

BLDE ASSOCIATION'S  
S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR



DEPARTMENT OF MATHEMATICS

**ADVANCE LEARNERS**

**For the Academic Year : 2017-18**

**BLDE ASSOCIATION'S**  
**S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR**



**DEPARTMENT OF MATHEMATICS**

**SEMINAR REPORT**  
**2017-18 (ODD SEMESTER)**

BLDE Association's  
S.B.Arts And K.C.P Science College, Vijayapur  
**DEPARTMENT OF MATHEMATICS**

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**NOTICE:**

The UG Department of Mathematics is conducting seminar for B.Sc students for the academic year 2017-18(Odd Semester)



**IQAC, Co-ordinator**  
S.B.Arts & K.C.P.Science  
Vijayapur.



**Principal,**  
S.B. Arts and KCP Science College  
VIJAYAPUR



**Head of Department**

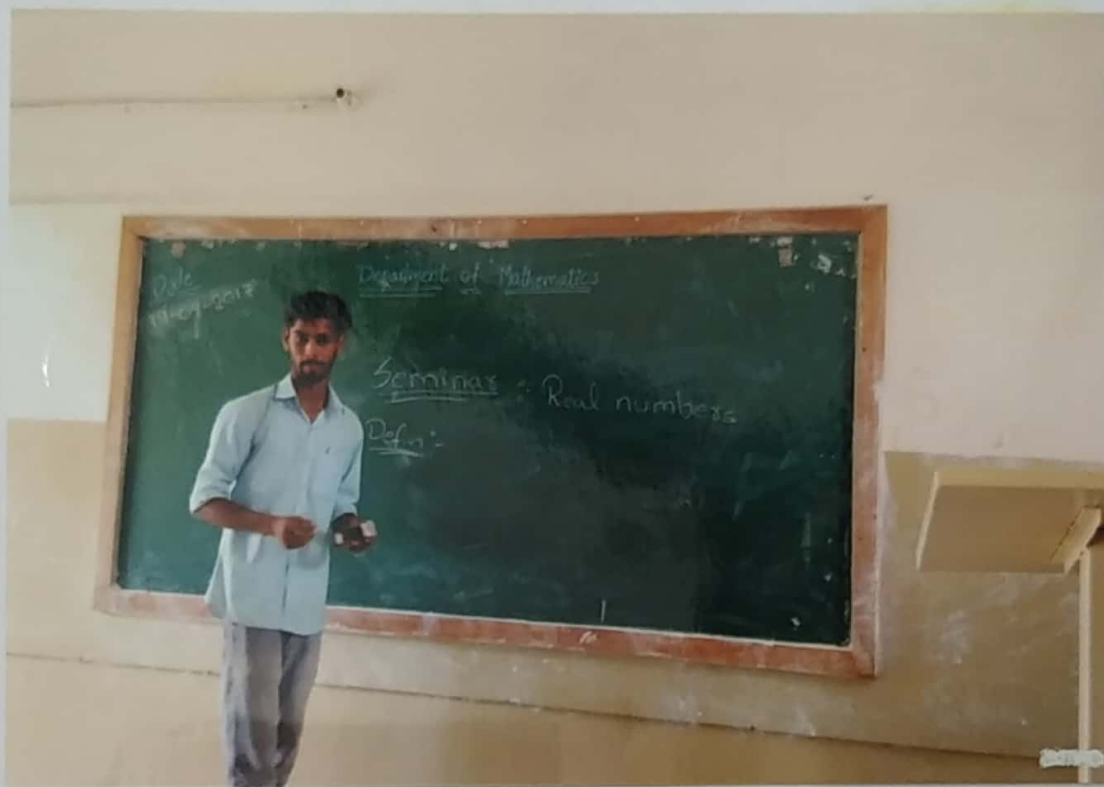
H. O. D:  
Department of Mathematics,  
S. B. Arts & K. C. P. Science  
College, BIJAPUR.

B.L.D.E.A's S.B.Arts & K.C.P Science College , Vijayapura

Department of Mathematics

SEMINAR – 2017-18 (ODD SEMESTER)

Sl.No	Semester	Name of the students	Topics	Date
1.	I	Akash Miragi	Real Numbers	19/09/2017
2.	III	Sushmita Mahindrakar	Properties of Jacobians	25/09/2017
3.	V	Soumya Muttagi	Kinematics	28/09/2017



Name : Akash Miragi

Semester : I

Topic : Real Numbers



# Mathematics Seminar

Topic :- Real Numbers

Prepared By :-

Name :- Akash. Miragi

Roll No :- 76

RCUB No :- S1721620

Sem : II<sup>Sem</sup> BSc

If  $a, b, c \in \mathbb{R}$  and  $a+b=a+c$  then  $b=c$

Solution:  $\forall a, b, c \in \mathbb{R}$  and  $a+b=a+c$

$$b=0+b$$

$$b=(-a+a)+b$$

$$b=-a+(a+b)$$

$$b=-a+(a+c)$$

$$b=(-a+a)+c \Rightarrow b=c$$

$\forall a, b, c \in \mathbb{R}$   $\& a+b=a+c \Rightarrow b=c$ .

Law of Trichotomy:-

Statement: If  $a, b \in \mathbb{R}$  then one and only one of the following three statements is true  $a < b, a = b, a > b$

Proof:  $a, b \in \mathbb{R} \Rightarrow a-b \in \mathbb{R}$

$$a-b > 0, a-b = 0, a-b < 0$$

$$a > b, a = b, a < b$$

$\therefore a, b \in \mathbb{R}$  one and only one of the following three statements is true.

Transitive Law:-

Statement: If  $a, b, c \in \mathbb{R}$  then  $a < b$  and  $b < c \Rightarrow a < c$

Proof

$$a, b, c \in \mathbb{R}$$

$$a < b \& b < c$$

$$a-b > 0 \quad b-c > 0$$

$$[(a-b) + (b-c)] > 0$$

$$(c-b) + (b-a) > 0$$

$$c-a > 0$$

$$c > a$$

$$\therefore a < c$$

If  $a > 0$  &  $b > 0$  then prove that  $a^2 + b^2 > 2ab$

Solution:-

To prove that  $a^2 + b^2 > 2ab$

We know that,  $a - b > 0$

$$(a - b)^2 > 0$$

$$a^2 + b^2 - 2ab > 0$$

$$a^2 + b^2 > 2ab.$$

If  $a, b, c \in \mathbb{R}$  the P.T  $a^2 + b^2 + c^2 \geq ab + bc + ca$ .

Solution:-

$$a \geq b$$

$$a - b \geq 0$$

$$(a - b)^2 \geq 0$$

$$a^2 + b^2 \geq 2ab \rightarrow (1)$$

$$b \geq c$$

$$(b - c)^2 \geq 0$$

$$b^2 + c^2 \geq 2bc \rightarrow (2)$$

$$c \geq a$$

$$(c - a)^2 \geq 0$$

$$c^2 + a^2 - 2ca \geq 0$$

$$c^2 + a^2 \geq 2ca \rightarrow (3)$$

From (1) & (2) & (3)

$$a^2 + b^2 + c^2 + a^2 + b^2 + c^2 \geq 2ab + 2bc + 2ca$$

$$2a^2 + 2b^2 + 2c^2 \geq 2(ab + bc + ca)$$

$$(a^2 + b^2 + c^2) \geq ab + bc + ca.$$



If  $a, b, c$  are all +ve S.T  $\frac{a+b+c}{3} > \sqrt[3]{abc}$  If  $a=b=c$   
 $\forall a, b, c \in \mathbb{R}^+$

Solution:  $a=b=c$

$$a > 0, b > 0, c > 0$$

$$a = x^3, b = y^3, c = z^3$$

$$x > 0, y > 0, z > 0$$

$$x > 0 \ \& \ y > 0 \Rightarrow (x-y)^2 > 0 \Rightarrow x^2 + y^2 + 2xy > 0 \rightarrow \textcircled{1}$$

$$y > 0 \ \& \ z > 0 \Rightarrow (y-z)^2 > 0 \Rightarrow y^2 + z^2 + 2yz > 0 \rightarrow \textcircled{2}$$

$$z > 0 \ \& \ x > 0 \Rightarrow (z-x)^2 > 0 \Rightarrow z^2 + x^2 - 2xz > 0 \rightarrow \textcircled{3}$$

Adding above Equation

$$x^2 + y^2 + y^2 + z^2 + z^2 + x^2 \geq 2xy + 2yz + 2xz$$

$$2x^2 + 2y^2 + 2z^2 \geq 2xy + 2yz + 2xz$$

$$x^2 + y^2 + z^2 > xy + yz + xz$$

$$(x^3 + y^3 + z^3) = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx) > 0$$

By eq<sup>n</sup>  $\textcircled{1}$  &  $\textcircled{2}$

$$x^3 + y^3 + z^3 - 3xyz > 0$$

$$x^3 + y^3 + z^3 > 3xyz$$

$$x^3 + y^3 + z^3 > 3\sqrt[3]{xyz}$$

$$\frac{a+b+c}{3} > \sqrt[3]{xyz}$$

For any  $a, b \in \mathbb{R}$  then Prove that  $\frac{a+b}{2} \geq \sqrt{ab}$

Solution:  $\forall a, b \in \mathbb{N}$   
 $a = x^2$   $b = y^2$  and  $x = \sqrt{a}$ ,  $y = \sqrt{b}$   
 $x > 0$ ,  $y > 0$

$$(x-y)^2 > 0 \Rightarrow x^2 + y^2 - 2xy > 0$$

$$x^2 + y^2 > 2xy$$

$$\frac{x^2 + y^2}{2} > \sqrt{x^2 y^2}$$

$$\frac{a+b}{2} > \sqrt{ab}$$

If  $a, b, c, d \in \mathbb{R}^+$  then P.T

$$a^2b + b^2c + c^2a + ab^2 + bc^2 + ca^2 \geq 6abc$$

Solution:  $a > 0$ ,  $b > 0$ ,  $c > 0$

$$a > 0, b > 0 \Rightarrow (a-b)^2 \geq 0$$

$$b > 0, c > 0 \Rightarrow (b-c)^2 \geq 0$$

$$c > 0, a > 0 \Rightarrow (c-a)^2 \geq 0$$

$$(a-b)^2 \geq 0$$

$$a^2 + b^2 - 2ab \geq 0$$

$$a^2 + b^2 \geq 2ab$$

$$(b-c)^2 \geq 0 \Rightarrow b^2 + c^2 - 2bc \geq 0$$

$$(c-a)^2 \geq 0 \Rightarrow c^2 + a^2 - 2ca \geq 0$$

$$a(c-b)^2 \geq 0 \rightarrow (1)$$

$$b(c-a)^2 \geq 0 \rightarrow (2)$$

$$c(a-b)^2 \geq 0 \rightarrow (3)$$

From equation (1) (2) & (3)

$$a(b^2+c^2-2bc) + b(c^2+a^2-2ca) + c(a^2+b^2-2ab) \geq 0$$

$$ab^2+ac^2-2abc+bc^2+ba^2-2abc+ca^2+cb^2-2abc \geq 0$$

$$ab^2+ac^2+bc^2+ba^2+ca^2+cb^2 \geq 6abc.$$

\* If  $x, y \in \mathbb{R}$  then P.T  $|x-y| \geq ||x|-|y||$

Solution:-  $|x-y| = \sqrt{x^2+y^2-2xy}$

$$|x-y| = \sqrt{|x|^2+|y|^2-2|x||y|}$$

But  $x < |x|$   $y < |y|$

$$|x-y| \geq \sqrt{|x|^2+|y|^2-2|x||y|}$$

$$|x-y| \geq \sqrt{(|x|-|y|)^2} \quad (\sqrt{x^2}=|x|)$$

$$|x-y| \geq ||x|-|y||$$

### Archimedean Property of Real Number.

Theorem: State and Prove Archimedean Property of real number.

Statement: If  $x, y \in \mathbb{R}$ ,  $x > 0$   $\exists n \in \mathbb{N}$  then  $nx > y$

Proof: Since  $y \in \mathbb{R}$

By Trichotomy law

$$y < 0, y = 0, y > 0$$



Case I:- If  $y \leq 0$

Since  $x > 0, n > 0 \Rightarrow nx > 0$

$$\begin{aligned} nx > 0 &\geq y \\ \Downarrow \\ nx &> y \end{aligned}$$

Case II:- If  $y > 0$

Assume the result is false, then

$$nx \leq y, \forall n \in \mathbb{N}$$

The set  $A = \{nx \mid n \in \mathbb{N}\}$  is bounded above

l.u.b of  $A = u$

$$nx \leq u, \forall n \in \mathbb{N}$$

$$\Rightarrow (n+1)x \leq u$$

$$\Rightarrow nx + x \leq u$$

$$nx \leq u - x < u$$

$u - x$  is less than  $u$

$\therefore$  Contradiction to hypothesis

$\therefore$  Our Assumption  $nx \leq y$  is Wrong

$$\therefore nx > y$$





Name : Sushmita Mahindrakar

Semester : III

Topic : Properties of Jaccobians

S. B. Arts & K. C. P Science  
College Vijayapur.

Mathematics Seminar

Topic & Properties of Jacobian.

Name & Sushmita Mahindrakar

Roll NO & 408

RCU NO & S16 22491

Sem & 3<sup>rd</sup> Sem. BSc.

## Properties of Jacobian

Inverse Rule: If  $u, v$  are f's of  $x, y$  then  $\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)} = 1$ .

or if  $J = \frac{\partial(u, v)}{\partial(x, y)}$ ,  $J' = \frac{\partial(x, y)}{\partial(u, v)}$  then

$$J \cdot J' = 1$$

Proof

If  $u = f_1(x, y)$   $v = f_2(x, y)$  then we can express  $x, y$  as f's of  $u, v$ .  
differentiating  $u = f_1(x, y)$   $v = f_2(x, y)$  partially w.r.t.  $u, v$ .

$$u = f_1(x, y)$$

$$1 = \frac{\partial u}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial u}$$

$$0 = \frac{\partial u}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial v}$$

$$v = f_2(x, y)$$

$$0 = \frac{\partial v}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial v}{\partial y} \frac{\partial y}{\partial u}$$

$$1 = \frac{\partial v}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial v}{\partial y} \frac{\partial y}{\partial v}$$

Now,

$$\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)} = 1$$



$$= \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial u}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial u} & \frac{\partial u}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial v} \\ \frac{\partial v}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial v}{\partial y} \frac{\partial y}{\partial u} & \frac{\partial v}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial v}{\partial y} \frac{\partial y}{\partial v} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 1 - 0$$

$$\therefore \frac{\partial(x, y)}{\partial(u, v)} \cdot \frac{\partial(u, v)}{\partial(x, y)} = 1 - 0$$

$$J \cdot J' = 1 \quad J = \frac{\partial(x, y)}{\partial(u, v)}$$

Example

1) If  $x = u(1-v)$  &  $y = uv$  then P.T

Soln

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$



$$\begin{vmatrix} 1 & -v \\ v & u \end{vmatrix}$$

$$= u - uv + uv$$

$$= u$$

$$\begin{aligned} x &= u - uv & y &= uv \\ x &= u - y & v &= \frac{y}{u} \\ u &= x + y & v &= \frac{y}{x+y} \end{aligned}$$

$$\frac{\partial(u, v)}{\partial(x, y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

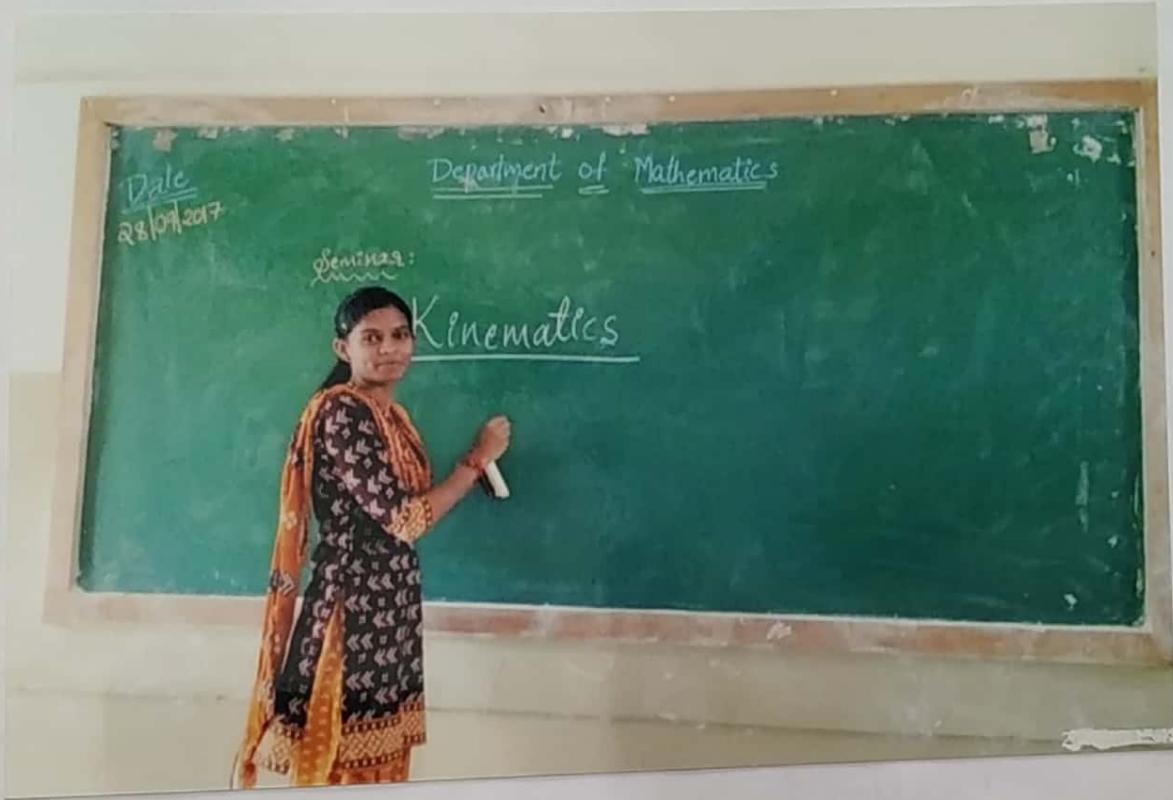
$$= \begin{vmatrix} -y & x \\ \frac{x}{(x+y)^2} & \frac{-y}{(x+y)^2} \end{vmatrix}$$

$$= \frac{-y}{(x+y)^2} + \frac{xy}{(x+y)^2}$$

$$= \frac{-y + xy}{(x+y)^2}$$

$$= \frac{-y(1-x)}{(x+y)^2}$$

$$= \frac{-y(1-x)}{u^2}$$



Name : Soumya Muttagi  
Semester : V  
Topic : Kinematics

S.B. Arts & K.C.P

Science College

Bijapur

Department of Mathematics

Seminar : Kinematics

Name : Soumya . S . Mutlagi

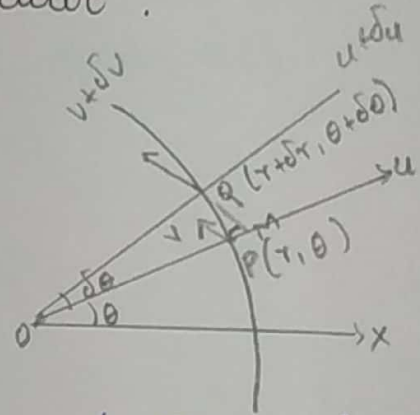
Sem : B.Sc V Sem

Date : 28-09-2017

Reg.No : S1528125



Expression for Radial and Transverse velocities of a particle moving in a plane curve.



Let  $P(r, \theta)$  and  $Q(r+dr, \theta+d\theta)$  be the positions of the particle in times  $t$  and  $t+dt$  respectively.

From right angled triangle,

$\Delta^{le} OQM$

$$\cos d\theta = \frac{OM}{OQ}$$

$$OQ \cos d\theta = OM$$

$$OM = (r+dr) \cos d\theta \quad \text{--- (1)}$$

$$\sin d\theta = \frac{QM}{OQ}$$

$$\sin d\theta OQ = QM$$

$$QM = (r+dr) \sin d\theta \quad \text{--- (2)}$$

The radial velocity at P =  $\lim_{dt \rightarrow 0} \frac{\text{displacement at P along OP}}{dt}$

$$= \lim_{dt \rightarrow 0} \frac{PM}{dt}$$

$$= \lim_{dt \rightarrow 0} \left[ \frac{(r+dr) \cos d\theta - r}{dt} \right]$$

$$= \lim_{dt \rightarrow 0} \left[ (r+dr) \left[ 1 - \frac{d^2\theta}{2!} + \frac{d^4\theta}{4!} \dots \right] - r \right]$$



Neglecting higher powers of  $d\theta$ , we have

$$= \lim_{dt \rightarrow 0} \frac{(r + dr)(1 - r)}{dt}$$

$$= \lim_{dt \rightarrow 0} \frac{dr}{dt}$$

$$= \frac{dr}{dt}$$

Radial velocity =  $\frac{dr}{dt}$

Transverse velocity at P =  $\lim_{dt \rightarrow 0} \frac{\text{displacement at P } \perp \text{ OP}}{dt}$

$$= \lim_{dt \rightarrow 0} \frac{QM}{dt}$$

$$= \lim_{dt \rightarrow 0} \frac{(r + dr) \sin d\theta}{d\theta}$$

Expanding  $\sin d\theta$  and neglecting higher powers of  $d\theta$ ,

$$= \lim_{dt \rightarrow 0} \frac{(r + dr) d\theta}{dt}$$

$$= \lim_{dt \rightarrow 0} \frac{r d\theta}{dt} + \frac{dr}{dt} d\theta$$

$$= \lim_{dt \rightarrow 0} \frac{r d\theta}{dt}$$

$$= \frac{r d\theta}{dt}$$

Transverse velocity at P =  $\frac{r d\theta}{dt}$

A point 'P'.

The radial velocity is proportional to its transverse velocity. Show that the path is an equiangular spiral.

∴ Given:

Radial velocity  $\propto$  Transverse velocity

$$\frac{dr}{dt} \propto r \frac{d\theta}{dt}$$

where  $k$  is constant

$$\frac{dr}{dt} = k r \frac{d\theta}{dt}$$

$$\frac{dr}{dt} = k r \frac{d\theta}{dt}$$

$$\frac{dr}{r} = k d\theta$$

∴ Integrating the above equation, we get

$$\log r = k\theta + \log a$$

where  $\log a$  is integrating constant

$$\log r - \log a = k\theta$$

$$\log\left(\frac{r}{a}\right) = k\theta$$

$$\frac{r}{a} = e^{k\theta}$$

$$\boxed{r = a e^{k\theta}}$$

which is eq<sup>n</sup> for equiangular spiral

show that the intrinsic equation of the path is  $s = ae^\psi + B$ , also show that the resultant velocity varies as  $e^\psi$ .

$$w) \text{ angular velocity} = k = \frac{d\psi}{dt}$$

$$\text{Tangential velocity} = \frac{ds}{dt} = \dot{s}$$

$$\text{normal velocity} = 0$$

$$\text{Tangential acc}^n = \text{Normal acc}^n$$

$$v \cdot \frac{dv}{ds} = \frac{v^2}{\rho}$$

$$\frac{dv}{ds} = v \cdot \frac{d\psi}{ds}$$

$$\frac{dv}{v} = d\psi$$

on integrating we get,

$$\log v = \psi + C_1$$

$$v = e^{\psi + C_1}$$

$$v = e^\psi \cdot e^{C_1}$$

$$v = e^\psi \cdot C$$

$$v \propto e^\psi$$

$\therefore$  Velocity varies as  $e^\psi$ .



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DEPARTMENT OF MATHEMATICS

SEMINAR REPORT: 2017-18  
(Odd Semester)

The UG Department of Mathematics has conducted seminar for B.Sc students.

Name of the student	Seminar Topic
1) Akash Mirgē	Real numbers
2) Sushmita Mahindrakar	Properties of Jacobian
3) Soumya Muttagi	Kinematics

Head of Department

H. O. D.  
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Principal

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**DEPARTMENT OF MATHEMATICS**

**SEMINAR REPORT**

**2017-18 (EVEN SEMESTER)**


**BLDE Association's**  
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**DEPARTMENT OF MATHEMATICS**


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S. B. Arts & K. C. P. Science College.

  
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H. O. D.  
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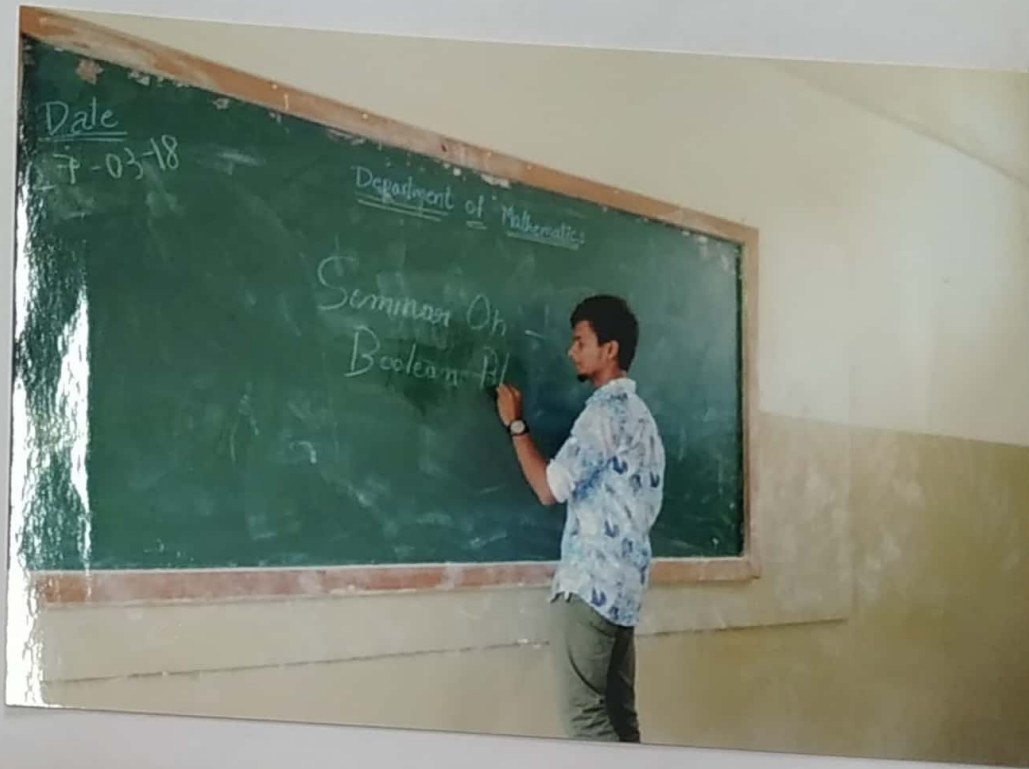


B.L.D.E.A's S.B.Arts & K.C.P Science College , Vijayapura

Department of Mathematics

SEMINAR – 2017-18 (EVEN SEMESTER)

Sl.No	Semester	Name of the students	Topics	Date
1.	II	Sandeep Ganni	Boolean Algebra	27/03/2018
2.	IV	Shruti Yadawad	Infinite Series - II	20/03/2018



Name : Sandeep. Ganni

Semester : II

Topic : Boolean Algebra

S.B. Arts & R.C.P Science college  
Bijapur

Department :- Mathematics  
Seminar on :- Boolean Algebra

Name :- Sandeep Sheni  
Stel :- Bsc III<sup>rd</sup> Sem  
Roll No :- 333

# SEMINAR ON BOOLEAN ALGEBRA :-

## Introduction :-

In mathematical logic Boolean Algebra is a branch of algebra in which we can use values of variables are truth values true & false, usually denoted by 1 & 0 respectively.

The basic laws of Boolean algebra that relate to the commutative law allowing a change in position for addition & multiplication, the associative law allowing the removal of brackets for addition & multiplication or useful as the factoring of an expression are same.

## Boolean Expression :-

$$A + 1 = 1$$

$$A + 0 = A$$

$$A \cdot 1 = A$$

$$A \cdot 0 = 0$$

$$A + A = A$$

$$A \cdot A = A$$

$$\bar{\bar{A}} = A$$



$$A + \bar{A} = 1$$

$$A \cdot \bar{A} = 0$$

$$A + B = B + A$$

$$A \cdot B = B \cdot A$$

$$\overline{A + B} = \bar{A} \bar{B}$$

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

These are the laws in Boolean Algebra :-

Identity law

Null law

Idempotent Law

Commutative law

Inverse law

Commutative law

Associative law

Absorption law

Distributive law

Demorgan's law

### 1] Identity law :-

Identity is a equal relation  $A = B$ , such that A & B contain some variable & A & B produce the same values as each other.

### 2] Null Law :-

Null law is defined as the expression which does not consider any values is considered as Null.

### 3] Idempotent law :-

It is the property of certain operation in Boolean Algebra that it can be explained multiple times without changing the result.

### 4] Commutative law :-

It is defined as either of true laws relating to number operations of addition & Multiplication.

### 5] Inverse law :-

It is type of a conditional which is opposite of another expression.

6] Associative law :-

It is defined as either true laws relating to nor operations such as addition & multiplication.

7] Distributive law :-

The law relating operation of multiplication & addition.

8] Absorption law :-

It is law for identifying or pair of binary operation.

9] Demorgan's law :-

This are pair of transformation rule that are both valid rules of Inference.





Name : Shruti. Yadavadi

Semester : IV

Topic : Infinite Series-II



B.L.D.E. Association

S.B. Arts and KCP Science

College Vijayapur

Seminar-Topic : Infinite Series - II

Shruti. M. Yadavadi.

Roll No: 404, IV Sem

R.C.U No: S1622433

Theorem :-

## De Alembert's Ratio Test.

statement :- If  $\sum u_n$  be a series of positive terms and  $\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = l$  then

i) The series converges if  $l < 1$

ii) The series diverges if  $l > 1$

iii) the series test fails if  $l = 1$ .

proof :- Since  $\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = l$

$\therefore$  By definition of limit

given  $\epsilon > 0$ , however small, then  $\exists$  integer  $m$ , such that

$$\left| \frac{u_{n+1}}{u_n} - l \right| < \epsilon \quad \forall n > m$$

$$- \epsilon < \frac{u_{n+1}}{u_n} - l < \epsilon \quad \forall n > m$$

$$l - \epsilon < \frac{u_{n+1}}{u_n} < l + \epsilon \quad \forall n > m \quad \text{--- (1)}$$

case (i) when  $l < 1$

Choose  $\epsilon > 0$  such that

$$l + \epsilon < 1$$

put  $l + \epsilon = r$  then  $0 < r < 1$

from (1)  $\frac{u_{n+1}}{u_n} < r \quad \forall n > m$

Now,

$$u_n = \frac{u_n}{u_{n-1}} \cdot \frac{u_{n-1}}{u_{n-2}} \cdot \frac{u_{n-2}}{u_{n-3}} \dots \frac{u_2}{u_1} \cdot u_1$$

$$u_n < (r \cdot r \cdot r \dots r) \cdot u_1$$

$$u_n < r^{n-1} u_1$$

If  $r < 1$ , then by geometric series

$\sum r^{n-1}$  is converges

$\therefore \sum u_n$  is converges

Hence the series is converges if  $l < 1$ .

Case (ii) When  $l > 1$ ,

Choose  $\epsilon > 0$ , s.t. that

$$l - \epsilon = R > 1$$

from (i),  $\frac{u_{n+1}}{u_n} > R$

Now

$$u_n = \frac{u_n}{u_{n-1}} \cdot \frac{u_{n-1}}{u_{n-2}} \cdot \frac{u_{n-2}}{u_{n-3}} \dots \frac{u_2}{u_1} \cdot u_1$$

$$u_n > (R \cdot R \cdot R \dots R) u_1$$

$$u_n > R^{n-1} u_1$$

If  $R > 1$  then by geometric series

$\therefore \sum R^{n-1}$  is divergent

$\therefore \sum u_n$  is also divergent



Hence the series diverges if  $l > 2$ .

Case (iii) If  $l = 1$  then series ratio test fails i.e. no conclusion can be drawn about the convergence or divergence of the series, the series may converge or it may diverge.

Practical form of De-Alembert's ratio test.

In practical ratio test is used in the following form if  $\sum u_n$  is a positive term series and  $\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} = l$  then.

- i) The series converges if  $l > 1$ .
- ii) The series diverges if  $l < 1$ .
- iii) The series test fails when  $l = 1$ .

Examples

1) Discuss the convergence of the following series.

$$\frac{1}{2^2} + \frac{2!}{3^3} + \frac{3!}{4^4} + \dots$$

Sol:

Here  $u_n = \frac{n!}{n^n}$

and  $u_{n+1} = \frac{(n+1)!}{(n+1)^{n+1}}$

$$\frac{u_n}{u_{n+1}} = \frac{n!}{n^n} \cdot \frac{(n+1)^{n+1}}{(n+1)!}$$



$$= \frac{n^{n+1} (1+1/n)^{n+1}}{n^n \cdot n (1+1/n)}$$

$$= \frac{n^n \cdot n (1+1/n)^n (1+1/n)}{n^n \cdot n (1+1/n)}$$

$$= (1+1/n)^n$$

$$\therefore \lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} = \lim_{n \rightarrow \infty} (1+1/n)^n$$

$$= e > 1 \quad \therefore 2 < e < 3$$

$\therefore$  By De-Alembert's ratio test

$\sum u_n$  is convergent

$$2) \quad 1 + \frac{2^p}{2!} + \frac{3^p}{3!} + \frac{4^p}{4!} + \dots \quad (p > 0)$$

Sol<sup>n</sup>: Here  $u_n = \frac{n^p}{n!}$

and

$$u_{n+1} = \frac{(n+1)^p}{(n+1)!}$$

$$= \frac{n^p (1+1/n)^p}{(n+1)n!}$$

$$\frac{u_n}{u_{n+1}} = \frac{n^p}{n!} \times \frac{(n+1)n!}{n^p (1+1/n)^p}$$

$$= \frac{n+1}{(1+1/n)^p}$$

$$\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} = \lim_{n \rightarrow \infty} \frac{n+1}{(1+1/n)^n}$$

$$= \infty > 1$$

∴ By de-Alembert's ratio test

Sum is convergent.

$$3) \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{2^{n-1} + 1} + \dots$$

∴ Here  $u_n = \frac{1}{2^{n-1} + 1}$

$$u_{n+1} = \frac{1}{2^n + 1}$$

$$\frac{u_n}{u_{n+1}} = \frac{1}{2^{n-1} + 1} \cdot \frac{2^n + 1}{1}$$

$$= \frac{2^n (1 + 1/2^n)}{2^{n-1} (1 + 1/2^{n-1})}$$

$$= \frac{2^n (1 + 1/2^n)}{2^n 2^{-1} (1 + 1/2^{n-1})}$$

$$= \frac{2 (1 + 1/2^n)}{(1 + 1/2^{n-1})}$$

$$\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} = \lim_{n \rightarrow \infty} \frac{2(1+1/2^n)}{(1+1/2^{n-1})}$$

$$= 2 > 1$$



∴ By De-Alembert's ratio test.

$\sum u_n$  is convergent.

$$4) \frac{1^2 \cdot 2^2}{1!} + \frac{2^2 \cdot 3^2}{2!} + \frac{3^2 \cdot 4^2}{3!} + \frac{4^2 \cdot 5^2}{4!} + \dots$$

Sol<sup>n</sup>: Here  $u_n = \frac{n^2(n+1)^2}{n!}$

$$u_{n+1} = \frac{(n+1)^2(n+2)^2}{(n+1)!}$$

$$= \frac{(n+1)^2(n+2)^2}{(n+1)n!}$$

$$\frac{u_n}{u_{n+1}} = \frac{n^2(n+1)^2 \cdot (n+1)n!}{n! \cdot (n+1)^2(n+2)^2}$$

$$= \frac{n^2(n+1)}{(n+2)^2}$$

$$= \frac{n^2 \cdot n(1+1/n)}{n^2(1+2/n)^2}$$

$$\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} = \lim_{n \rightarrow \infty} \frac{n(1+1/n)}{(1+2/n)^2}$$

$$= 1 > 1$$

∴ By De-Alembert's ratio test

$\sum u_n$  is convergent.

5)  $\frac{2}{1^2+1} + \frac{2^2}{2^2+1} + \frac{2^3}{3^2+1} + \dots$

Sol<sup>n</sup>: Here  $u_n = \frac{2^n}{n^2+1}$

$$u_{n+1} = \frac{2^{n+1}}{(n+1)^2+1}$$

$$= \frac{2^n \cdot 2}{n^2(1+1/n)^2 + 1/n^2}$$

$$\frac{u_n}{u_{n+1}} = \frac{2^n}{n^2+1} \times \frac{n^2(1+1/n)^2 + 1/n^2}{2^n \cdot 2}$$

$$= \frac{n^2(1+1/n^2) + 1/n^2}{2[n^2(1+1/n^2)]}$$

$$= 1/2 < 1$$

∴ By De-Alembert's ratio test

$\sum u_n$  is divergent.

6)  $\frac{1}{3} + \frac{8}{9} + \frac{27}{27} + \frac{64}{81} + \frac{125}{243} + \dots$

Sol<sup>n</sup>: The given series is

$$\frac{1^3}{3^1} + \frac{2^3}{3^2} + \frac{3^3}{3^3} + \frac{4^3}{3^4} + \frac{5^3}{3^5} + \dots$$

Here  $u_n = \frac{n^3}{3^n}$  and  $u_{n+1} = \frac{(n+1)^3}{3^{n+1}}$



$$\frac{u_n}{u_{n+1}} = \frac{n^3}{3^n} \cdot \frac{3^{n+1}}{(n+1)^3}$$

$$= \frac{n^3 \cdot 3}{n^3 (1 + 1/n)^3}$$

$$= \frac{3}{(1 + 1/n)^3}$$

$$\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} = \lim_{n \rightarrow \infty} \frac{3}{(1 + 1/n)^3}$$

$$= 3 > 1$$

∴ By De-Alembert's ratio test.  
∑ u<sub>n</sub> is convergent.

7)  $\frac{2!}{3} + \frac{3!}{3^2} + \frac{4!}{3^3} + \dots$

Sol<sup>n</sup> Here  $u_n = \frac{(n+1)!}{3^n}$

$$u_{n+1} = \frac{(n+2)!}{3^{n+1}}$$

$$= \frac{(n+1)n!}{3^n} \cdot \frac{3^{n+1}}{(n+1)(n+2)n!}$$

$$= \frac{3}{n+2}$$

$$\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} = \lim_{n \rightarrow \infty} \frac{3}{n+1}$$

$$= \frac{3}{\infty} = 0 < 1$$

∴ By De-Alembert's ratio test.

∑ u<sub>n</sub> is divergent.

$$8) \frac{1}{5} + \frac{2!}{5^2} + \frac{3!}{5^3} + \dots$$

sol<sup>n</sup>: Here  $u_n = \frac{n!}{5^n}$

$$u_{n+1} = \frac{(n+1)!}{5^{n+1}}$$

$$\frac{u_n}{u_{n+1}} = \frac{n!}{5^n} \cdot \frac{5^{n+1}}{(n+1)n!}$$

$$= \frac{5}{(n+1)}$$

$$\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} = \lim_{n \rightarrow \infty} \frac{5}{(n+1)}$$

$$= \frac{5}{\infty} = 0 < 1$$

∴ By de-Alembert's ratio test.

∑ u<sub>n</sub> is divergent.

BLDE ASSOCIATION'S  
S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR  
DEPARTMENT OF MATHEMATICS

SEMINAR REPORT: 2017-18  
(Even Semester)

The UG Department of Mathematics has conducted seminar for B.Sc students.

Name of the student

- 1) Sandeep Garri
- 2) Shreeti Yadavciad
- 3)


Seminar Topic

Boolean Algebra  
Infinite series - II


  
Head of Department

H. O. D.

Department of Mathematics,  
S. B. Arts & K. C. P. Science  
College, BIJAPUR.

  
IQAC, Co-ordinator

S.B.Arts & K.C.P.Science College,  
Vijayapur.

  
Principal

Principal,

S. B. Arts & KCP Sc. College,  
Bijapur