_1)(x-r_2)(x-r_3)\dots(x-r_n)]$$

$$= a_0[x^n - \Sigma r_1 x^{n-1} + \Sigma r_1r_2x^{n-2} - \Sigma r_1r_2r_3x^{n-3} + \dots + (-1)^n \Sigma r_1r_2r_3\dots r_n]$$

$$= a_0x^n - a_0\Sigma r_1x^{n-1} + a_0\Sigma r_1r_2x^{n-2} - a_0\Sigma r_1r_2r_3x^{n-3} + \dots + a_0(-1)^n \Sigma r_1r_2r_3\dots r_n \quad \dots\dots (1)$$

re Σr_1 = sum of all roots.

Σr_1r_2 = sum of roots taken two at a time.

$\Sigma r_1r_2r_3$ = sum of roots taken three at a time.

$\Sigma r_1r_2r_3\dots r_n$ = sum of roots taken 'n' number of roots at a time.

consider the like powers of x in eqn (1)

ating like powers of x

ow

$a_1x^{n-1} = a_0\Sigma r_1x^{n-1}$

$a_1 = -a_0\Sigma r_1$

$$\Sigma r_1 = -\frac{a_1}{a_0}$$

$a_2 x^{n-2} = a_0 \sum r_1 r_2 x^{n-2}$

$a_2 = a_0 \sum r_1 r_2$

$\sum r_1 r_2 = \frac{a_2}{a_0}$

Illy

$\sum r_1 r_2 r_3 = \frac{-a_3}{a_0}$

Continuing in this way we get

$\sum r_1 r_2 r_3 \cdots\cdots r_k = (-1) \frac{a_k}{a_0}$

Finally we have

$\sum r_1 r_2 r_3 \cdots\cdots r_n (-1)^n \frac{a_n}{a_0}.$

The rational roots of the form P/q of the polynomial $f(x)$

Proof: since $\frac{p}{q}$ is a root of $f(x)=0$

then we have

$$a_0\left(\frac{p}{q}\right)^n + a_1\left(\frac{p}{q}\right)^{n-1} + \cdots + a_{n-1}\left(\frac{p}{q}\right) + a_n = 0$$

$$a_0\frac{p^n}{q^n} + a_1\frac{p_1^{n-1}}{q_1^{n-1}} + \cdots + a_{n-1}\frac{p}{q} + a_n = 0 \quad \text{--- (1)}$$

Now multiplying eq^n (1) by q^n on both sides we get

$$a_0p^n + a_1\frac{p^{n-1}}{q^n - q^{-1}} + \cdots + a_{n-1}\frac{p}{q} + a_nq^n = 0$$

$$a_0p^n + a_1p^{n-1}q + \cdots + a_{n-1}pq^{n-1} + a_nq^n = 0 \quad \text{---- (2)}$$

Now dividing eq^n (2) by p we get

$$a_0p^{n-1} + a_1p^{n-2}q + \cdots + a_{n-1}q^{n-1} = \frac{-a_nq^0}{p} \quad \text{--- (3)}$$

since $a_0, a_1, a_2 \cdots a_{n-1}$ and p, q are all integers.

The left side of eq^n (3) is the sum of the integers

$\therefore$ The right side of eq^n (3) is also an integer.

This is possible only iff p is exact divisor of a_n

Next from equation (2)

$$a_nq^n + a_{n-1}q^{n-1}p + \cdots + a_1p^{n-1}q + a_0p^n = 0$$

Now dividing the above eq^n by 'q' on both sides

$$a_nq^{n-1} + a_{n-1}q^{n-2} + \cdots + a_1p^{n-1} = \frac{-a_0p^n}{q} \quad \text{--- (4)}$$

Since $a_0, a_1, a_2 \cdots a_n$ and p, q are all integers.

The left side of eq^n (4) is the sum of the integers.

The right side of equation (4) is the sum of the integer,

The right side of equation (4) is also an integer this is possible is only iff q is an exact divisor of a_n.

-: SEMINAR :- Topic - Homogeneous Equations

NAME : APURVA .L. MIRAJKAR

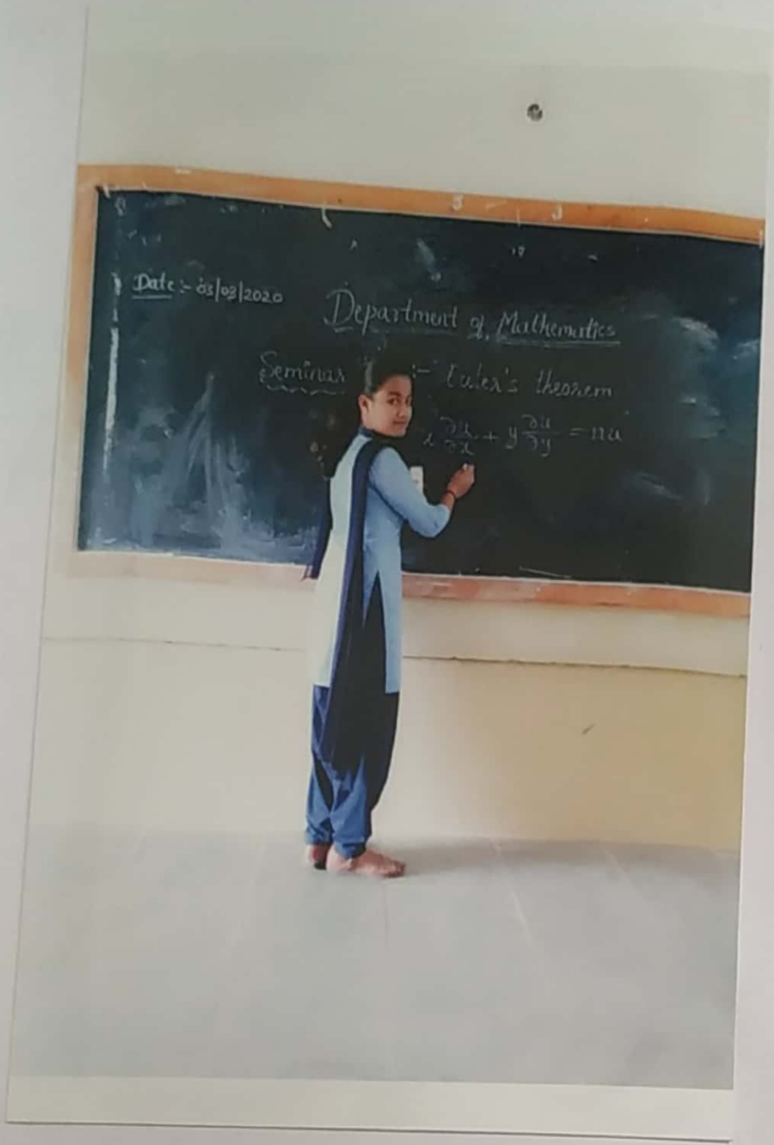
SUBJECT : MATHEMATICS - II

RCU SEAT NO: S1824673

ROLL NO : 428 (c-div)

CLASS : BSc III sem

DATE : 20-09-2019


Date :- 05/03/2020
Department of Mathematics
Seminar :- Euler's theorem
$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu$

DIFFERENTIAL EQUATION - I Seminar Topic

HOMOGENEOUS EQUATIONS (PROBLEMS)

1) Solve $(x^2+y^2)dx - 2xy\,dy = 0$

→ Given : $(x^2+y^2)dx - 2xy\,dy = 0$

$$(x^2+y^2)dx = 2xy\,dy$$

$$\frac{dy}{dx} = \frac{x^2+y^2}{2xy} \longrightarrow ①$$

put $y = vx$ or $v = y/x$

Now, Diff wrt 'x' $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

from ①

$$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2x^2v}$$

$$x.\frac{dv}{dx} = \frac{x^2(1+v^2)}{2x^2v} - v$$

$$x\frac{dv}{dx} = 1+v^2-2v^2$$

$$x.\frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$\frac{2v}{1-v^2}dv = \frac{1}{x}dx$$

Integrating above equation, we have

$$\int\frac{2v}{1-v^2}dv = \int\frac{1}{x}dx$$

$$\frac{\log(1-v^2)}{-1} = \log x + \log c$$

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$$\log(1-v^2) = -\log x - \log c$$

$$\log(1-v^2) + \log x = \log c^{-1}$$

$$\log(1-v^2).x = \log c^{-1}$$

$$\left(1-\frac{y^2}{x^2}\right)x = \frac{1}{c}$$

$$\frac{x^2-y^2}{x} \times x = \frac{1}{c}$$

$$x^2-y^2 = \frac{x}{c}$$

$$c(x^2-y^2) = x$$

$$\boxed{x = (x^2-y^2)c}$$ //

---

2) Solve $(x^2-y^2)dx + 2xy\,dy = 0$

Sol: Given : $(x^2-y^2)dx + 2xy\,dy = 0$

$$2xy\,dy = -(x^2-y^2)dx$$

$$\frac{dy}{dx} = \frac{y^2-x^2}{2xy} \longrightarrow (1)$$

put $y = vx$ or $v = y/x$

Diff wrt 'x'

$$\frac{dy}{dx} = v + x.\frac{dv}{dx}$$

from (1)

$$v + x\frac{dv}{dx} = \frac{v^2x^2-x^2}{2x^2v}$$

$$x.\frac{dv}{dx} = \frac{x^2(v^2-1)}{2x^2v} - v$$

$$x.\frac{dv}{dx} = \frac{v^2-1-2v^2}{2v}$$

$$x \cdot \frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$\frac{2v}{-(v^2+1)} dv = \frac{1}{x} dx$$

Integrating above Equation, we have

$$\int \frac{2v}{-(v^2+1)} dv = \int \frac{1}{x} dx$$

$$-\log(v^2+1) = \log x + \log c$$

$$-\log\left(\frac{y^2}{x^2}+1\right) = \log x c$$

$$\log \frac{y^2}{x^2}+1 = -\log x c$$

$$\log \frac{y^2}{x^2}+1 = \log (xc)^{-1}$$

$$\frac{y^2}{x^2}+1 = x^{-1}c^{-1}$$

$$\frac{y^2+x^2}{x^2} = x^{-1}c^{-1}$$

$$(y^2+x^2) = xc$$

$$\therefore \boxed{x^2+y^2 = xc}.$$

This is the required solution//.

3) Solve $(x^3+3xy^2)dx + (y^3+3x^2y)\,dy = 0$.

→ Given: $(x^3+3xy^2)dx + (y^3+3x^2y)dy = 0$

$$(y^3+3x^2y)dy = -(x^3+3xy^2)dx$$

$$\frac{dy}{dx} = -\frac{(x^3+3xy^2)}{(y^3+3x^2y)} \rightarrow ①$$

put $y = vx$ or $v = y/x$

Diff w.r.t 'x'

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

from ① we have

$$v + x\frac{dv}{dx} = \frac{-(x^3+3xy^2)}{y^3+3x^2y}$$

$$= -\frac{(x^3+3x^3v^2)}{x^3v^3+3x^3v}$$

$$x\frac{dv}{dx} = \frac{-(1+3v^2)}{v^3+3v} - v$$

$$= \frac{-1-3v^2-v^4-3v^3}{v^3+3v}$$

$$x\frac{dv}{dx} = -\frac{(v^4+6v^2+1)}{v^3+3v}$$

$$\frac{v^3+3v}{v^4+6v^2+1}\,dv = -\frac{dx}{x}$$

$$\frac{4(v^3+3v)\,dv}{v^4+6v^2+1} = -4\frac{dx}{x}$$

Integrating we get.

$$\int \frac{4v^3+12v}{v^4+6v^2+1}\,dv = -\int \frac{4}{x}\,dx$$

$$\log(v^4+6v^2+1) = -4\log x + \log c$$

$$\log\left(\frac{y^4}{x^4}+\frac{6y^2}{x^2}+1\right) = -\log x^4 + \log c$$

$$\log\left(\frac{y^4}{x^4}+\frac{6y^2}{x^2}+1\right) + \log x^4 = \log c$$

$$\log\left(\frac{y^4}{x^4}+\frac{6y^2}{x^2}+1\right)(x^4) = \log c$$

$$\boxed{y^4 + 6x^2y^2 + x^4 = C}$$

4) Solve $(x^3 - 2y^3)dx + 3xy^2dy = 0$

→ Given: $(x^3 - 2y^3)dx + 3xy^2dy = 0$

$$3xy^2dy = -(x^3 - 2y^3)dx$$

$$\frac{dy}{dx} = -\frac{(x^3 - 2y^3)}{3xy^2} \longrightarrow (1)$$

put $y = vx$ or $v = y/x$

Diff wrt (x)

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

from (1)

$$v + x\frac{dv}{dx} = -\frac{(x^3 - 2v^3x^3)}{3x^3v^2}$$

$$x\frac{dv}{dx} = -\frac{(1-2v^3)}{3v^2} - v$$

$$x\frac{dv}{dx} = \frac{-1+2v^3-3v^3}{3v^2}$$

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$$x\frac{dv}{dx} = \frac{-1-v^3}{3v^2}$$

$$x\frac{dv}{dx} = -\frac{(1+v^3)}{3v^2}$$

$$\frac{3v^2}{(1+v^3)}\,dv = -\frac{dx}{x}$$

Integrating we get

$$\int \frac{3v^2}{(1+v^3)}\,dv = -\log x + \log c$$

$$\log(1+v^3) = -\log x + \log c$$

$$\log(1+v^3) + \log x = \log c$$

$$\log (1+v^3)(x) = \log c$$

$$\left[1+\frac{y^3}{x^3}\right]x = c$$

$$x + \frac{y^3}{x^2} = c$$

$$\frac{x^3+y^3}{x^2} = c$$

$$\boxed{x^3+y^3 = cx^2}$$

This is the required solution.

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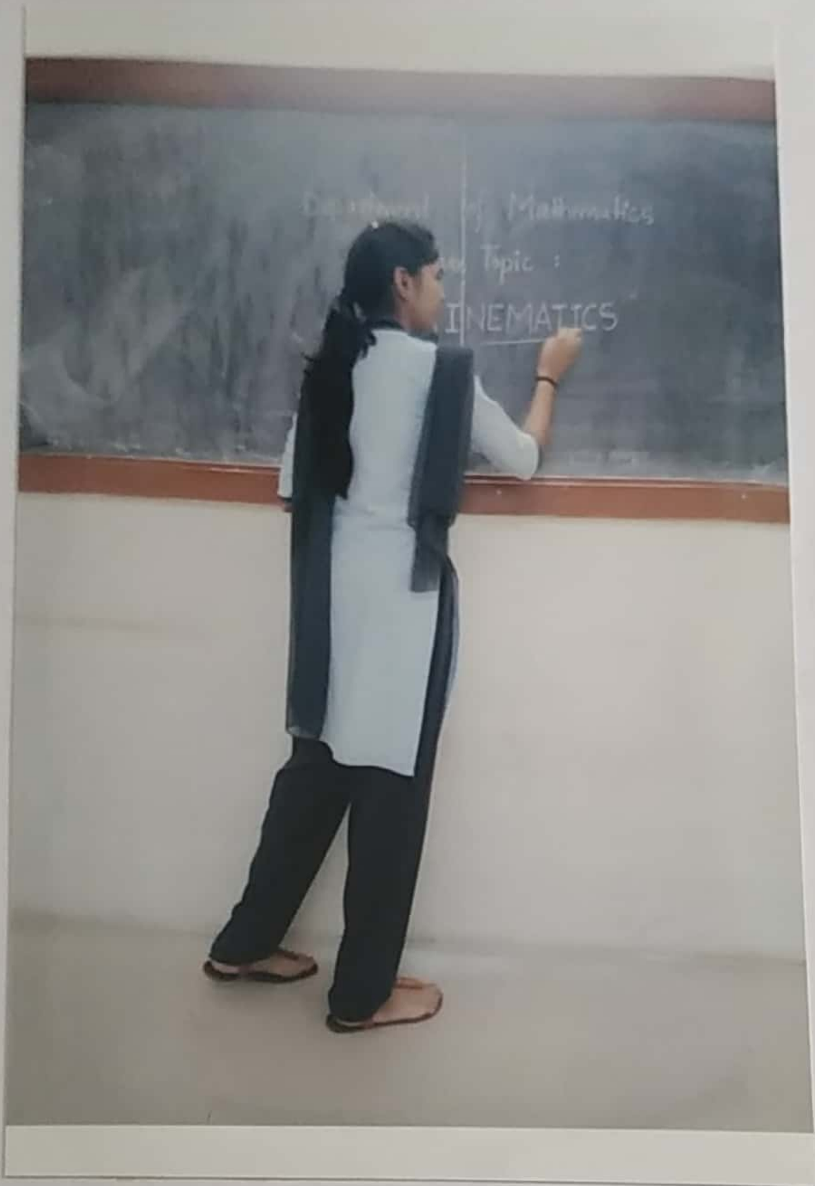
SEMINAR ON KINEMATICS

MATHS PAPER - III

Pallavi. A. Akkiwad
Roll No : 43
Uni. No : S1731028
Guided by : Pavitra Mam
Class : B. Sc V - Div C


Mathematics
Topic :

Q1. A particle moves along a circle $r = 2a\cos\theta$ in such a way that its acceleration towards origin is always zero. PT $\frac{d^2\theta}{dt^2} = -2\cot\theta\left(\frac{d\theta}{dt}\right)^2$

Soln: $r = 2a\cos\theta$ ——— (1)

Radial acceleration $= 0$

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = 0$$

$$\frac{d^2r}{dt^2} = r\left(\frac{d\theta}{dt}\right)^2 \quad \text{——— (2)}$$

from (1)

$$\frac{dr}{dt} = -2a\sin\theta.\frac{d\theta}{dt}$$

$$\frac{d^2r}{dt^2} = -2a\left[\sin\theta.\frac{d^2\theta}{dt^2} + \frac{d\theta}{dt}\cos\theta.\frac{d\theta}{dt}\right]$$

$$\frac{d^2r}{dt^2} = -2a\sin\theta\frac{d^2\theta}{dt^2} - 2a\cos\theta\left(\frac{d\theta}{dt}\right)^2 \quad \text{——— (3)}$$

(2) & (3)

$$r\left(\frac{d\theta}{dt}\right)^2 = -2a\sin\theta\frac{d^2\theta}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$$

$$2r\left(\frac{d\theta}{dt}\right)^2 = -2a\sin\theta\frac{d^2\theta}{dt^2}$$

$$-\frac{2r\left(\frac{d\theta}{dt}\right)^2}{2a\sin\theta\frac{d^2\theta}{dt^2}} = 1$$

$$-\frac{2\times \cancel{2a}\cos\theta\left(\frac{d\theta}{dt}\right)^2}{\cancel{2a}\sin\theta} = \frac{d^2\theta}{dt^2}$$

$$\boxed{-2a\cot\theta\left(\frac{d\theta}{dt}\right)^2 = \frac{d^2\theta}{dt^2}}$$

2] The component of velocity of a particle moving along a plane curve & $\perp^r$ to radius vector from a fixed point are λr^n & $\mu\theta^n$ respectively. Derive the equation for path.

Soln: Radial velocity $= \frac{dr}{dt} = \lambda r^n$ —— ①

Transverse velocity $= \frac{r\,d\theta}{dt} = \mu\theta^n$ —— ②

①/②

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{\lambda}{\mu}\cdot\frac{r^n}{\theta^n}$$

$$\mu\frac{dr}{r^{n+1}} = \lambda\frac{d\theta}{\theta^n}$$

On integrating b.s

$$\int \mu\frac{dr}{r^{n+1}} = \int \lambda\frac{d\theta}{\theta^n}$$

$$\mu\frac{r^{-(n+1)+1}}{-(n+1)+1} = \lambda\frac{\theta^{-n+1}}{-n+1} + c$$

$$\cancel{-}\mu\frac{1}{nr^n} = \cancel{-}\lambda\frac{1}{(1-n)\theta^{n-1}} + c. \quad \text{(taking '−' common)}$$

$$\boxed{\mu(1-n)\theta^{n-1} = \lambda n r^n - cnr^n(1-n)\theta^{n-1}}$$

8. A particle is moving in a parabola ($p^2 = ar$) with uniform angular velocity about the focus. Prove that its normal acceleration at any point is proportional to the radius of curvature of its path at that point.

Soln:

$$p^2 = ar \quad \text{——— (1)}$$

$$2p\frac{dp}{dr} = a$$

$$\frac{dp}{dr} = \frac{a}{2p}$$

$$\because \quad \rho = \frac{r\,dr}{dp}$$

$$\rho = r\frac{2p}{a} = \frac{2pr}{a} \quad \text{——— (2)}$$

$$\frac{d\theta}{dt} = k \text{ (say)} \quad \text{——— (3)}$$

$$\frac{d\theta}{dt} = \frac{vp}{r^2}$$

$$v = \frac{kr^2}{p}$$

$$\frac{\text{Normal Acceleration}}{\rho} = \frac{v^2}{\rho^2}$$

$$= \frac{k^2r^4}{p^2} \times \frac{a^2}{4p^2r^2}$$

$$= \frac{k^2r^2a^2}{4p^4}$$

$$= \frac{k^2\cancel{r^2}\cancel{a^2}}{4\cancel{a^2}\cancel{r^2}}$$

$$\Rightarrow \frac{k^2}{4} = \text{Constant}$$

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SEMINAR REPORT: 2019-20

(Odd Semester)

The UG Department of Mathematics has conducted seminar for B.Sc students.

Name of the student	Seminar Topic
1) Kiran .G. Watharkar	Theory of equations
2) Apoorva Mirajkar	Homogeneous Eq^ns
3) Pallavi Akkiwaad	kinematics

Head of Department

H. O. D.
Department of Mathematics
S. B. Arts & K. C. P. Science
College, BIJAPUR.

Principal

Principal,
S. B. Arts & KCP Sc. College,
Bijapur

IQAC, Co-ordinator
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SEMINAR REPORT

2019-20 (EVEN SEMESTER)

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The UG Department of Mathematics is conducting seminar for B.Sc students for the academic year 2019-20(Even Semester)

IQAC, Co-ordinator
.Arts & K.C.P.Science College,
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Head of Department

H. O. D.
Department of Mathematics,
S. B. Arts & K. C. P. Science
College, Bijapur.

Principal,
S.B.Arts & K.C.P. Science College,
VIJAYAPUR.

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DEPARTMENT OF MATHEMATICS

Seminar Report-2019-20

Even Seminar

Sl. No	Semester	Name of the Student	Topics	Date
1	II	Swati Malaghan	Eulers Theorem	03-03-2020
2	IV	Aishwarya Biradar	Differential Equations	11-03-2020
3	VI	Lingaraj Mattihal	Power Series solution	16-03-2020

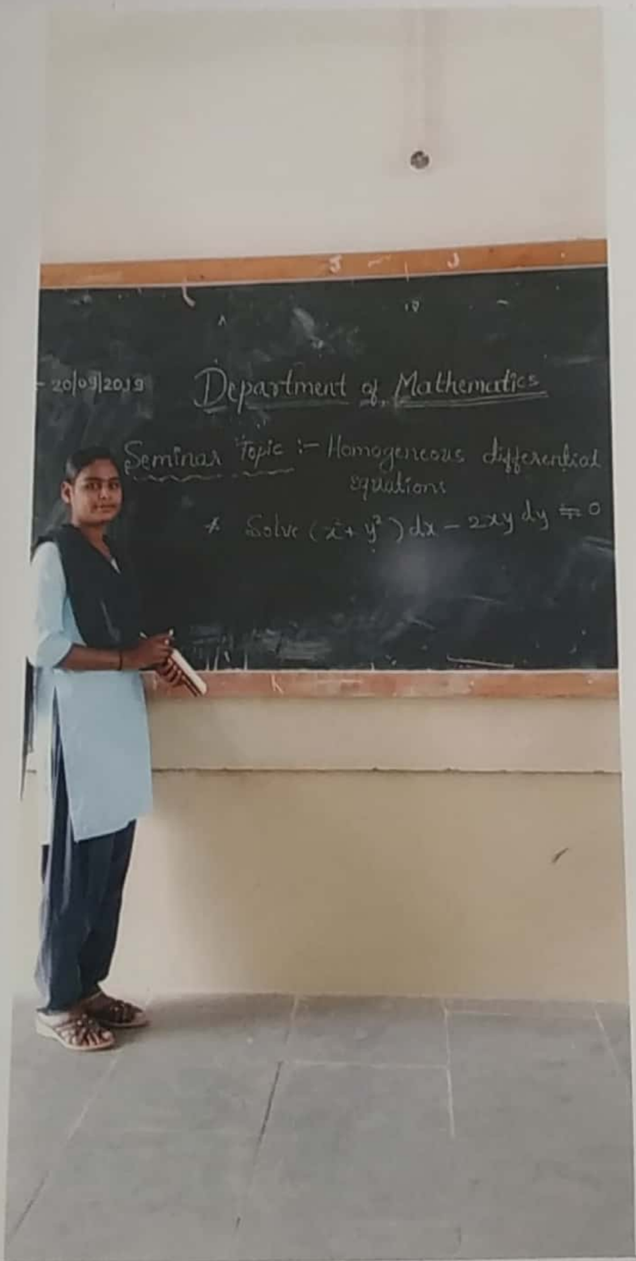
Name : Swati R Malegaon

Class : Bsc IInd Sem

Roll no : 183

Sub : Mathematics

Uni No : S1925313



Homogenous Functions and Euler's Theorem

Consider the function,

$$f(x,y) = a_0 x^n + a_1 x^{n-1} y + a_2 x^{n-2} y^2 + \cdots + a_{n-1} x y^{n-1} + a_n y^n \quad \text{--- (1)}$$

this Expression is polynomial in (x,y) such that the degree of Each of the terms is same, i.e, n.

since f is a homogeneous function of degree n, it is Expressible as, $x^n f\left(\frac{y}{x}\right)$

The polynomial function (1) which can be written as

$$x^n \left[a_0 + a_1 \frac{y}{x} + a_2 \left(\frac{y}{x}\right)^2 + \cdots + a_n \left(\frac{y}{x}\right)^n\right]$$

is a homogenous Expression of degree n.

- **Homogenous function of two Variables**

An Expression $f(x,y)$ in x and y is said to be homogenous function of degree n.

$$\text{If } f(x,y) = x^n f\left(\frac{y}{x}\right)$$

Example: $f(x,y) = \dfrac{x^3+y^3}{x^3-y^3}$

$$= \frac{x^3\left(1+\frac{y^3}{x^3}\right)}{x\left(1-\frac{y}{x}\right)} = x^2 \left[\frac{1+\frac{y^3}{x^3}}{1-\frac{y}{x}}\right]$$

$$f(x,y) = x^2 f\left(\frac{y}{x}\right)$$

It is homogeneous function of degree 2.

Homogeneous function of three Variables

An Expression $f(x,y,z)$ in x, y and z is said to be homogenous function of degree n.

If $f(x,y,z) = x^n f_1\left(\frac{y}{x}, \frac{z}{x}\right) = y^n f_2\left(\frac{x}{y}, \frac{z}{y}\right) = z^n f_3\left(\frac{x}{z}, \frac{y}{z}\right)$

Examples : $f(x,y,z) = \dfrac{x^4+y^4+z^4}{x+y+z}$

$$= \frac{x^4\left\{1+\left(\frac{y}{x}\right)^4+\left(\frac{z}{x}\right)^4\right\}}{x\left\{1+\left(\frac{y}{x}\right)+\left(\frac{z}{x}\right)\right\}}$$

$$= x^3 f\left(\frac{y}{x}, \frac{z}{x}\right)$$

Youngs Theorem for Homogeneous Function

Statement : If 'u' is a homogeneous function in x and y then $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$

proof : u is homogenous function in x and y

Let, $u(x,y) = x^n f\left(\frac{y}{x}\right)$ — (1)

Differentiating eqn (1) partially w.r.to 'y'

$$\frac{\partial u}{\partial y} = x^n f'\left(\frac{y}{x}\right) \frac{\partial}{\partial y}\left(\frac{y}{x}\right)$$

$$= x^n f'\left(\frac{y}{x}\right)\left(\frac{1}{x}\right)$$

$$= x^{n-1} f'\left(\frac{y}{x}\right)$$

Again differentiating w.r.to 'x'

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = x^{n-1} f''\left(\frac{y}{x}\right)\left(-\frac{y}{x^2}\right) + (n-1)\, x^{n-2} f'\left(\frac{y}{x}\right)$$

$$= -y x^{n-1} x^{-2} f''\left(\frac{y}{x}\right) + (n-1) x^{n-2} f'\left(\frac{y}{x}\right)$$

$$\frac{\partial^2 u}{\partial x \partial y} = -y x^{n-3} f''\left(\frac{y}{x}\right) + (n-1) x^{n-2} f'\left(\frac{y}{x}\right) \quad \text{—(2)}$$

Differentiating Eqn (1) p. w.r.t 'x'

$$\frac{\partial u}{\partial x} = x^n f'\left(\frac{y}{x}\right)\left(-\frac{y}{x^2}\right) + n x^{n-1} f\left(\frac{y}{x}\right)$$

$$\frac{\partial u}{\partial x} = -y x^{n-2} f'\left(\frac{y}{x}\right) + n x^{n-1} f\left(\frac{y}{x}\right)$$

Again differentiate partially w.r.to 'y'

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = -x^{n-2} \frac{\partial}{\partial y}\left\{y f'\left(\frac{y}{x}\right)\right\} + n x^{n-1} \frac{\partial}{\partial y}\left\{f\left(\frac{y}{x}\right)\right\}$$

$$= -x^{n-2}\left\{y f''\left(\frac{y}{x}\right)\left(\frac{1}{x}\right) + f'\left(\frac{y}{x}\right)\right\} + n x^{n-1} f'\left(\frac{y}{x}\right)\left(\frac{1}{x}\right)$$

$$= -x^{n-2} \frac{y}{x} f''\left(\frac{y}{x}\right) - x^{n-2} f'\left(\frac{y}{x}\right) + n x^{n-2} f'\left(\frac{y}{x}\right)$$

$$= -x^{n-2} x^{-1} y f''\left(\frac{y}{x}\right) - x^{n-2} f'\left(\frac{y}{x}\right) + n x^{n-2} f'\left(\frac{y}{x}\right)$$

$$= -x^{n-3} y f''\left(\frac{y}{x}\right) - x^{n-2} f'\left(\frac{y'}{x}\right) + n x^{n-2} f'\left(\frac{y}{x}\right)$$

$$\frac{\partial^2 u}{\partial x \partial y} = -x^{n-3} y f''\left(\frac{y}{x}\right) + (n-1) x^{n-2} f'\left(\frac{y}{x}\right) \quad -(3)$$

Now, from Equation (2) and (3)

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x}$$

Euler's Theorem for Homogeneous Function

Statement : If $u = f(x,y)$ is a homogeneous function of degree 'n' then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

proof : Given that $u = f(x,y)$ is a homogeneous function of degree 'n'.

$$u = x^n g\left(\frac{y}{x}\right) \quad \text{— (1)}$$

Differentiate Equation (1) partially w.r.to 'x'

$$\frac{\partial u}{\partial x} = x^n g'\left(\frac{y}{x}\right)\left(-\frac{y}{x^2}\right) + n x^{n-1} g\left(\frac{y}{x}\right)$$

$$= -y\, x^n x^{-2} g'\left(\frac{y}{x}\right) + n x^{n-1} g\left(\frac{y}{x}\right)$$

$$\frac{\partial u}{\partial x} = -y\, x^{n-2} g'\left(\frac{y}{x}\right) + n x^{n-1} g\left(\frac{y}{x}\right)$$

Multiplig 'x' on both side, we have

$$x\frac{\partial u}{\partial x} = -y\, x^{n-2} x \cdot g'\left(\frac{y}{x}\right) + n x^{n-1} \cdot x\, g\left(\frac{y}{x}\right)$$

$$= -y\, x^{n-1} g'\left(\frac{y}{x}\right) + n x^n g\left(\frac{y}{x}\right) \quad -(2)$$

Differentiate Equation (1) partially w.r.to 'y'

$$\frac{\partial u}{\partial y} = x^n g'\left(\frac{y}{x}\right)\left(\frac{1}{x}\right)(1)$$

$$= x^n g'\left(\frac{y}{x}\right) x^{-1}$$

$$\frac{\partial u}{\partial y} = x^{n-1} g'\left(\frac{y}{x}\right)$$

Multiplying 'y' on b.s, we have

$$y\frac{\partial u}{\partial y} = y\, x^{n-1} g'\left(\frac{y}{x}\right)$$

Adding equations (2) and (3), we get

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -y\, x^{n-1} g'\left(\frac{y}{x}\right) + n x^n g\left(\frac{y}{x}\right) + y\, x^{n-1} g'\left(\frac{y}{x}\right)$$

$$= n x^n g\left(\frac{y}{x}\right)$$

$$= nu \qquad \left|\because u = x^n g\left(\frac{y}{x}\right)\right.$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu.$$

Extensions of Euler's Theorem

Theorem 1: If $u = f(x,y)$ is a homogeneous function of degree n, then show that,

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u$$

proof: given that, $u = f(x,y)$ is a homogeneous function of degree n.

By Euler's theorem,

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu \quad \text{--- (1)}$$

Differeiantiate Eqn (1) partially w.r.to 'x'

$$x \frac{\partial^2 u}{\partial x} + \frac{\partial u}{\partial x}(1) + y\left(\frac{\partial^2 u}{\partial x \partial y}\right) = n \frac{\partial u}{\partial x}$$

$$x \frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 u}{\partial x \partial y} = n \frac{\partial u}{\partial x} - \frac{\partial u}{\partial x}$$

$$x \frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 u}{\partial x \partial y} = (n-1) \frac{\partial u}{\partial x}$$

Multiplying 'x' on b.s, we have

$$x^2 \frac{\partial^2 u}{\partial x^2} + xy \frac{\partial^2 u}{\partial x \partial y} = x(n-1) \frac{\partial u}{\partial x} \quad \text{--- (2)}$$

Differentiating Eqn (1) partially w.r.to 'y'

$$x \frac{\partial^2 u}{\partial y \partial x} + y \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}(1) = n \frac{\partial u}{\partial y}$$

$$x \frac{\partial^2 u}{\partial y \partial x} + y \frac{\partial^2 u}{\partial y^2} = n \frac{\partial u}{\partial y} - \frac{\partial u}{\partial y}$$

$$x \frac{\partial^2 u}{\partial y \partial x} + y \frac{\partial^2 u}{\partial y^2} = (n-1) \frac{\partial u}{\partial y}$$

Differentiating Equation (1) partially w.r.to 'y'

$$\frac{x\,\partial^2 u}{\partial y\,\partial x} + y\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}(1) = n\frac{\partial u}{\partial y}$$

$$\frac{x\,\partial^2 u}{\partial y\,\partial x} + y\frac{\partial^2 u}{\partial y^2} = n\frac{\partial u}{\partial y} - \frac{\partial u}{\partial y}$$

$$\frac{x\,\partial^2 u}{\partial y\,\partial x} + y\frac{\partial^2 u}{\partial y^2} = (n-1)\frac{\partial u}{\partial y}$$

Multiplying 'y' on both side, we have

$$xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (n-1)\,y\frac{\partial u}{\partial y} \rightarrow (3) \quad \Big|\because \frac{\partial^2 u}{\partial y\,\partial x} = \frac{\partial^2}{\partial x\,\partial y}$$

Now adding Equations (2) and (3), we have

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = x(n-1)\frac{\partial u}{\partial x} + (n-1)y\frac{\partial u}{\partial y}.$$

$$= (n-1)\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right]$$

$$= (n-1)\,nu \quad \Big|\because \text{from Eq}^n(1)$$

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

Theorem 2:

If $u = f(x,y,z)$ is a Homogeneous function of degree n, then show that.

$$x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + z^2\frac{\partial^2 u}{\partial z^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y}$$
$$+ 2yz\frac{\partial^2 u}{\partial y\,\partial z} + 2zx \times \frac{\partial^2 u}{\partial z\,\partial x} = (n-1)nu.$$

proof: Let $u = f(x,y,z)$ be a homogeneous function of degree n.

By using Euler's theorem for n^{th} degree,

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = nu \quad —(1)$$

Differentiate equation (1) partially w.r.to (x)

$$x\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + y\frac{\partial^2 u}{\partial x \partial y} + z\frac{\partial^2 u}{\partial x \partial z} = n\frac{\partial u}{\partial x}$$

$$x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} + z\frac{\partial^2 u}{\partial x \partial z} = n\frac{\partial u}{\partial x} - \frac{\partial u}{\partial x}$$

$$x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} + z\frac{\partial^2 u}{\partial x \partial z} = (n-1)\frac{\partial u}{\partial x}$$

Multiply (x) on both side, we have.

$$x^2\frac{\partial^2 u}{\partial x^2} + xy\frac{\partial^2 u}{\partial x \partial y} + xz\frac{\partial^2 u}{\partial x \partial z} = x(n-1)\frac{\partial u}{\partial x} \quad —($$

Differentiate equation (1) partially w.r.to (y)

$$x\frac{\partial^2 u}{\partial y \partial x} + y\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} + z\frac{\partial^2 u}{\partial y \partial z} = n\frac{\partial u}{\partial y}$$

$$x\frac{\partial^2 u}{\partial y \partial x} + y\frac{\partial^2 u}{\partial y^2} + z\frac{\partial^2 u}{\partial y \partial z} = n\frac{\partial u}{\partial y} - \frac{\partial u}{\partial y}$$

$$x\frac{\partial^2 u}{\partial y \partial x} + y\frac{\partial^2 u}{\partial y^2} + z\frac{\partial^2 u}{\partial y \partial z} = (n-1)\frac{\partial u}{\partial y}$$

put, $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial y z}$

Multiply (y) on both side, we have

$$xy\frac{\partial^2 u}{\partial y \partial x} + y^2\frac{\partial^2 u}{\partial y^2} + yz\frac{\partial^2 u}{\partial y \partial z} = y(n-1)\frac{\partial u}{\partial y} \quad —(3)$$

Differentiate Eqn (1) partially w.r.to (x)

$$x\frac{\partial^2 u}{\partial z \partial x} + y\frac{\partial^2 u}{\partial z \partial y} + z\frac{\partial^2 u}{\partial z^2} + \frac{\partial u}{\partial z} = n\frac{\partial u}{\partial z}$$

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$$x\frac{\partial^2 u}{\partial z\partial x} + y\frac{\partial^2 u}{\partial z\partial y} + z\frac{\partial^2 u}{\partial z^2} = n\frac{\partial u}{\partial z} - \frac{\partial u}{\partial z}$$

$$x\frac{\partial^2 u}{\partial z\partial x} + y\frac{\partial^2 u}{\partial z\partial y} + z\frac{\partial^2 u}{\partial z^2} = (n-1)\frac{\partial u}{\partial z}$$

Multiply 'z' on both side, we have

$$xz\frac{\partial^2 u}{\partial z\partial x} + yz\frac{\partial^2 u}{\partial z\partial y} + z^2\frac{\partial^2 u}{\partial z^2} = (n-1)z\frac{\partial u}{\partial z} \quad \text{— (4)}$$

Adding Equation (2) (3) and (4), we get

$$x^2\frac{\partial^2 u}{\partial x^2} + xy\frac{\partial^2 u}{\partial x\partial y} + xz\frac{\partial^2 u}{\partial x\partial z} + xy\frac{\partial^2 u}{\partial y\partial x} + y^2\frac{\partial^2 u}{\partial y^2}$$
$$+ yz\frac{\partial^2 u}{\partial y\partial z} + xz\frac{\partial^2 u}{\partial z\partial x} + yz\frac{\partial^2 u}{\partial z\partial y} + z^2\frac{\partial^2 u}{\partial z^2}$$

$$= x(n-1)\frac{\partial u}{\partial x} + (n-1)y\frac{\partial u}{\partial y} + (n-1)z\frac{\partial u}{\partial z}$$

$$= (n-1)\left\{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right\} = (n-1)nu$$

$$x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + z^2\frac{\partial^2 u}{\partial z^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + 2yz\frac{\partial^2 u}{\partial y\partial z} + 2zx\frac{\partial^2 u}{\partial z\partial x} = n(n-1)u.$$

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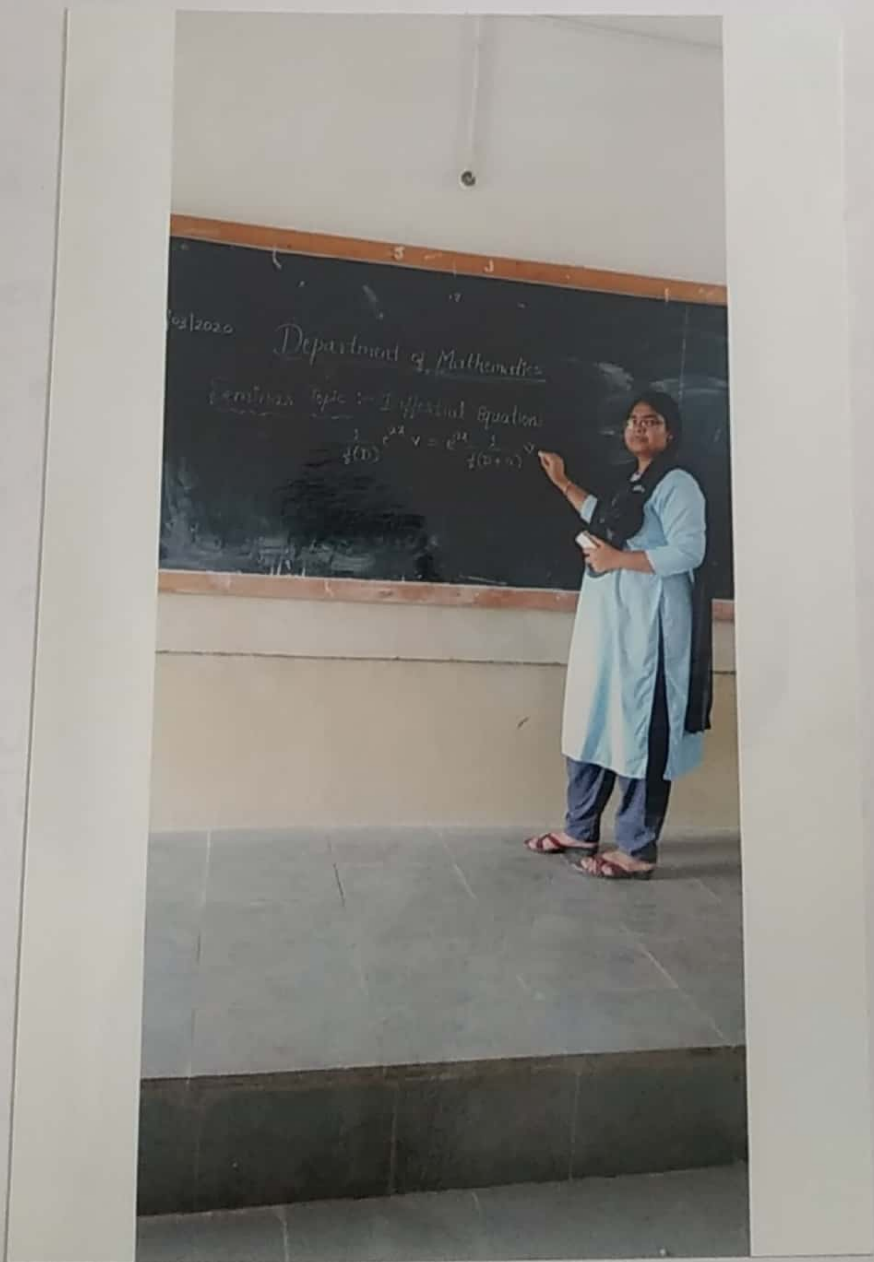

eorem:-

With usual notation prove that $\frac{1}{f(D)} e^{ax} V = e^{ax} \frac{1}{f(D+a)} V.$

where 'a' is a function of 'x'

oof:-

By the successive differentiation of $e^{ax} v$ gives

$$D[e^{ax} v] = e^{ax} Dv + a e^{ax} v$$

$$= e^{ax} [D+a] v$$

$$D^2 [e^{ax}.v] = D[D e^{ax}.v]$$

$$= D[e^{ax} (D+a) v]$$

$$D^2 [e^{ax}.v] = e^{ax} [D(D+a) + a(D+a)] v$$

$$D^2 [e^{ax}.v] = e^{ax} [(D+a)^2] v$$

In General

$$D^n [e^{ax}.v] = e^{ax} [(D+a)^n] v$$ treating $n=1$

$$f(D) e^{ax} v = e^{ax} f(D+a) v.$$

Put $f(D+a) v = v_1$ then Replace $v = \frac{v_1}{f(D+a)}$

$$e^{ax} \frac{v_1}{f(D+a)} = \frac{1}{f(D)} \cancel{f(D+a)} \frac{v_1}{\cancel{f(D+a)}}$$

$$e^{ax} \frac{v_1}{f(D+a)} = \frac{1}{f(D)} e^{ax} v_1$$

Replace v_1 by v

$$\frac{1}{f(D)} e^{ax} v = e^{ax} \frac{1}{f(D+a)} v.$$

Example.

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2x}\sin 2x.$$

The above d.E can be written as

$$D^2y - 2Dy + 5y = e^{2x}\sin 2x$$

$$y(D^2-2D+5)y = e^{2x}\sin 2x$$

$$A.E = m^2-2m+5$$

$$m = \frac{2 \pm \sqrt{4-4(1)(5)}}{2(1)}$$

$$m = \frac{2\pm\sqrt{4-20}}{2}$$

$$m = \frac{2\pm\sqrt{-16}}{2} = 1\pm 2i$$

$$C.F = e^{x}(C_1\cos 2x + C_2\sin 2x)$$

$$P.I = \frac{1}{D^2-2D+5} e^{2x}\sin 2x$$

$$= e^{2x}\frac{1}{D^2-2D+5}\sin 2x$$

$$= e^{2x}\frac{1}{(D+2)^2-2(D-2)+5}\sin 2x$$

$$= e^{2x}\frac{1}{D^2+4+4D-2D-4+5}\sin 2x$$

$$= e^{2x}\frac{1}{D^2+2D+5}\sin 2x$$

$$= e^{2x}\frac{1}{-2^2+2D+5}\sin 2x$$

$$= e^{2x}\frac{1}{2D+1}\sin 2x$$

$$= e^{2x} \frac{1}{2D+1} \frac{2D-1}{2D-1} \sin 2x$$

$$= e^{2x} \frac{2D-1}{4D^2-1} \sin 2x$$

$$= e^{2x} \frac{2D-1}{4(-2^2)-1} \sin 2x$$

$$= e^{2x} \frac{2D-1}{-16-1} \sin 2x$$

$$= e^{2x} \frac{2D-1}{-17} \sin 2x$$

$$= \frac{e^{2x}}{-17} [2\cos 2x . 2 - \sin 2x]$$

$$= \frac{e^{2x}}{-17} [4\cos 2x - \sin 2x.$$

solution is

$$y = C.F + P.I$$

$$= e^{x}(c_1 \cos 2x + c_2 \sin 2x) +$$

$$\frac{e^{2x}}{-17} [4\cos 2x - \sin 2x].$$

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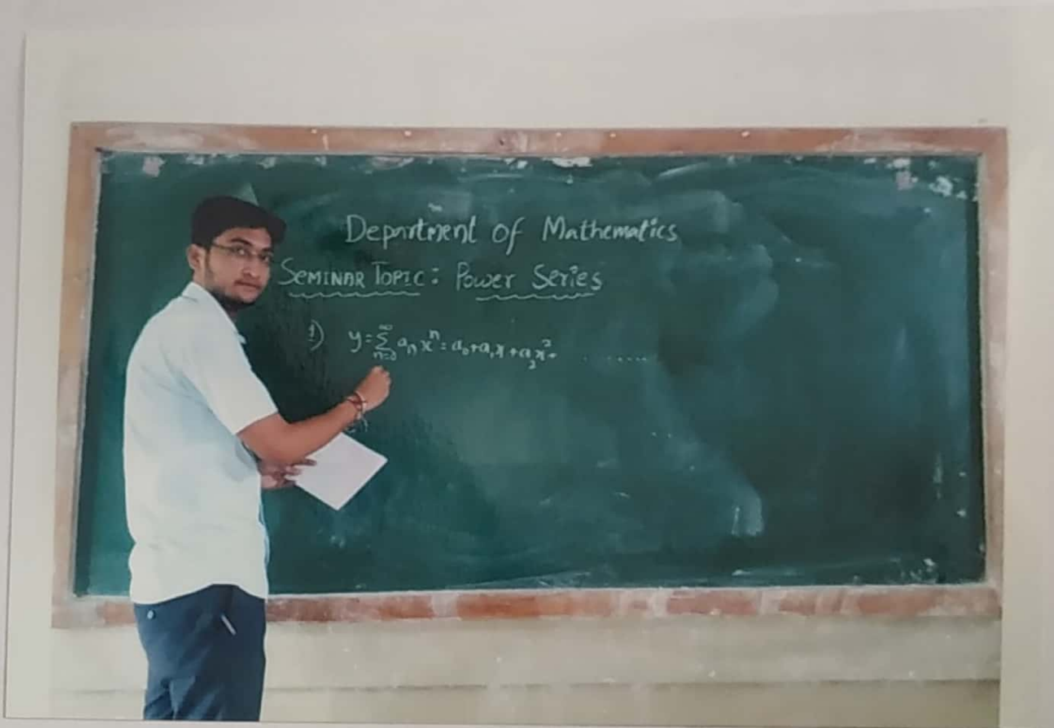

Series solution of Ordinary differential equations

Power series

An infinite series of the form $y = a_0 + a_1 x + a_2 x^2 + \dots = \sum_{n=0}^{\infty} a_n x^n$ where $a_0, a_1, a_2 \dots$ are constants is called a power series in x.

Convergence of Power series

We know that power series are functions of x & we know that not every series will infact exist for all x. This question is answered by looking at the convergence of the power series.

We say that a power series converges for $x = c$ if the series $\sum_{n=0}^{\infty} a_n (x - x_0)^n$ converges.

Every power series in x falls into one of three categories

Category-1 : The power series converges only for $x = 0$.

Category-2 : The power series converges for $|x| < R$ & diverges (that is, fails to converge) for $|x| > R$ (where R is some positive number).

Category-3 : The power series converges for all x.

Since the power series converges only for $x = 0$ are essentially useless, only those power series that fall into category 2 or category 3 will be discussed here.

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$\sum_{n=0}^{\infty} \frac{x^n}{5^n}$ where

Soln: Given series is $\sum_{n=0}^{\infty} \frac{x^n}{5^n}$ & comparing it with $\sum_{n=0}^{\infty} a_n x^n$.

we get $a_n = \frac{1}{5^n}$ & $a_{n+1} = \frac{1}{5^{n+1}}$

Now, $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{5^n}{5^{n+1}}\right| = \left|\frac{5^n}{5^n \cdot 5^1}\right| = \frac{1}{5}$

$\therefore \frac{1}{R} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{1}{5} = \frac{1}{5}$.

$\therefore$ Radius of convergence, $R = 5$.

2) Solve. $\sum_{n=0}^{\infty} (n+1)^2 x^n$.

Soln: Given series is $\sum_{n=0}^{\infty} (n+1)^2 x^n$ comparing with $\sum_{n=0}^{\infty} a_n x^n$.

we get $a_n = (n+1)^2$ & $a_{n+1} = (n+2)^2$

& $\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+2)^2}{(n+1)^2} = \frac{\cancel{n^2}(1+2/n)^2}{\cancel{n^2}(1+1/n)^2}$.

$\therefore \frac{1}{R} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left(\frac{1+2/n}{1+1/n}\right)^2$

$= \left(\frac{1+0}{1+0}\right)^2 = 1$

$\therefore$ The radius of convergence is $= 1$

Analytical function

If $f(x)$ is a function which is defined in a nbd of the point x_0 is said to be analytic at the point x_0. if it can be represented by a power series $\sum_{n=0}^{\infty} a_n (x-x_0)^n$ with centre at x_0 & radius of convergence $R > 0$.

Ordinary & Singular points of differential eq^n

Ordinary point :- Consider the point

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0 \rightarrow (1)$$

or $$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$$

where P & Q are polynomials in x.

If both $P(x)$ & $Q(x)$ are analytical at x_0 then the point x_0 is said to be ordinary point of diff. eq^n (1)

Singularity point :- Consider the eq^n

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$

Either $P(x)$ or $Q(x)$ or both $P(x)$ & $Q(x)$ are not analytic at $x = x_0$, then the point x_0 is called singular point of differential eq^n.

Ex 1 :- Show that $x = 0$ is an ordinary point of D.E.

$$(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - xy = 0.$$

Sol^n $$(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - xy = 0$$ we

dividing the eq^n by (x^2+1)

$$\frac{d^2y}{dx^2} + \frac{x}{(x^2+1)}\frac{dy}{dx} - \frac{x}{(x^2+1)}y = 0$$

Comparing this with standard eq^n i.e.

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + xy = 0,$$ we have

$$P(x) = \frac{x}{x^2+1} \text{ \& } Q(x) = \frac{-x}{x^2+1}$$

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Both nbd of $x=0$

i.e if $x=0 \Rightarrow P(x) = \frac{x}{x^2+1} = \frac{0}{0+1} = \frac{0}{1} = 0$

if $x=0 \Rightarrow Q(x) = \frac{-x}{x^2+1} = \frac{-0}{0+1} = \frac{0}{1} = 0$

Both $P(x)$ & $Q(x)$ ~~of~~ are analytic at $x=0$.

$\therefore x=0$ is an ordinary point of the given.

» Show that $x=0$ is a regular singular point & $x=1$ is an irregular singular point of the differential eqn

$$x(x-1)^3 \frac{d^2y}{dx^2} + 2(x-1)^2 \frac{dy}{dx} + 3y = 0.$$

Soln: Given differential eqn is

$$x(x-1)^3 \frac{d^2y}{dx^2} + 2(x-1)^2 \frac{dy}{dx} + 3y = 0$$

Dividing it by $x(x-1)^3$ in order to make the coefficient of $\frac{d^2y}{dx^2}$ unity, we get.

$$\frac{d^2y}{dx^2} + \frac{2}{x(x-1)} \frac{dy}{dx} + \frac{3}{x(x-1)^2} y = 0$$

On comparing it with $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$.

we get $P(x) = \frac{2}{x(x-1)}$ & $Q(x) = \frac{3}{x(x-1)^3}$

Both $P(x)$ & $Q(x)$ are not defined at $x=0$ & $x=1$.

$\therefore$ Both $P(x)$ & $Q(x)$ are not analytic at $x=0$ & $x=1$.

Hence $x=0$ & $x=1$ are singular points.

Now $(x-0)P(x) = \frac{2}{x-1}$ & $(x-0)^2 Q(x) = \frac{3x}{(x-1)^3}$

which shows that both $(x-0)P(x)$ & $(x-0)^2 Q(x)$ are analytic at $x=0$

$\therefore x=0$ is a regular singular point.

Also $(x-0)P(x) = 2/x$ & $(x-0)^2 Q(x) = 3$.

which shows that $(x-0)^2 Q(x)$ is not analytic at $x=1$

$\therefore x=1$, is an irregular singular point.

Power series sol^n of differential eq^n

If $x = x_0$ is an ordinary point of the differential eq^n

$$P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)y = 0$$

or

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Q(y) = 0$$

then every sol^n of this differential eq^n is analytic at $x = x_0$ & thus can be represented by a power series in power of x with radius of convergence $R>0$, as.

$$y = a_0 + a_1x + a_2x^2 + \dots + \sum_{n=0}^{\infty} a_n x^n.$$

Suppose the above differential eq^n can be expressed in power of $(x-x_0)$ with $R>0$ as

$$y = a_0 + a_1 x + a_2 x^2 + \dots + \sum_{n=0}^{\infty} a_n x^n$$

So where $a_1, a_2, a_3, \dots$ are constants.

If $x = x_0$ is a regular singularity then atleast one sol^n of the D.E can be expressed as.

$$y = (x-x_0)^n [a_0 + a_1(x-x_0) + a_2(x-x_0)^2 + \dots]$$

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Solving method to find the Power series (when x_0 is an ordinary point).

Consider the D.E

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$

Step 1:- Verify that x_0 is an ordinary point of the differential eq^n.

Step 2:- Assume the sol^n $y = a_0 + a_1 x + a_2 x^2 + \dots + \sum a_n x^n$

where $a_0, a_1, a_2 \dots$ are constants.

Step 3:- Find $\frac{dy}{dx} = \sum_{n=0}^{\infty} n a_n x^{n-1}$ ($n=0$ has no contribution to the sum) & again find. $\frac{d^2y}{dx^2} = \sum_{n=1}^{\infty} n(n-1) a_n x^{n-2}$

($m=0$ & $m=1$ have no contribution to the sum). & again find $\frac{d^2y}{dx^2} = \sum_{n=1}^{\infty} n(n-1) a_n x^{n-2}$

($m=0$ & $m=1$ have no contribution to sum).

Step-4:- Substitute the values of y, $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ in the given differential eq^n

Step-5 Write every summation in terms of x^n.

$$\sum^{\infty} a_n x^{n+2} = \sum_{n=2}^{\infty} a_{k-2} x^k \text{ (by putting } n+2=k)$$

$$= \sum_{n=1}^{\infty} a_{n-2} x^n \text{ (replacing } k \text{ by } n)$$

Step 6 : Equate to zero the coefficient of various power of x. to find out $a_2, a_3, \dots a_n$ in terms of a_0 & a_1

Step 7 :- Substitute $k = 0, 1, 2, \ldots$ in the recurrence relations to get the values of $a_2, a_3, \ldots a_n, \ldots$

Step 8 :- Substitute the values of $a_2, a_3 \ldots$ etc (obtained in step 6) in the solⁿ of given D.E when $x = 0$ the ordinary point.

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BLDE ASSOCIATION'S

S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

SEMINAR REPORT: 2019-20

(Even Semester)

The UG Department of Mathematics has conducted seminar for B.Sc students.

Name of the student	Seminar Topic
1) Swati Malegaum	Homogeneous f^n s
2) Aishwarya Biradar	Differential eq^n s
3) Lingaraj Matyal	Power series

Head of Department

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