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S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

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For the Academic Year : 2016-17

BLDE ASSOCIATION'S

S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

SEMINAR REPORT

2016-17 (ODD SEMESTER)

BLDE Association's

S.B.Arts And K.C.P Science College, Vijayapur

DEPARTMENT OF MATHEMATICS

NOTICE:

The UG Department of Mathematics is conducting seminar for B.Sc students for the academic year 2016-17(Odd Semester)

Head of Department

IQAC, Co-ordinator
S.B.Arts & K.C.P.Science College
Vijayapur.

Principal,
S.B. Arts and KCP Science College
VIJAYAPUR

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Department of Mathematics.
S. B. Arts & K. C. P. Science
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B .L.D.E.A's S.B.Arts & K.C.P Science College, Vijayapura

Department of Mathematics

SEMINAR – 2016 -17 (ODD SEMESTER)

Sl.No	Semester	Name of the students	Topics	Date
1.	I	Rashmi Naikodi	Successive Differentiation	19/09/2016
2.	III	Amruta Kashetti	Sequence – II	29/09/2016
3.	V	Banashri Biradar	Impact Transformation	21/09/2016

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Name : Rashmi Naikodi

Semester : I

Topic : Successive Differentiation

Mathematics Seminar

Topic: Successive Differentiation.

Prepared by:-

Name:- Rashmi Naikodi

Roll No:- 430

RCU. No:- S1622329

Sem:- 1^{st} Sem Bsc.

Successive Differentiation

Definition:

The process of differentiating the same function again and again is called successive differentiation.

If $y = f(x)$ is a differentiable function of x then its derivative is called the 1^{st} order derivative of y w.r.t x. and is denoted by $\frac{dy}{dx}$. If $\frac{dy}{dx}$ is differentiable w.r.t x then its derivative is called the second order derivative of y w.r.t x and is denoted by $\frac{d^2y}{dx^2}$:

i. e, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

Similarly,

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$$

and so, on.

The n^{th} order derivative of y w.r.t x denoted by $\frac{d^ny}{dx^n}$

i. e., $\frac{d^ny}{dx^n} = \frac{d}{dx}\left(\frac{d^{n-1}y}{dx^{n-1}}\right)$

Notation:

If $y=f(x)$ be a function of x, its succesive [d]erivatives are denoted by

$$\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}, \ldots, \frac{d^ny}{dx^n}$$

or $y_1, y_2, y_3 \ldots\ldots y_n$
or $y', y'', y''', \ldots, y^{(n)}$
or $Dy, D^2y, D^3y \ldots, D^n(y)$

Standard Results:

To find the nth derivative of x^m:

Let $y=x^m$

then $y_1 = mx^{m-1}$

$y_2 = m(m-1)\,x^{m-1-1}$

$y_2 = m(m-1)\,x^{m-2}$

Similarly, $y_3 = m(m-1)(m-2)\,x^{m-3}$

$y_n = [m(m-1)(m-2) \text{ --- upto } n \text{ factors}]\,x^{m-n}$

$\therefore y_n = [m(m-1)(m-2) \ldots (m-n+1)]\,x^{m-n}$, where $n<m$.

[N]ote: If m be a positive integer and $n=m$.

then, $y_m = m(m-1)(m-2) \ldots (m-m+1)\,x^{m-m}$

$y_m = m(m-1)(m-2) \ldots x^0$

$y_m = m(m-1)(m-2) \ldots 1$

$y_m = m!$

As $y_m = m!$ is constant

then $y_{m+1} = 0$.

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2) To find the n^{th} derivative of a^{mx}:

Let $y = a^{mx}$

then $y_1 = a^{mx}(\log a) \times m$

$y_1 = m a^{mx}(\log a)$

$y_2 = m(\log a) m a^{mx}(\log a)$

$y_2 = m^2 a^{mx}(\log a)^2$

$y_3 = m^3 \cdot a^{mx} (\log a)^3$

$y_n = m^n \cdot a^{mx} \cdot (\log a)^n$

Note: If $y = a^x$, $m = 1$

Then $y_n = 1^n \cdot a^x (\log a)^n$

$\therefore y_n = a^x \cdot (\log a)^n$

3) To find the n^{th} derivative of e^{mx}.

Let $y = e^{mx}$

$y_1 = m e^{mx}$

$y_2 = m \cdot m e^{mx}$

$y_3 = m^3 e^{mx}$

$y_n = m^n e^{mx}$

Note: If $m = 1$, $y = e^x$

then $y_n = 1^n \cdot e^x$

$\therefore y_n = e^x$

To find the n^{th} derivative of $(ax+b)^m$?

Let $y = (ax+b)^m$

then $y_1 = m(ax+b)^{m-1} a$

$y_1 = am(ax+b)^{m-1}$

$y_2 = am(m-1)(ax+b)^{m-1-1} \cdot a$

$y_2 = m(m-1)(ax+b)^{m-2} a^2$

$y_3 = m(m-1)(m-2)(ax+b)^{m-3} a^3$

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$y_n = [m(m-1)(m-2) \text{ --- upto } n \text{ factors}] \times (ax+b)^{m-n} \cdot a^n$

$y_n = [m(m-1)(m-2) \cdots (m-n+1)] \cdot (ax+b)^{m-n} a^n$

To find the n^{th} derivative of e^{ax+b}.

Let $y = e^{ax+b}$

$y_1 = e^{ax+b} \cdot a$

$y_2 = a \cdot e^{ax+b} \cdot a$

$y_2 = a^2 e^{ax+b}$

$y_3 = a^3 \cdot e^{ax+b}$

⋮

$y_n = a^n e^{ax+b}$ //

Leibnitz Theorem:

Leibnitz Theorem helps us to find the n^{th} derivative of the product of two functions

Statement:

If $y=uv$, where u and v are functions of x having derivatives of n^{th} order, then.

$$y_n = {}^nC_0 u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + \cdots + {}^nC_r u_{n-r} v_r + \cdots + {}^nC_n u v_n.$$

Proof:

We shall prove the theorem by Mathematical Induction.

Step-I:

Let $y=uv$

By actual differentiation, we have

$$y_1 = u_1 v + u v_1$$

$$y_1 = {}^1C_0 u_1 v + {}^1C_1 u v_1$$

Thus, the theorem is true for $n=1$

Step-II:

Let us assume that the theorem is true for $n=m$.

$$y_m = {}^mC_0 u_m v + {}^mC_1 u_{m-1} v_1 + {}^mC_2 u_{m-2} v_2 + \cdots + {}^mC_r u_{m-r} v_r + \cdots + {}^mC_m u v_m.$$

Differentiating bothside w.r.t x, we have.

$y_{m+1} = {}^mC_0(u_{m+1}v + u_m v_1) + {}^mC_1(u_m v_1 + u_{m-1}v_2) + {}^mC_2(u_{m-1}v_2 + u_{m-2}v_3) + \cdots + {}^mC_{r-1}(u_{m-r+2}v_{r-1} + u_{m-r+1}v_r) + {}^mC_r(u_{m-r+1}v_r + u_{m-r}v_{r+1}) + \cdots + {}^mC_m(u_1 v_m + u v_{m+1})$.

$y_{m+1} = {}^mC_0 u_{m+1}v + ({}^mC_0 u_m v_1 + {}^mC_1 u_m v_1) + ({}^mC_1 u_{m-1}v_2 + {}^mC_2 u_{m-1}v_2) + \cdots + ({}^mC_{r-1}u_{m-r+1}v_r + {}^mC_r u_{m-r+1}v_r + \cdots + {}^mC_m u v_{m+1}$.

Since, ${}^mC_0 = 1 = {}^{(m+1)}C_0$, ${}^mC_m = 1 = {}^{(m+1)}C_{m+1}$

Also, ${}^mC_{r-1} + {}^mC_r = {}^{(m+1)}C_r$.

Putting $r = 1, 2, 3 \cdots\cdots$

${}^mC_0 + {}^mC_1 = {}^{(m+1)}C_1$

${}^mC_1 + {}^mC_2 = {}^{(m+1)}C_2$ and so on.

$\therefore y_{m+1} = {}^{m+1}C_0 u_{m+1}v + {}^{m+1}C_1 u_m v_1 + {}^{m+1}C_2 u_{m-1}v_2 + \cdots + {}^{m+1}C_r u_{m-r+1}v_r + \cdots + {}^{m+1}C_{m+1} u v_{m+1}$.

$\therefore$ Theorem is true for $n = m+1$ whenever $n = m$ is true.

Hence, by principle of mathematical Induction, the theorem is true for all positive integer n.

Name : Amruta kashetti

Semester : III

Topic : Sequence - II

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DEPARTMENT OF MATHEMATICS

Seminar on :- Sequence - II.

Cauchy's Theorems.

NAME : AMRUTA. L. KASHETTI.

CLASS : B.Sc III sem.

DATE : 29/09/2016.

SUBJECT : MATHEMATICS.

SEQUENCE - II

Cauchy's First Theorem on Limits :-

Statement :- If (a_n) converges, to l, then the sequence (x_n) where $x_n = \frac{a_1 + a_2 + \cdots + a_n}{l}$ also converges to l.

Proof :- Let $b_n = a_n - l$ then $\lim_{n\to\infty} b_n = \lim_{n\to\infty} a_n - l$.

but $\lim_{n\to\infty} a_n = l$.

$\therefore \lim_{n\to\infty} b_n = l - l = 0 \Rightarrow b_n \to 0$ —— (1)

Now. $b_n = a_n - l \Rightarrow a_n = b_n + l$.

$$x_n = \frac{(b_1 + l) + (b_2 + l) + \cdots + (b_n + l)}{n}$$

$$= \frac{(b_1 + b_2 + \cdots + b_n) + nl}{n}$$

$$\therefore x_n = \frac{b_1 + b_2 + \cdots + b_n}{n} + l$$

In order to prove $x_n \to l$ it is enough to prove that

$$\frac{b_1 + b_2 + \cdots + b_n}{n} \to 0 \text{ —— (2)}$$

Now, from (1) $b_n \to 0$.

$\therefore$ Given $\varepsilon > 0$, $\exists$ a positive integer m such that

$|b_n - 0| < \varepsilon/2 \Rightarrow |b_n| < \varepsilon/2$, $\forall n \geq m$ —— (3)

Again $b_n \to 0$ the sequence $\langle b_n \rangle$ is convergent.

$\Rightarrow \langle b_n \rangle$ is bounded.

$\Rightarrow \exists$ a real number $M > 0$, such that $|b_n| < M$, $\forall n$ ---(4)

Now, to prove (2)

Let $\left| \frac{b_1 + b_2 + \cdots + b_n}{n} - 0 \right|$

$$= \left| \frac{b_1 + b_2 + \cdots + b_n}{n} \right| = \frac{|b_1 + b_2 + \cdots + b_n|}{|n|}$$

$$= \frac{1}{n} \cdot |b_1 + b_2 + \cdots + b_m + b_{m+1} + \cdots + b_n|$$

$\because n \in \mathbb{N} \Rightarrow |n| = n$.

$\leq \frac{1}{n}\left[(|b_1|+|b_2|+\cdots+|b_m|)+(|b_{m+1}|+|b_{m+2}|+\cdots+|b_n|)\right]$

$< \frac{1}{n}\left[(M+M+\cdots m \text{ times})+\left(\varepsilon/2+\varepsilon/2+\cdots+(n-m) \text{ times}\right)\right]$

$| \because$ from (3) & (4)

$< \frac{1}{n}\left[m.M+(n-m)\varepsilon/2\right]$

$\therefore \left|\frac{b_1+b_2+\cdots+b_n}{n}-0\right| < \frac{1}{n}\left[m.M+(n-m)\frac{\varepsilon}{2}\right]$

$\Rightarrow \left|\frac{b_1+b_2+\cdots+b_n}{n}-0\right| < \frac{m}{n}.M+\frac{(n-m)}{n}.\frac{\varepsilon}{2} < \frac{mM}{n}+\varepsilon/2$

$\because \frac{(n-m)}{n}=1-m/n<1$

$\Rightarrow \left|\frac{b_1+b_2+\cdots+b_n}{n}-0\right| < \frac{mM}{n}+\varepsilon/2$

but $\frac{mM}{n} < \varepsilon/2$ if $\frac{n}{mM} > \frac{2}{\varepsilon}$. i.e, if $n > \frac{2mM}{\varepsilon}$

If p is natural number $> \frac{2mM}{\varepsilon}$ and $q=\max(m.p)$

then, $\left|\frac{b_1+b_2+\cdots+b_n}{n}-0\right| < \varepsilon/2+\varepsilon/2, \forall n \geq p$

$\Rightarrow \left|\frac{b_1+b_2+\cdots+b_n}{n}-0\right| < \varepsilon, \forall n\geq p.$

$\Rightarrow \frac{b_1+b_2+\cdots+b_n}{n} \to 0.$

from (2) we have

$x_n = \frac{b_1+b_2+\cdots+b_n}{n}+l$

$x_n \to l$ as $n\to\infty$.

i.e, $\frac{a_1+a_2+\cdots+a_n}{n} \to l$

Cauchy's Second Theorem on Limits:-

Statement:- If $\langle a_n\rangle$ is a sequence of positive terms, then $\lim_{n\to\infty}(a_n)^{1/n} = \lim_{n\to\infty}\frac{a_{n+1}}{a_n}$ provided the limit on the right hand side exists, whether finite or infinite

proof:- To prove the theorem, it is given that the right hand side of the given limit exists which is finite or infinite.

we consider two cases.

Case 1:

Let $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=l$ where l is finite

The given $\varepsilon>0$, $\exists$ positive integer m such that

$\left|\frac{a_{n+1}}{a_n} - l\right| < \varepsilon/2, \ \forall n \geq m$ | $\because$ by df^n

$\Rightarrow l - \varepsilon/2 < \frac{a_{n+1}}{a_n} < l + \varepsilon/2, \ \forall n \geq m.$

put $n = m, m+1, m+2 \ldots n-1$

we have $l - \varepsilon/2 < \frac{a_{m+1}}{a_m} < l + \varepsilon/2$

$l - \varepsilon/2 < \frac{a_{m+2}}{a_{m+1}} < l + \varepsilon/2$

$l - \varepsilon/2 < \frac{a_{m+3}}{a_{m+2}} < l + \varepsilon/2$

$\ldots \ldots \ldots \ldots \ldots \ l - \varepsilon/2 < \frac{a_n}{a_{n+1}} < l + \varepsilon/2.$

multiplying all $(n-m)$ inequalities,

$(l - \varepsilon/2)^{n-m} < \frac{a_{m+1}}{a_m} \cdot \frac{a_{m+2}}{a_{m+1}} \cdot \frac{a_{m+3}}{a_{m+2}} \cdots \frac{a_n}{a_{n-1}} < (l + \varepsilon/2)^{n-m}$

$\Rightarrow (l - \varepsilon/2)^{n-m} < \frac{a_n}{a_m} < (l + \varepsilon/2)^{n-m}$

$\Rightarrow (l - \varepsilon/2)^{n(1-m/n)} < \frac{a_n}{a_m} < (l + \varepsilon/2)^{n(1-m/n)}$

$\Rightarrow (l - \varepsilon/2)^{l-m/n} < \left(\frac{a_n}{a_m}\right)^{1/n} < (l + \varepsilon/2)^{1-m/n}$

$\Rightarrow (a_m)^{1/n} \cdot (l - \varepsilon/2)^{1-m/n} < (a_n)^{1/n} < (a_m)^{1/n} \cdot (l + \varepsilon/2)^{1-m/n}$ —— (1)

Now,

as $n \to \infty$, $(a_m)^{1/n}, (l - \varepsilon/2)^{l-m/n} \to l - \varepsilon/2$ $\left[\because a^{1/n} \to 1 \text{ as } n \to \infty \text{ and } a > 0.\right.$

and $(a_m)^{1/n} \cdot (l + \varepsilon/2)^{l-m/n} \to l + \varepsilon/2.$

for above $\varepsilon > 0$, $\exists$ positive integers m_1 & m_2 such that.

$\left|(a_m)^{1/n} \cdot (l - \varepsilon/2)^{l-m/n} - (l - \varepsilon/2)\right| < \varepsilon/2, \ \forall n \geq m_1$

and $\left|(a_m)^{1/n} \cdot (l + \varepsilon/2)^{1-m/n} - (l - \varepsilon/2)\right| < \varepsilon/2, \ \forall n \geq m_2$

$\Rightarrow (l - \varepsilon/2) - \varepsilon/2 < (a_m)^{1/n} \cdot (l - \varepsilon/2)^{l-m/n} < (l - \varepsilon/2) + \varepsilon/2$

$\Rightarrow l - \varepsilon < (a_m)^{1/n} \cdot (l - \varepsilon/2)^{1-m/n} < l, \ \forall n \geq m_1$ —— (2)

and also we have,

$(l + \varepsilon/2) - \varepsilon/2 < (a_m)^{1/n} \cdot (l + \varepsilon/2)^{1-m/n} < (l + \varepsilon/2) + \varepsilon/2, \ \forall n \geq m_2$

$\Rightarrow l < (a_m)^{1/n} \cdot (l + \varepsilon/2)^{1-m/n} < l + \varepsilon, \ \forall n \geq m_2$ —— (3)

let $P = \max(m_1, m_2)$ then combining eq^n (2) & (3) we get.

$l - \varepsilon < (a_m)^{1/n} \cdot (l - \varepsilon/2)^{l-m/n} < (a_m)^{1/n} \cdot (l + \varepsilon/2)^{l-m/n} < l + \varepsilon, \ \forall n \geq P$

—— (4)

from eq^n ② & ④ we get,

$l-\varepsilon < (am)^{1/n} \cdot (l-\varepsilon/2)^{l-m/n} < (an)^{1/n} < (am)^{1/n}(l+\varepsilon/2)^{l-m/n} < l+\varepsilon$.

$\Rightarrow l-\varepsilon < (an)^{1/n} < l+\varepsilon, \forall n \geq p$.

$\Rightarrow |(an)^{1/n} - l| < \varepsilon, \forall n \geq p$.

$\Rightarrow \lim_{n\to\infty} (an)^{1/n} = l$.

Case 2: Let $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \infty$

$$\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = 0.$$

$$\lim_{n\to\infty} \frac{1/a_{n+1}}{1/a_n} = 0 \text{ (finite)}$$

from case (i) we have

$$\lim_{n\to\infty} (a_n)^{1/n} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} \text{ (finite)}$$

$$\Rightarrow \lim_{n\to\infty} \left(\frac{1}{a_n}\right)^{1/n} = 0.$$

$$\Rightarrow \lim_{n\to\infty} (a_n)^{1/n} = \infty.$$

Hence the theorem.

Show that the sequence $\langle n^{1/n} \rangle$ converges to limit 1.

Soln:- Let $a_n = n$.

$$\frac{a_{n+1}}{a_n} = \frac{n+1}{n} = 1 + 1/n.$$

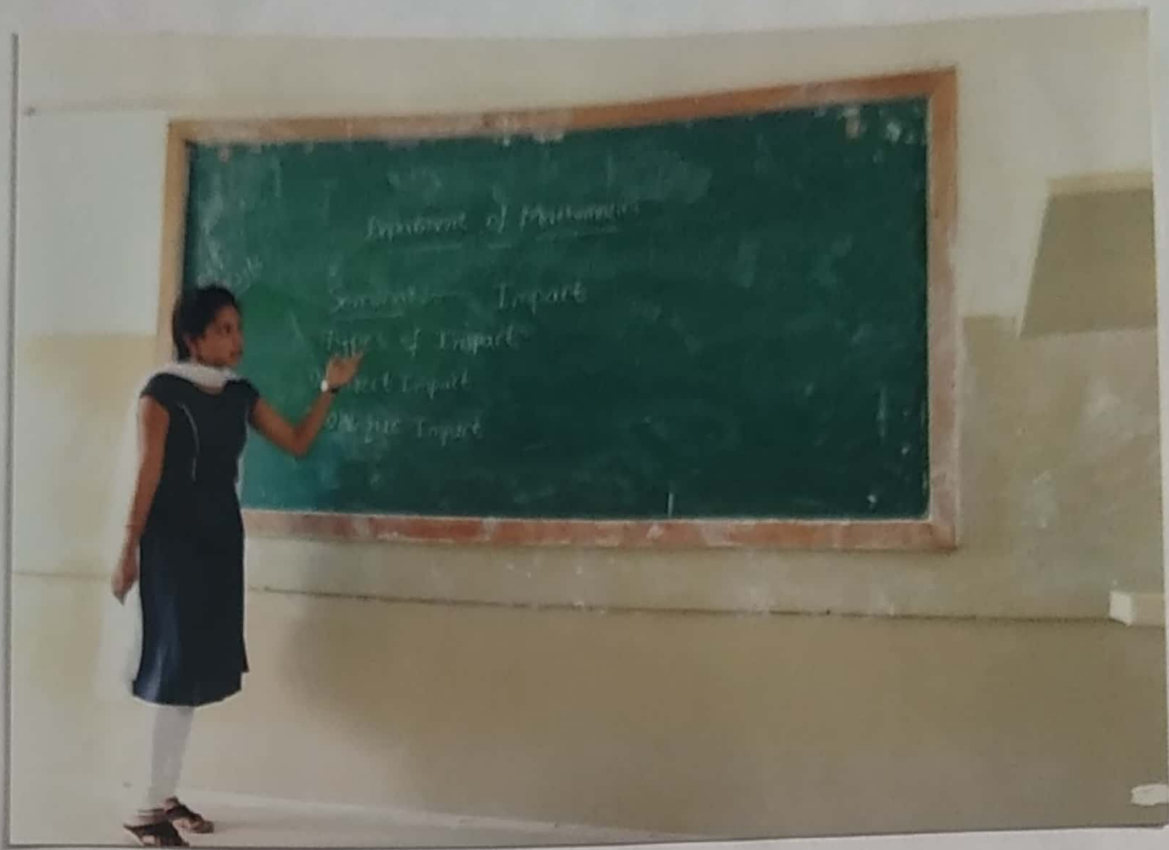
$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} (1 + 1/n)$$

$$= 1.$$

Now, By cauchy's second theorem on limits.

we have, $\lim_{n\to\infty} (a_n)^{1/n} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$

$$\Rightarrow \lim_{n\to\infty} n^{1/n} = 1$$



Name : Banashri Biradar

Semester : V

Topic : Impact

S.B. Arts And K.C.P. Science College

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SEMINAR

TOPIC :- Impact

Name : Banashree. Biradar.

Class : Bsc V Sem

Subject : Mathematics

Date : 21/09/16

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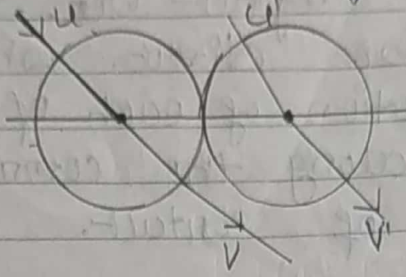
1) Oblique Impact ⇒
Impact between two bodies is said to be oblique when the direction of motion of either or both them before impact is not along the common normal at that point of contact.

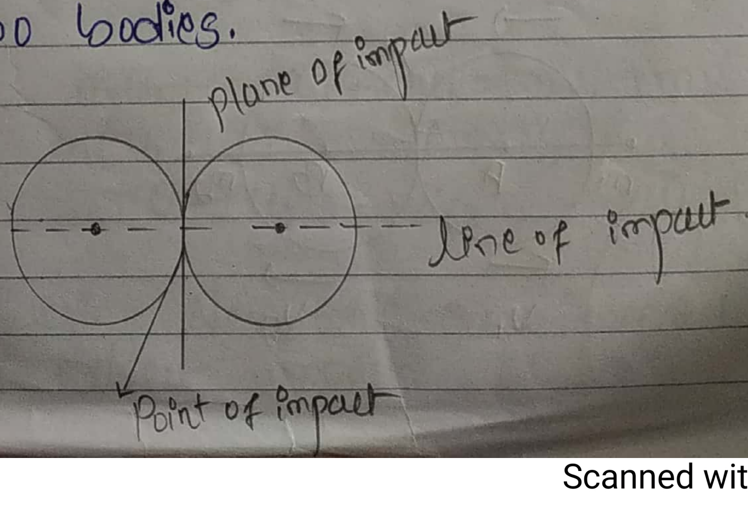


⊛ Coefficient of Restitution (e) ⇒
When two bodies strike or collide against each other their relative velocity after the impact is in a constant ratio of their relative velocity before the impact and is in opposite direction. This constant ratio is called Coefficient of Restitution and It is denoted by e.

If e = 1 the bodies are perfectly elastics.
If e = 0 the bodies are inelastic.

⊛ Elastic Impact ⇒

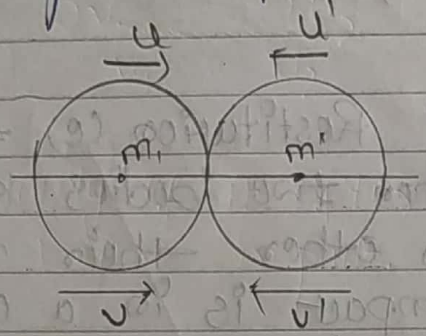
Impact → A Sudden short time act betn. two bodies.



There are two types of Impact.

① Direct Impact.
② Oblique Impact.

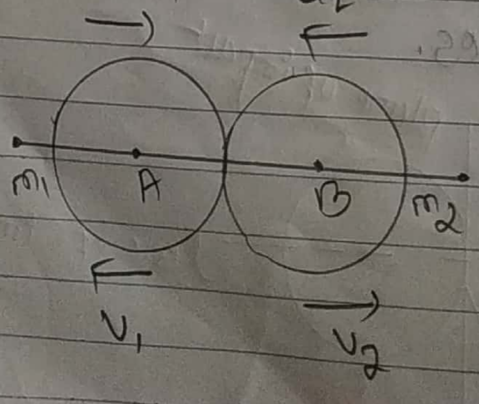
1) Direct Impact -
Impact betn. two bodies is said to be direct. when the d-ction of motion of each of them b- impact is along the common normal their point of contact.



⊛ Principle of conservation of linear momentum of direct impact.

When the two bodies masses m_1 with velocity u_1 & u_2 collide with each other then after colliding with the velocity v_1 & v_2 the sum of momentum before impact is equal to the sum of momentum after impact.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2.$$



* Newoton's Experimental law in case of direct impact ⇒

Statement ⇒

When two bodies collide (impinge) directly their relative velocity after impact is in constant ratio to the relative velocity before impact and it is in apposite direction.

$$v_1 - v_2 = -e(u_1 - u_2)$$

* Theorem ⇒

If two equal & perfectly elastic bodies moving in the same direction collide (impinge) directly. S.T. They can interchange their velocity after impact.

⇒ Let us consider ~~two do~~ bodies A & B with masses m_1 & m_2 (equal mass) collide directly which are moving with velocity u_1 & u_2 before impact & v_1 & v_2 are the velocities after impact.

Then by law of conservation of momentum by direct impact.

i.e. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

But two ~~of~~ bodies are same mass.

i.e. $m_1 = m_2 = m$.

So that. $\cancel{m_1u_1+}\ mu_1 + mu_2 = mv_1 + mv_2$

$$m(u_1 + u_2) = m(v_1 + v_2)$$

$$u_1 + u_2 = v_1 + v_2 \rightarrow (1)$$

By using Newoton's experimental law of direct impact.

$\therefore v_1 - v_2 = -e(u_1 - u_2)$

But bodies are perfectly elastic.
(i.e $e = 1$)

$\therefore v_1 - v_2 = -u_1 + u_2 \rightarrow (2)$

Adding equation (1) & (2) we get.

$v_1 + v_2 + v_1 - v_2 = u_1 + u_2 - u_1 + u_2$.

$v_1 + v_1 = u_2 + u_2$

$2v_1 = 2u_2$

$v_1 = u_2$

$\therefore$ Velocity of A after impact = velo of B. before impact.

Now substracting eq^n. (1) & (2) we get.

$v_1 + v_2 - v_1 + v_2 = u_1 + u_2 + u_1 - u_2$

$v_2 + v_2 = u_2 + u_2$

$2v_2 = 2u_2$

$v_2 = u_1$

$\therefore$ velocity of B after impact = ve of A before impact.

$\therefore$ The velocities are interchange.

(*) Theorem (2) ⇒

Two elastic sphere of masses m_1 & m_2 moving with velocity u_1 & u_2 impinge directly. If e is the coefficient of the restitution. Find their velocity after impact.

⇒ Let v_1 & v_2 be the velocities of the two sphere after impact.

By Newton experimental law, we have.

$$v_1 - v_2 = -e(u_1 - u_2) \rightarrow (1)$$

By the principal of conservation of linear momentum we have,

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 \rightarrow (2)$$

Multiplying equation (1) by m_2 & adding to the eqn (2) we get.

$$\therefore (m_1 + m_2) v_1 = (m_1 - e m_2) u_1 + m_2 (1+e) u_2$$

$$v_1 = \frac{(m_1 - e m_2) u_1 + m_2 (1+e) u_2}{m_1 + m_2} \rightarrow (3)$$

Multiplying eqn. (1) by m_1 & substracting from eqn. (2) we get.

$$(m_1 + m_2) v_2 = m_1 (1+e) u_1 + (m_2 - e m_1) u_2$$

$$v_2 = \frac{m_1 (1+e) u_1 + (m_2 - e m_1) u_2}{m_1 + m_2} \rightarrow (4)$$

Eqn. (3) & (4) are the velocity after the impact.

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S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

SEMINAR REPORT: 2016-17

(Odd Semester)

The UG Department of Mathematics has conducted seminar for B.Sc students.

Name of the student	Seminar Topic
1) Rashmi Naikodi	Successive differentiation
2) Amruta kashetti	Sequence - II
3) Banashri Biradar	Impact

Head of Department — H. O. D. Department of Mathematics, S. B. Arts & K. C. P. Science College, BIJAPUR.

IQAC, Co-ordinator — S.B.Arts & K.C.P.Science College, Vijayapur.

Principal — Principal, S. B. Arts & KCP Sc. College, Bijapur

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S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

SEMINAR REPORT

2016-17 (EVEN SEMESTER)

BLDE Association's

S.B.Arts And K.C.P Science College, Vijayapur

DEPARTMENT OF MATHEMATICS

NOTICE:

The UG Department of Mathematics is conducting seminar for B.Sc students for the academic year 2016-17(Even Semester)

IQAC, Co-ordinator
S.B.Arts & K.C.P.Science College,
Vijayapur.

Principal,
S.B.Arts & K.C.P. Science College,
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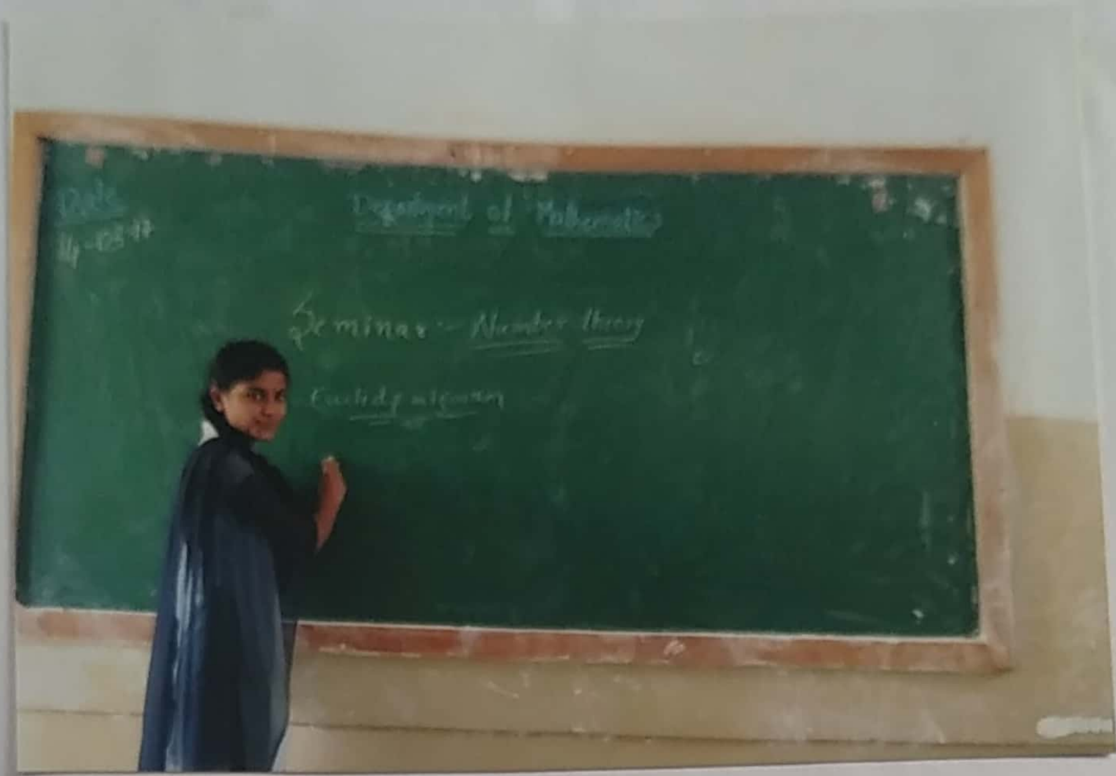
B.L.D.E.A's S.B.Arts & K.C.P Science College , Vijayapura

Department of Mathematics

SEMINAR – 2016 -17 (EVEN SEMESTER)

Sl.No	Semester	Name of the students	Topics	Date
1.	II	Pooja Sank	Number Theory	14/03/2017
2.	IV	Manjula Godhihal	Vector Analysis	22/03/2017
3.	VI	Vishwanath Keshagond	Differential Equation	29/03/2016



Name : Pooja . Sank

Semester : II

Topic : Number Theory

B.L.D.E. Association.

S.B Arts and K.C.P Science college Vijayapur.

Name :- Pooja. S. Sank.
Roll no :- 223
R.C.U Seat no :- S1622265
Subject :- Mathematics.
class :- Bsc. II sem
Topic :- Number Theory

NUMBER THEORY

Introduction:-

Number theory is an interesting topic to learn and teach Basically it is branch of pure mathematics, specially it is called the "queen of mathematics", Number theorists study prime number as well as the properties of objects made out of integer's. The older term for number theory in Arithmetic. By the early twentieth century it had been superseded by "number theory"

Here Fundamenal theorem of Arithmetic plays an important role in number theory, Division Algorithm, congruences and its properties are also an interesting facts. The important theory of number theory are wilson's and Fermat's therom for prime. p.

Divisibility

An integer $a \neq 0$ is said to be divide an integer b, if $k \in z$. Such that $b = ak$. It is denoted by a/b thus if a/b thus if a/b then $b = ka$, $k \in z$ and $a, b \in z$ where $a \neq 0$ iff $b = ka$, $k \in z$ then a/b, it can be read in many ways.

i.e a is divisor of b

a is factor of b.

b is divisible by a

b is multiple of a.

Properties of Divisibility

1) If n/m, then there exist 'q' such that
$m = q \times n$
2) The absolute value of both 'q' and 'n' are less than the absolute value of 'm'
i.e $|n| < |m|$ and $|q| < |m|$

G.C.D (Greatest common driver).

common divisor :- If a number 'c' divides any two number a and b
i.e, if c/a and c/b then c is known as a common divisor of a & b.
If a number a d divides a and b and is divisble by all the common divisors of a and b., then d is known as greatest common divisor (G.C.D) of a and b.

Properties of G.C.D. :- The G.C.D of a and b is unique. positive integer d, then the following properties are satisfied

i) $\frac{d}{a}$ and $\frac{d}{b}$

ii). If $\frac{c}{a}$ and $\frac{c}{b}$ then $\frac{c}{d}$.

i.e., $(a, b) = d$.

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Euclids algorithm

The method of finding G.C.D of two integers by successive division to obtain a sequence of remainder. $r_1, r_2, r_3 \ldots r_n$ and such that $r_1 > r_2 > r_3 > \ldots > r_n > 0$. This is called euclids Algorithm.

Bracket function or step function or Integral part of a Real number.

def^n :- If 'x' is any real number then the largest integral which does not exceed 'x' is called integral part of x. It is denoted by $[x]$.

The function $f: R \rightarrow R$ associates with zero number 'x' then the integer $[x]$ is called Bracket function.

Ex :- 1] $[x] = [3.2] = 3$.

2] $[x] = [4.7] = 5$

3] $[x] = [5.5] = 6$

Fundamental theoryem of Arithmetic.

Statement : Each natural number greater than 1 can be expressel as product of primes and the factorising of any positive integer 'n' into primes is unique apart from the order of primes.

Proof :- Let 'n' be a natural number grater than 1 (i.e, $n>1$) and n has a prime factors say P_1 i.e, P_1/n.

$\therefore$ P_1 must divide some one of these primes say q_1, But q_1 is prime.

$\therefore$ Either $P_1 = 1$ or $P_1 = q_1$

$\therefore$ But $P_1 = 1$ is impossible because P_1 being prime > 1

$P_1 = q_1$ therefore, there exist an integer n_1 such that $n = n_1 P_1$

$$n = n_1 P_1 \longrightarrow (1)$$

where $n > n_1$

If $n_1 = 1$, $n = P_1$, i.e, n is prime P_1

If $n_1 > 1$, then n_1 has prime factor P_2.

Therefore, there exists a positive integer n_2 such that $n_1 = P_2 n_2$

where $n_1 > n_2$

Putting the value of n_1 in eqn (1) we get

$$n = P_1 P_2 n_2$$

where $n > n_1 > n_2$.

If $n_2 = 1$ then $n = P_1 P_2$.

i.e n is the product of primes P_1 and P_2.

If $n_2 > 1$, we continue the above process, but this process must end after a finite number of steps.

($\because$ n is finite must end after a finite number of steps. ($\because$ n is finite and $n > n_1 > n_2 > \ldots$).

$\therefore$ $\exists$ primes $P_1, P_2, P_3 \ldots P_r \rightarrow (3)$

Now to show that it is unique.

Let if possible.

$n = q_1 q_2 \ldots q_s \longrightarrow (4)$.

be an alternative expression of n as as a product of prime from (3) and (4).

$P_1, P_2, P_3 \ldots P_r = q_1, q_2, q_3, \ldots q_s \rightarrow (5)$.

Now equation (5) show that prime P_1 is a factor of the product

$q_1, q_2 \ldots q_n$

$\therefore$ P_1 must divide some one of these primes say q_1

But q_1 is prime.

$\therefore$ Either $P_1 = 1$ Or $P_1 = q_1$

$\therefore$ But $P_1 = 1$ is impossible because P_1 being prime > 1 $\quad P_1 = q_1$

$\therefore$ By cancelled law, (5) becomes

$P_2 P_3 \ldots P_r = q_2 q_3 \ldots q_s \longrightarrow (6)$

As before, we have that P_2 must divide some one of the primes say q_2

with out loss of generality, we have $P_2 = q_2$

eq^n (6)

$P_3 . P_4, \ldots P_r = q_3 q_4 \ldots q_s$.

proceeding similarly, we have

$P_3 = q_3, \quad P_4 = q_4$ etc.

Finally to prove that $r = s$.

If possible, Let $r > s$.

$\therefore$ Eqn (5) can be written as

$P_1 P_2 \ldots P_s P_{s+1} \ldots P_r = q_1 q_2 \ldots q_s$

putting,

$q_1 = P_1, q_2 = P_2 \ldots q_s = P_s$ we have.

$P_1 P_2 \ldots P_s P_{s+1} \ldots P_r = P_1 P_2 \ldots P_s$.

Dividing both side by $P_1, P_2 \ldots P_s$, we have

$P_{s+1} \ldots P_r = 1$.

But this is imposibble because each of the factors on L.H.S being prime > 1

$\therefore r \ngtr s$.

similarly $r \nless s$.

$\therefore r = s$.

Hence the two decomposition of n namely $P_1 P_2 \ldots P_r$ and $q_1 q_2 \ldots q_s$ are identical.

hence proof.

Euler's function

statment :-

The number of integers $\leq n$ and co-prime to n is called euler's function for n and it is denoted by $\phi(n)$.

examples :- $\phi(1) = 1$, $\phi(2) = 1$, $\phi(5) = 4$, $\phi(16) = 8$, etc.

Fermatt's Theorem

statment.

If P is prime number and n is positive integer and relatively prime to each other, than prove that $n^{P-1} \equiv 1 \pmod P$.

Proof. we shall first prove that
$$(a+b)^P \equiv (a^P + b^P) \pmod P.$$
where 'p' is prime number.

Now, expanding $(a+b)^P$ by using biomomial theorem, we get

$$(a+b)^P = a^P + {}^PC_1 a^{P-1} b + {}^PC_2 a^{P-2} b^2 + \cdots + {}^PC_{P-2} a^2 b^{P-2} + {}^PC_{P-1} a b^{P-1} + b^P.$$

$$(a+b)^P - (a^P + b^P) = {}^PC_1 a^{P-1} b + {}^PC_2 a^{P-2} b^2 + {}^PC_3 a^{P-3} b^3 \cdots$$
$$\cdots + {}^PC_{P-2} a^2 b^{P-2} + {}^PC_{P-1} a b^{P-1}$$

$$(a+b)^P - (a^P + b^P) = \sum_{r=1}^{P-1} {}^PC_r a^{P-r} b^r \longrightarrow ①$$

Now $${}^PC_r = \frac{P!}{(P-r)!\, r!} \quad \text{where } 1 \leq r \leq P-1$$

But $P!$ is divisible by P and P is co-prime. [$\because$ P is co-prime to $1, 2, 3 \ldots r$ ($r < P$ and p is prime) and also p is co-prime to their product $= r!$]

Also for the some reasons p is co-prime to $(P-r)!$

$\therefore$ $${}^PC_r = \frac{P!}{(P-r)!\, r!} \text{ is divisible by 'p'}$$

$\therefore$ $\exists$ and integer k_r such that ${}^PC_r = P(K_r)$.

But ${}^PC_r = P(k_r)$ in eqn ①, we have.

$$(a+b)^P - (a^P + b^P) = P \sum_{r=1}^{P-1} k_r a^{P-r} b^r.$$

which is divisible by p.

$\therefore p \mid (a+b)^p - (a^p + b^p)$.

or $(a+b)^p \equiv (a^p + b^p) \pmod p$

similarly, $(a+b+c)^p \equiv (a^p + b^p + c^p) \pmod p$.

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Generally, we can write in this way, we have.

$(a_1 + a_2 + a_3 + \cdots + a_n)^p \equiv (a_1^p + a_2^p + \cdots + a_n^p) \pmod p$.

Put $a_1 = a_2 = a_3 = \cdots = a_n = 1$.

$\therefore (1+1+1+1+\cdots+1)^p \equiv (1^p + 1^p + 1^p + \cdots + 1^p) \pmod p$.

$$n^p \equiv n \pmod p$$

$n^{+1}, n^{-1}, n^p \equiv n \pmod p$. $\quad 1 \because n^{+1} . n^{-1} = 1$

$n(n^{p-1}) \equiv n \pmod p$ and $(n, p) = 1$

$\therefore n^{p-1} \equiv 1 \pmod p$.

hence proof.

Name : Manjula. Godihal

Semester : IV

Topic : Vector Analysis

Departement of Mathematics

S. B. Arts & K. C. P Science
college Vijayapur.

Name : Manjula. Godihal
R. NO : 09 ; RCU NO : S1522161
Seminar topic : Vector Analysis.
Semester : VI Semester

Definition of vector : A quantity which has both magnitude & direction is called vector.

Theorem : If $\vec{A}(t)$ and $\vec{B}(t)$ are two differentiable function of a scalar variable t, then prove that $\frac{d}{dt}\{\bar{A}(t)+\bar{B}(t)\} = \frac{d}{dt}\bar{A}(t) + \frac{d}{dt}\bar{B}(t)$

proof : Given $\bar{A}(t)$ & $\bar{B}(t)$ are two differentiable vector functions of a scalar variable t

Let $\vec{f}(t) = \vec{A}(t) + \vec{B}(t)$

and $\vec{f}(t+\delta t) = \vec{A}(t+\delta t) + \vec{B}(t+\delta t)$

then by difinition of derivatives

$$\frac{d}{dt}\{\vec{f}(t)\} = \lim_{\delta t \to 0} \frac{\vec{f}(t+\delta t) - \vec{f}(t)}{\delta t}$$

$$\frac{d}{dt}\{\bar{A}(t)+\bar{B}(t)\} = \lim_{\delta t \to 0} \frac{\{\vec{A}(t+\delta t)+\vec{B}(t+\delta t)\} - \{\vec{A}(t)+\vec{B}(t)\}}{\delta t}$$

$$= \lim_{\delta t \to 0} \frac{\vec{A}(t+\delta t) - \vec{A}(t)}{\delta t} + \lim_{\delta t \to 0} \frac{\vec{B}(t+\delta t) - \vec{B}(t)}{\delta t}$$

single, Algebra of limits,

$$\therefore \frac{d}{dt}\{\vec{A}(t)+\vec{B}(t)\} = \frac{d}{dt}\vec{A}(t) + \frac{d}{dt}\vec{B}(t)$$

Theorem:2 :- prove that necessary & sufficient condition for the vector function $\vec{f}(t)$ to have constant magnitude.

proof: Necessary condition: Given $\vec{f}(t)$ is a vector function of const magnitude then to prove that, $\vec{f}\cdot\frac{d\vec{f}}{dt} = 0$.

Let. $|\vec{f}(t)| = \text{const} = k$.

$$|\vec{f}(t)|^2 = k^2$$

$$\vec{f}(t)\cdot\vec{f}(t) = k^2 \qquad [\because |\vec{a}|^2 = \vec{a}\cdot\vec{a}$$

$$\frac{d}{dt}\{\vec{f}(t)\cdot\vec{f}(t)\} = k^2$$

$$\vec{f}\cdot\frac{d\vec{f}}{dt} + \vec{f}\cdot\frac{d\vec{f}}{dt} = 0.$$

$$2\vec{f}\cdot\frac{d\vec{f}}{dt} = 0 \Rightarrow \vec{f}\cdot\frac{d\vec{f}}{dt} = 0$$

sufficient condition: Given that $\vec{f}\cdot\frac{d\vec{f}}{dt} = 0$

then prove that $|\vec{f}|$ is constant vector.

$$\therefore \vec{f}\cdot\frac{d\vec{f}}{dt} = 0.$$

$$\vec{f}\,\frac{d\vec{f}}{dt} + \vec{f}\cdot\frac{d\vec{f}}{dt} = 0 + 0 = 0$$

$$\frac{d}{dt}[\vec{f}\cdot\vec{f}] = 0.$$

$$\vec{f}\cdot\vec{f} = \text{constant}$$

$$[\vec{f}]^2 = \text{constant}$$

$$[\vec{f}] = \text{const} \Rightarrow \therefore [\vec{f}] \text{ is const}$$

Theorem 3: prove that necessory and sufficient condition for the vector functions $\vec{f}(t)$ to have constant direction is $\vec{f} \times \frac{d\vec{f}}{dt} = 0$.

proof: Let $\vec{f}(t) = \phi(t)\vec{F}(t)$, where $\vec{F}(t)$ is a unit vector in the direction $\vec{f}(t)$ & $\phi(t)$ is scalar function of scalar variable t.

$$= (\phi\vec{F}) \times \frac{d}{dt}(\phi\vec{F})$$

Let $$\vec{f} \times \frac{d\vec{f}}{dt} = \phi(\vec{F}) \times \left[\phi \cdot \frac{d\vec{F}}{dt} + \frac{d\phi}{dt} \cdot \vec{F}\right]$$

$$= \phi^2 \left[\vec{F} \times \frac{d\vec{F}}{dt}\right] + \phi \cdot \frac{d\phi}{dt} [\vec{F} \times \vec{F}]$$

$$= \phi^2 \left[\vec{F} \times \frac{d\vec{F}}{dt} + \phi \cdot \frac{d\phi}{dt} \cdot (0)\right] \quad | \because \vec{a} \times \vec{a} = 0$$

$$\vec{f} \times \frac{d\vec{f}}{dt} = \phi^2 \left[\vec{F} \times \frac{d\vec{F}}{dt}\right] \quad \text{---} (1)$$

__Necessory condition__: Given $\vec{f}(t)$ is a vector function of const direct^n, to P.T $\vec{f} \times \frac{d\vec{f}}{dt} = 0$

Since, $\vec{f}$ is a const direct^n

$\therefore \vec{F}(t)$ is a const vector

($\because \vec{F}$ has const direct^n & magnitude)

$$\therefore \frac{d\vec{F}}{dt} = 0 \quad \text{---} (2)$$

$$\vec{f} \times \frac{d\vec{F}}{dt} = \phi^2(\vec{F} \times 0) \quad | \because \text{from (1) \& (2)}$$

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$\therefore \vec{f} \times \frac{d\vec{f}}{dt} = 0$

Sufficient condition: Given that $\vec{f} \times \frac{d\vec{f}}{dt} = 0$

then to prove that the vector fun^n. $\vec{f}(t)$ has a const direction.

Since. $\vec{f} \times \frac{d\vec{f}}{dt} = 0$

from equation (1)

$$\phi^2 \left[\vec{F} \times \frac{d\vec{F}}{dt} \right] = 0$$

$$\vec{F} \times \frac{d\vec{F}}{dt} = 0 \quad \text{- - - - (3)} \quad \Big| \because \phi \text{ is not zero.}$$

Also $\vec{F}$ is a const magnitude.

$$\therefore \vec{F} \times \frac{d\vec{F}}{dt} = 0 \quad \text{- - - (4)}$$

Now, from equation (3) & (4). we get

$$\frac{d\vec{F}}{dt} = 0$$

$\Rightarrow \vec{F}$ is a const vector function both in magnitude & direction

But magnitude of F is unity & direction $\vec{F}$ is also constant.

Since, $\vec{f} = \phi\vec{F}$

But direction of $\vec{F}$ is const. then the direct^n of $\vec{f}$ is also const.

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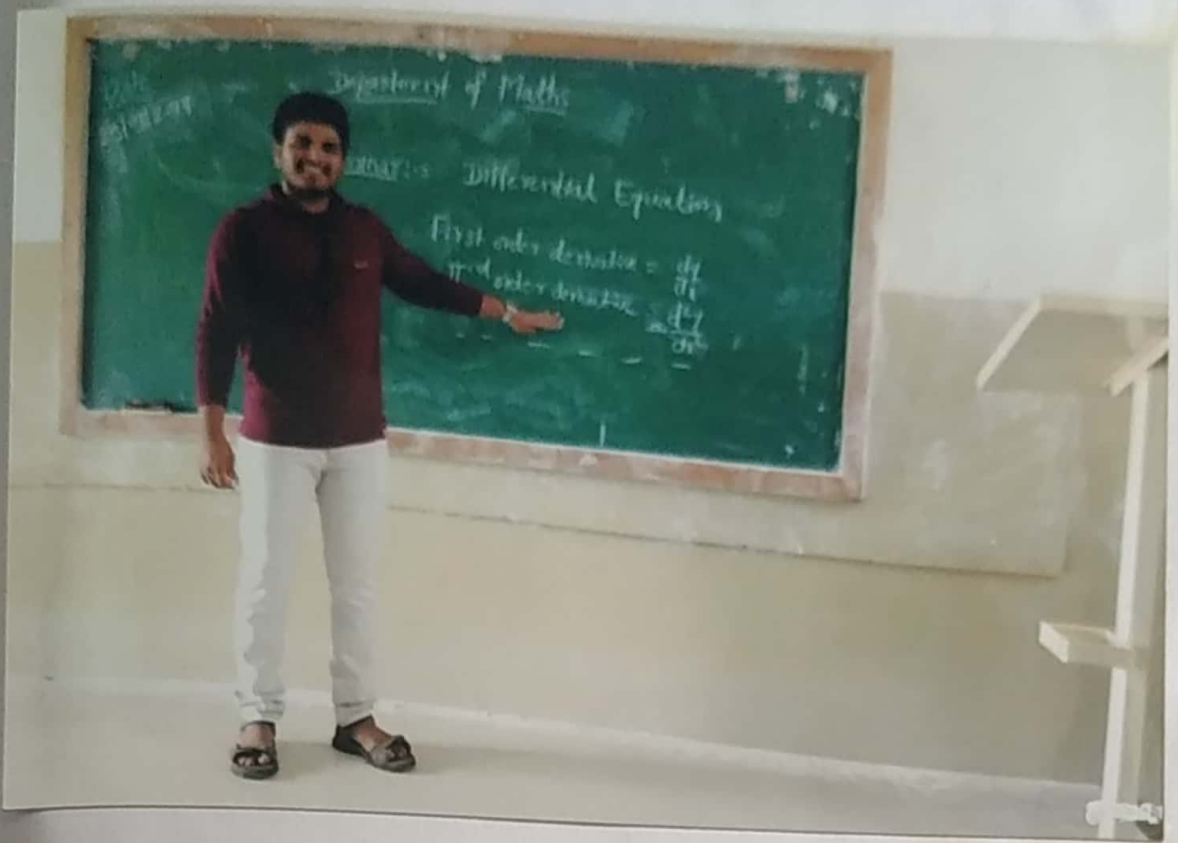

Name : Vishwanath . Keshagond

Semester : VI

Topic : Differential Equation

Name :- Vishwanath M. Keshagond

std :- B.SC VIth sem

Roll. No :- 12 (A)

RCU No :- S1420769

Sub :- M-3 (Seminar) Simultaneous linear Equations

college :- SB Arts & KCP science college Vijaypura

Method of solving simultaneous Linear Equations with const. coefficients:→

Let x, y be dependent variables & t be the independent variable. Given two simulteneous equations we have to establish relations between x, y & t. Let the simulteneous eq^n be

$$\left.\begin{aligned} \frac{dx}{dt} &= a_1x + b_1y + \phi_1(t) \\ \frac{dy}{dt} &= a_2x + b_2y + \phi_2(t) \end{aligned}\right\} \text{——} ①$$

If $\phi_1(t)$ & $\phi_2(t)$ are identically zero in equation ① then the system is called homogeneous. otherwise it is said to be non-homogeneous.

By removing $\phi_1(t)$ & $\phi_2(t)$ from eq^n ① we get

$$\left.\begin{aligned} \frac{dx}{dt} &= a_1x + b_1y \\ \frac{dy}{dt} &= a_2x + b_2y \end{aligned}\right\} \text{——} ②.$$

obviosuly $x = y = 0$ is a trivial sol^n of eq^n ②. Our main interest is in finding non-trivial solution. We state in that proof the following theorem:.

Differential Equations - III.
(Simulteniony & Total differtial eqn)

Theorem:=>

If $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ & $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ are two solns

~~then~~ of $\frac{dx}{dt} = a_1x + b_1y$ & $\frac{dy}{dx} = a_2x + b_2y$ — (*)

then $\begin{cases} x = c_1x_1(t) + c_2x_2(t) \\ y = c_1y_1(t) + c_2y_2(t) \end{cases}$ is also a solution of eqn (2).

we next consider the non-homogeneous system (1) & conclude our discussion with

Theorem:=> If $\begin{cases} x = x_p(t) \\ y = y_p(t) \end{cases}$ is any perticular soln of (1) then $\begin{cases} x = c_1x_1(t) + c_2x_2(t) \\ y = c_1y_1(t) + c_2y_2(t) \end{cases}$ is the

General solution (G.S) of eqn (1).

Examples:-> 1) Solve for x. $\frac{dx}{dt} + my = 0$, $\frac{dy}{dt} - mx = 0$

Solution:-> The given equations can be written as,

$$Dx + my = 0 \quad \text{——— (1)}$$

$$-mx + Dy = 0 \quad \text{where } D = \frac{d}{dt} \quad \text{——— (2)}$$

operating eqn (1) by D & ×ing (2) by m

$$D^2x + mDy = 0$$
$$-m^2x + mDy = 0$$

Substracting $(D^2+m^2)x = 0$.

The Auxiliuary eqn is

$$D^2 + m^2 = 0$$

i.e

$$D = \pm im$$

$\therefore x = c_1 \cos mt + c_2 \sin mt$ where $c_1 c_2$ are consts.

differentiating wrt t. $\frac{dx}{dt} = -mc_1 \sin mt + mc_2 \cos mt$

from eqn (1)

$$my = -\frac{dx}{dt}$$
$$= m(c_1 \sin mt - c_2 \cos mt)$$

$$\therefore y = c_1 \sin mt - c_2 \cos mt$$

② Solve $\frac{dx}{dt} - 8y = 0$, $\frac{dy}{dt} - 2x = 0$.

Sol^n:→ The equations are

$$Dx - 8y = 0 \qquad \text{——— ①}$$

$$-2x + Dy = 0 \quad \text{where } D = \frac{d}{dt} \qquad \text{——— ②.}$$

operating ① by D & multiplying ② by 8.

$$D^2x - 8Dy = 0$$

$$-16x + 8Dy = 0$$

Adding up $(D^2 - 16)\, x = 0$

Auxiliry eq^n (A.E) is $D^2 - 16 = 0 \Rightarrow D = \pm 4$

$\therefore x = c_1 e^{4t} + c_2 e^{-4t}$ $\quad c_1\ c_2$ are constants

from eq^n ①

$$8y = \frac{dx}{dt} = 4(c_1 e^{4t} + c_2 e^{-4t})$$

$$\therefore \quad y = \frac{1}{2}(c_1 e^{4t} - c_2 e^{-4t})$$

i.e $x = c_1 e^{4t} + c_2 e^{-4t}$; $y = \frac{1}{2}(c_1 e^{4t} - c_2 e^{-4t})$

together give the required sol^n.

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S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

SEMINAR REPORT: 2016-17

(Even Semester)

The UG Department of Mathematics has conducted seminar for B.Sc students.

Name of the student	Seminar Topic
1) Pooja Sark	Number theory
2) Manjula Godhihal	Vector Analysis
3) Vishwanath Keshagond	Differential eqns

Head of Department

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