BLDE ASSOCIATION'S

S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

ADVANCE LEARNERS

For the Academic Year : 2018-19

BLDE ASSOCIATION'S

S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

SEMINAR REPORT

2018-19 (ODD SEMESTER)

BLDE Association's

S.B.Arts And K.C.P Science College, Vijayapur

DEPARTMENT OF MATHEMATICS

NOTICE:

The UG Department of Mathematics is conducting seminar for B.Sc students for the academic year 2018-19(Odd Semester)

IQAC, Co-ordinator
S.B.Arts & K.C.P.Science College
Vijayapur.

Principal,
S.B. Arts and KCP Science College
VIJAYAPUR

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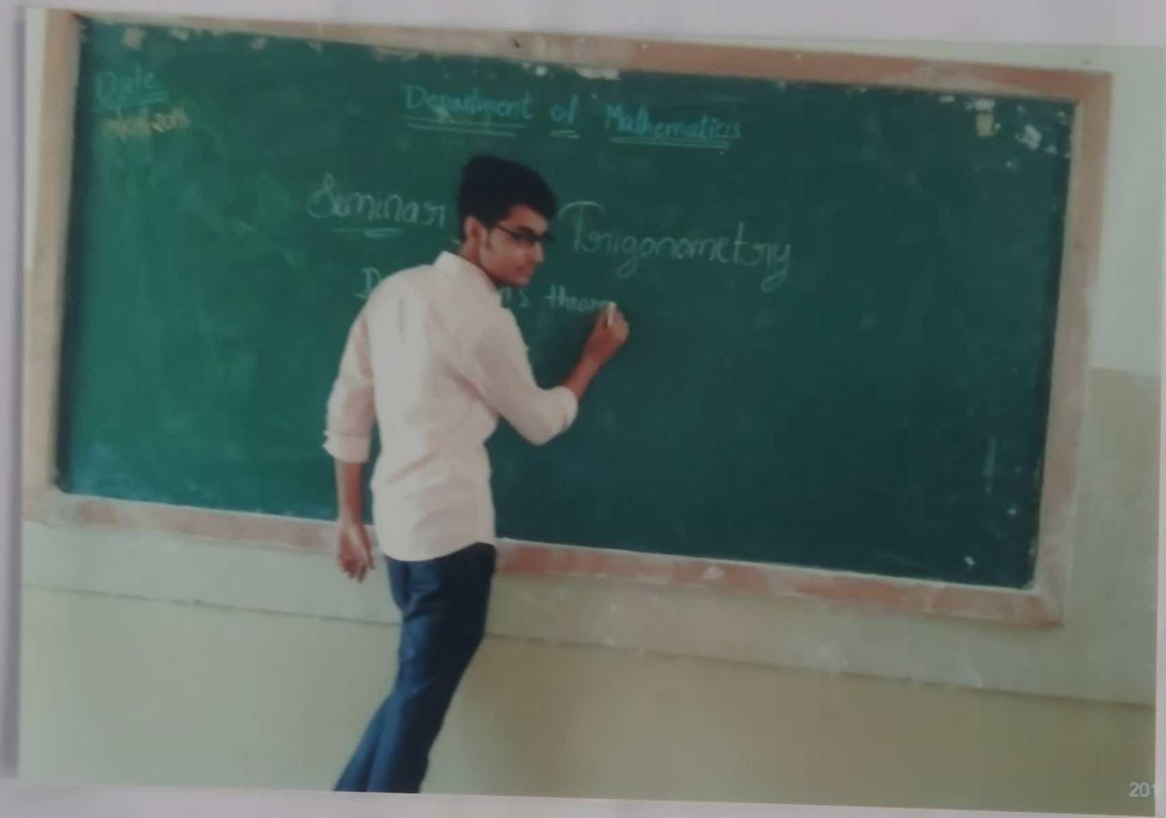
B.L.D.E.A's S.B.Arts & K.C.P Science College , Vijayapura

Department of Mathematics

SEMINAR – 2018-19 (ODD SEMESTER)

Sl.No	Semester	Name of the students	Topics	Date
1.	I	Praveen Rathod	Trignometry	19/09/2018
2.	III	Sudharani Biradar	Sequence - I	17/09/2018
3.	V	Sheela Salotagi	Range Kutta Method	05/09/2017



Name : Praveen Rathod

Semester : I

Topic : Trigonometry

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S. B. ARTS & K.C.P. SCIENCE COLLEGE

VIJAYAPUR.

Department of Mathematic's

Seminar on :- Trigonometry

De-Moiver's theorem.

Name :- Praveen H. Rathod.

Class :- B.Sc. I Sem.

Date :- 19/09/18

Subject :- Mathematic's.

-: TRIGONOMETRY :-

Introduction :-

-> It is a branch of mathematic's, which deals with the measurement of 3 sides or angles.

-> Aryabhata had given the 3 major trigonometric ratios are sine, cosine & tangent (sin, cos, tan).

Power's of complex numbers :-

If $z = r(\cos\theta + i\sin\theta)$. here r & θ determiny the 'z'. So. there exists the infinitely value of θ.

then satisfy the $x = |z|\cos\theta$, & $y = |z|\sin\theta$.

Demoiver's theorem :-

-> If $z = r(\cos\theta + i\sin\theta)$. then $z^n = r^n(\cos n\theta + i\sin n\theta)$. this theorem is known as the De-Moiver theorem. this theorem was given by a french mathematician.

-> This theorem was already proved. as when 'n' is an positive or negative integer and when 'n' is any ration number.

De-Moivers theorem :-

→ De-Moivers theorem was introduced as a consequ-ence of Euler's identity.

$[\cos\theta + i\sin\theta]^n = \cos(n\theta) + i\sin(n\theta)$, $\forall$ $\alpha \in \theta$ $n \in R$.

→ To prove this theorem we use to the mathematical induction elementary trigonometric identity.

→ Proof :-

Step 1 :- The given $[\cos\theta + i\sin\theta]^n = \cos n\theta + i\sin n\theta$ is true for $n=1$.

because $[\cos\theta + i\sin\theta]^1 = \cos(1\theta) + i\sin(1\theta)$

$\Rightarrow \cos\theta + i\sin\theta = \cos\theta + i\sin\theta$.

then assume that the De moivers theorem is true for some ~~so~~ positive integer 'n'.

Step 2 :- Then it implies that the it is true for 'n+1' i.e.

$[\cos\theta + i\sin\theta]^{n+1} = \cos[(n+1)\theta] + i\sin[(n+1)\theta]$ — ①

Since it is true the by hypothesis that.

$[\cos\theta + i\sin\theta]^n = \cos n\theta + i\sin n\theta$. — ②.

multiply by. $(\cos\theta + i\sin\theta)$ to eqn ②.

then $[\cos\theta + i\sin\theta]^n \cdot [\cos\theta + i\sin\theta] = [\cos n\theta + i\sin n\theta]$.
$(\cos\theta + i\sin\theta)$. — ③

$\Rightarrow$ by simplifying the eqn (3).
we get.

$$(\cos\theta + i\sin n\theta)^{n+1} = [\cos n\theta \cos\theta - \sin n\theta . \sin\theta] + i[\sin n\theta \cos\theta + \cos n\theta . \sin\theta]. \quad -(4)$$

$\Rightarrow$ w.k.t.

$$\sin(a+b) = \sin a \cos b + \cos a \sin b.$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b.$$

$\Rightarrow$ eqn (4). becomes,

$$= (\cos\theta + i\sin n\theta)^{n+1} = \cos(n\theta+\theta) + i\sin(n\theta+\theta).$$

$$= (\cos\theta + i\sin n\theta)^{n+1} = \cos(n+1)\theta. + i\sin(n+1)\theta.$$

Hence. the Euler's identity is proved

then the de-Moviers theorem also proved

and it is true for "n+1" nr's also.

Conclusion of theorem:-

$$z = r(\cos\theta + i\sin n\theta)$$

$$\frac{1}{z} = r(\cos\theta - i\sin n\theta)$$

$$z + \frac{1}{z} = 2\cos\theta.$$

$$\left[\left(z^n + \frac{1}{z^n}\right) = 2\cos n\theta\right] //$$

$$\left[\left(z^n - \frac{1}{z^n}\right) = i2\sin n\theta\right] //$$

Expansion's for $\sin n\theta$ & $\cos n\theta$:-

By the use of De-Moivres theorem.

w.k.t.

$$\cos n\theta + i\sin n\theta = [\cos\theta + i\sin\theta]^n$$

the RHS. $[\cos\theta + i\sin\theta]^n$

apply binomial theorem.

$$\Rightarrow {}^nC_0 \cos^n\theta + {}^nC_1 \cos^{n-1}\theta \sin\theta + i\,{}^nC_2 \cos^{n-2}\theta \sin^2\theta\, i^2 + \cdots$$

$$\Rightarrow \cos n\theta + i\sin n\theta = \cos^n\theta + i n \sin\theta + \frac{n(n-1)}{2!}\cos^{n-2}\theta\, i^2\sin^2\theta + \frac{n(n-1)(n-2)}{3!}\cos^{n-3}\theta\, i^3\sin^3\theta + \cdots$$

$$\Rightarrow \cos n\theta + i\sin n\theta = \cos^n\theta + i n\sin\theta - \frac{n(n-1)}{2!}\cos^{n-2}\theta\sin^2\theta - i\frac{n(n-1)(n-2)}{3!}\cos^{n-3}\theta\sin^3\theta + \cdots$$

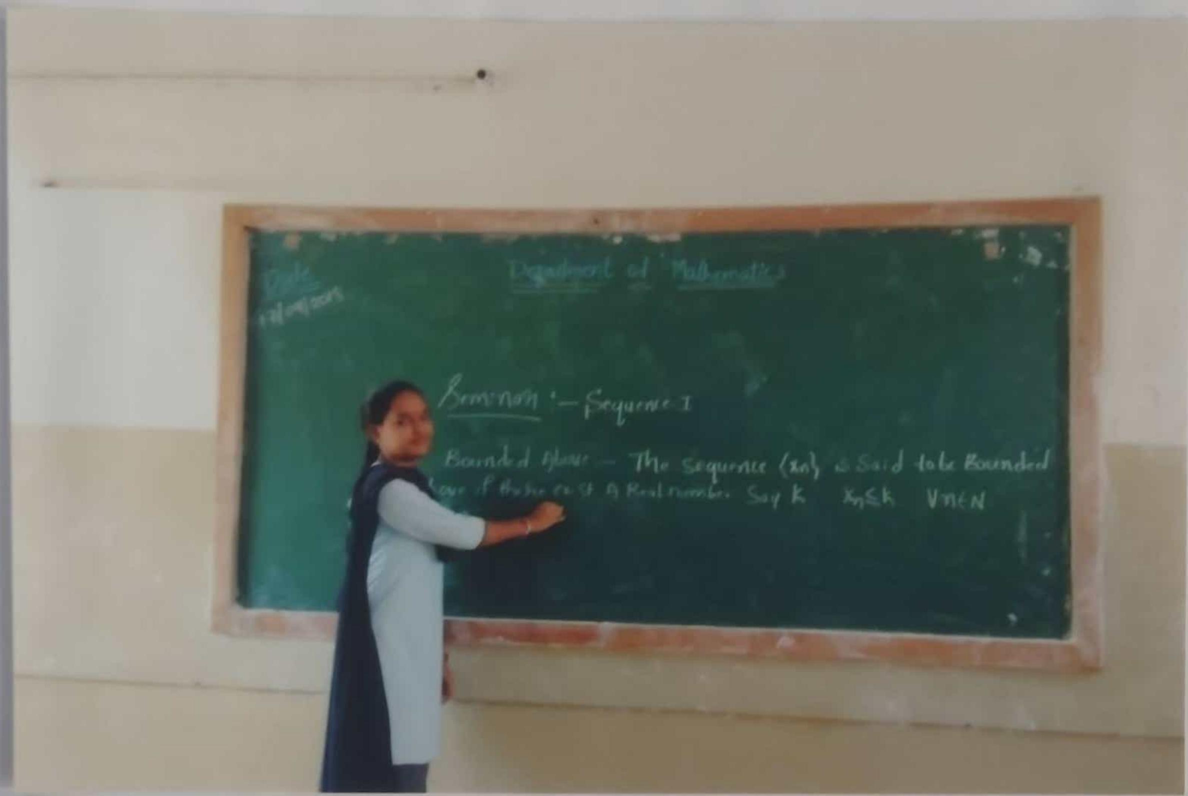
$\Rightarrow$ by simplyfing.

$$\left[\cos n\theta = \cos^n\theta - \frac{n(n-1)}{2!}\cos^{n-2}\theta\sin^2\theta + \cdots\right]$$

//

$$\left[\sin n\theta = n\cos^{n-1}\theta\sin\theta - \frac{n(n-1)(n-2)}{3!}\cos^{n-3}\theta\sin^3\theta + \cdots\right]$$

//



Name : Sudharani Biradar

Semester : III

Topic : Sequence - I

MATHEMATICS SEMINAR

TOPIC :- Sequence.

Prepared By

Name :- SUDHABANI. BIRADAR

Roll No :- 75

RCUB No :- S1722088

Sem :- BSC III

Sequence :-

Sequence is a mapping or a function f from the set of natural number to the set of Real number R. i.e $f: N \rightarrow R$.

* It is denoted by $\{x_n\}$

Bounded above

A sequence $\{x_n\}$ is said to be bounded above if there exist a real number k such that $x_n \leq k$. for all $n \in N$.

Ex :- Sequence $\{x_n\} = \{-n^2\}$.

$\therefore x_n \leq 1$.

$\therefore$ The sequence is Bounded above by -1.

Bounded Below

A Sequence $\{x_n\}$ is said to be Bounded Below if there exist a real number say k. Such that $x_n \geq k$. for all $n \in N$.

Ex :- Consider the Sequence $\{x_n\} = \{n^2\}$.

$\therefore x_n \geq 1$.

The sequence Bounded below by 1.

Bounded Sequence

A Sequence $\{x_n\}$ is said to be Bounded if it Bounded Both above and Below that is if there exist two Real numbers say k & k such that $k \leq x_n \leq k \;\; \forall \; n \in N$

Limit of a Sequence

A Real number 'l' is said to be Limit of a Sequence $\langle x_n \rangle$ as n tends to ∞ if for every $\varepsilon > 0$. However small. there exist a positive integer. 'm'.
Such that $|x_n - l| < \varepsilon \quad \forall\, n \geq m$.

Convergent Sequence

A Sequence $\langle x_n \rangle$ is said to be Convergent if the sequence tends to a Finite and Unique Value say 'l'

i.e $\lim_{n \to \infty} x_n = l$. [Finite and Unique]

Divergent Sequence

A Sequence $\langle x_n \rangle$ is said to be divergent if the limit of the Sequence is Unique and Infinite

i.e $\lim_{n \to \infty} \langle x_n \rangle = \pm \infty$.

Oscillatory Sequence.

A Sequence $\langle x_n \rangle$ is Said to be oscillatory if the sequence neither tends to a Unique finite [in Convergent] nor to $-\infty$ or ∞ [Divergent]

Infimum of Sequence [Greatest lower bound]

A Real number M is said to be infimum or glb of a sequence $\langle x_n \rangle$ if

1) $x_n \geq M \quad \forall \; n \in N$
2) $\forall \; \varepsilon > 0 \quad \exists$ atleast one term x_k. Such that $x_k < M + \varepsilon$.

Supremum of Sequence [least Upper bound]

A Real number 'm' is said to be Supremum of or the least upper bound of the Sequence $\langle x_n \rangle$ if

1) $x_n \leq m \quad \forall \; n \in N$
2) $\forall \; \varepsilon > 0. \quad \exists$ atleast one term x_k such that $x_k > m - \varepsilon$.

Monotonic Increasing Sequence

A Sequence $\langle x_n \rangle$ is said to be monotonically increasing. If $x_{n+1} > x_n \; \forall \; n \in N$.
i.e if $x_1 \leq x_2 \leq x_3 \leq - - - \; - - - \leq x_n \leq x_{n+1} \leq - -$

Monotonic decreasing Sequence

A Sequence $\langle x_n \rangle$ is said to be monotonically decreasing if $x_{n+1} < x_n \; \forall \; n \in N$.
i.e if $x_1 \geq x_2 \geq x_3 \geq - - - \; - - \geq x_n \geq x_{n+1} \geq - - -$

Theorem 1

Every Convergent Sequence is Bounded.

proof:- let $\{x_n\}$ be a Convergent Sequence and $\lim_{n \to \infty} x_n = l$.

$\forall \varepsilon > 0$. $\exists m \in N$ such that

$|x_n - l| < \varepsilon \quad \forall n \geq m$

$-\varepsilon < (x_n - l) < \varepsilon \quad \forall n \geq m$

$l - \varepsilon < (x_n) < l + \varepsilon \quad \forall n \geq m$

i.e the terms of the sequence after the m^{th} term are bounded between $l-\varepsilon$ & $l+\varepsilon$. but we have to show that $\forall n \in N$. $k \leq x_n \leq K$ & for some Real number k & K.

let $k = \min \{x_1, x_2, x_3 ------ x_{m-1} . l-\varepsilon . l+\varepsilon\}$

& $K = \max \{x_1, x_2, x_3 ---- x_{m-1} , l-\varepsilon . l+\varepsilon\}$.

Clearly. $k \leq x_i \quad i = 1.2, 3 - .. (m-1)$

& $k \leq l-\varepsilon . l+\varepsilon$.

Also $x_i \leq K$ & $l-\varepsilon . l+\varepsilon \leq K$.

we have $k \leq l-\varepsilon < x_n < l+\varepsilon \leq K \quad \forall n \in N$

$k \leq x_n \leq K \quad \forall n \in N$.

Theorems on Monotonic Sequence

* A monotonic Increasing Sequence which is Bounded above is always Convergent

Proof:- let x_n be the monotonic Increasing Sequence which is bounded above

Let M be its supremum or l.u.b

$\therefore x_n < M \ \forall n \in N.$

& $\forall \varepsilon > 0 \ \exists m \in N$ such that

$M - \varepsilon < x_m < M$

Since $\langle x_n \rangle$ is a monotonic increasing sequence

$x_n \geq x_m \ \forall n > m \quad \langle x_{n+1} > x_n$

$\Rightarrow x_n \geq x_m > M - \varepsilon \quad \forall n > m$

$\Rightarrow M - \varepsilon < x_m \leq x_n < M \leq M - \varepsilon \ \forall n > m$

$\Rightarrow M - \varepsilon < x_n < M + \varepsilon \quad \forall n > m$

$\Rightarrow -\varepsilon < x_n - M < \varepsilon \quad \forall n > m$

$\Rightarrow |x_n - M| < \varepsilon \quad \forall n > m$

$\therefore \lim_{n \to \infty} x_n = M$ (finite)

Hence $\langle x_n \rangle$ is convergent to its supremum

2) Monotonic decreasing sequence which is bounded below is always to convergent to its Infimum

let L be its infimum or g.l.b

$\therefore x_n \geq L \quad \forall n \in N.$

$\varepsilon \ \forall \varepsilon > 0 \ \exists m \in N$ such that $L < x_m < L + \varepsilon.$

Since $\langle x_n \rangle$ is a monotonic decreasing sequence

$\therefore x_n \leq x_m \ \forall \ n > m \quad \langle x_{n+1} \leq x_n$

$\Rightarrow x_n \leq x_m < L + \varepsilon \ \forall n > m$

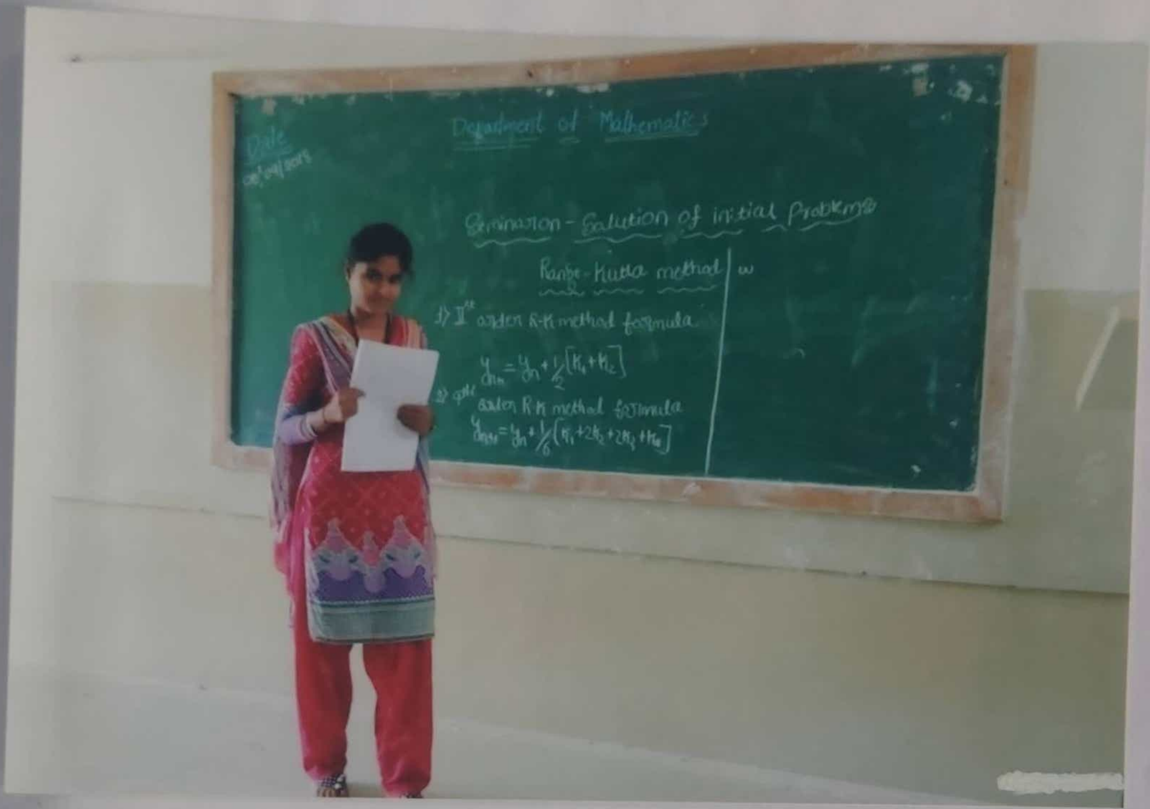
$\Rightarrow L - \varepsilon < L < x_n < x_m < L + \varepsilon \ \forall n > m$

$\Rightarrow L - \varepsilon < x_n < L + \varepsilon \quad \forall n > m$

$\Rightarrow -\varepsilon < x_n - L < \varepsilon \quad \forall n > m$

$\Rightarrow |x_n - L| < \varepsilon \quad \forall n > m$

$\therefore \lim_{n \to \infty} x_n = L$ [finite]



Name : Sheela Salotagi

Semester : V

Topic : Range kutta Method

STUDENT'S NAME

CLASS

ROLL NO.

SUBJECT

DATE

TOTAL MARKS OBTAINED

B.L.D.E Association

B. Arts & K.C.P Sc College Vijayapur

Mathematics

Name - Sheelavati . C. Salotagi

Class - B.Sc V th sem.

Rool no - 365

R.C.U NO - SI622392.

Topic : Range-Kutta method.

Date : 05/09/18

Maths

STUDENT'S NAME		TOTAL MARKS OBTAINED
CLASS	SUBJECT	
ROLL NO.	DATE	

Solution of initial problems

Range-Kutta Method

Range Kutta method are designed to given greater accuracy and a posses the advantages of requirment only the function values at some selected points on the subintervals

These method agree with taylor's series solution upto the term h^r method is called the order of the method of which the 2^{nd} order & 4^{th} order methods are widely used where 'r' is differ from method method to method is called the order of that method.

1) First order of Range-Kutta method

Let 'y' is $y' = \frac{dy}{dx} = f(x, y)$ as the initial conditions $y = y_0$ & $x = x_0$

By Euler's method

We know that $y_1 = y_0 + hf(x_0, y_0)$

$$y_1 = y_0 + hy_0'$$

Expanding by the taylor's series, we get

$$y_1 = y(x_0+h) = y_0 + hy_0' + \frac{h^2}{2!} y_0'' + \cdots$$

It shows that the Euler's method consider with the taylor's series solution up to the term in h.

Hence the Euler's method is Range-Kutta method of 1^{st} order

2) <u>Second order Range-Kutta method</u>

$$y' = \frac{dy}{dx} = f(x, y) \longrightarrow ①$$

We know that for modified Euler's method.

$$y_1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1)] \longrightarrow ②$$

putting $x_1 = x_0 + h$ & $y_1 = y_0 + hf(x_0, y_0)$ on the R.H.S of equation ②

we get

$$y_1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_0+h, y_0+hf(x_0, y_0))]$$

$$y_1 = y_0 + \frac{h}{2}[f_0 + f(x_0+h, y_0+hf_0)] \longrightarrow ③$$

Expanding L.H.S by taylor's series

$$y_1 = y(x_0+h)$$

$$= y_0 + hy_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \cdots$$

Sona Roopa

STUDENT'S NAME | TOTAL MARKS OBTAINED
CLASS | SUBJECT
ROLL NO. | DATE

Expanding $f(x_0+h, y_0+hf_0)$ by taylor's series for a function of two variables using eqn ③

$$y_1 = y_0 + \frac{h}{2}\left[f_0 + f(x_0, y_0) + h\left(\frac{\partial f}{\partial x}\right)_0 + hf_0\left(\frac{\partial f}{\partial y}\right)_0 + \text{order of } (h)^2 \text{-------}\right.$$

$$= y_0 + \frac{1}{2}\left[hf_0 + hf_0 + h^2\left\{\left(\frac{\partial f}{\partial x}\right)_0 + f_0\left(\frac{\partial f}{\partial y}\right)_0\right\}\right] + \text{order of } (h)^2 + \text{----}$$

$$= y_0 + hf_0 + \frac{h^2}{2}\left\{\left(\frac{\partial f}{\partial x}\right)_0 + f_0\left(\frac{\partial f}{\partial y}\right)_0\right\} + \text{order of } (h)^2 + \text{----}$$

$$= y_0 + hf_0 + \frac{h^2}{2} f_0' + \text{order of } (h^2) + \text{order of } (h^3)$$

$$+ \text{-----} \longrightarrow ⑤$$

$$\left[\because f_0' = \frac{\partial f}{\partial y}(x, y) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\right.$$

Eqn ③ and ⑤ are same shows that modified Euler's method agree with taylor's series solution up to the term in $(h)^2$. Hence the modified Eulers method in range kutta method of 2^{nd} order. Therefore the 2^{nd} order of range kutta method formula is given by

$$y = y_0 + \frac{1}{2}[K_1 + K_2]$$

where $K_1 = hf(x_0, y_0)$

$K_2 = hf(x_0+h, y_0+K_1)$

The general formula is

$$y_{n+1} = y_n + \frac{1}{2}[k_1 + k_2]$$

Note:

1) Range Kutta method 2^{nd} order formula.

$$y_{n+1} = y_n + \frac{1}{2}[k_1 + k_2]$$

2) Range Kutta method 4^{th} order formula.

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where $k_1 = hf(x_0, y_0)$

$k_2 = hf(x_0 + h/2, y_0 + \frac{k_1}{2})$

$k_3 = hf(x_0 + h/2, y_0 + \frac{k_2}{2})$

$k_4 = hf(x_0 + h, y_0 + k_3)$

3) Range Kutta method 3^{rd} order formula.

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 4k_2 + k_3)$$

where $k_1 = hf(x_0, y_0)$

$k_2 = hf(x_0 + h/2, y_0 + \frac{k_1}{2})$

$k_3 = hf(x_0 + h, y_0 + k_1)$

STUDENT'S NAME | TOTAL MARKS OBTAINED
CLASS | SUBJECT
ROLL NO. | DATE

Examples on Range-Kutta method.

01. Solve $\frac{dy}{dx} = x + y^2$ with initial condition $y = 1$ when $x = 0$ for $x = 0.4$ using Range-Kutta method.

Soln: Here $f(x, y) = x + y^2$, $x_0 = 0$, $y_0 = 1$ & $h = 0.2$.

The first appn is given by

$$y_1 = y_0 + \frac{1}{2}[K_1 + K_2]$$

Now $K_1 = hf(x_0, y_0)$

$$= 0.2(x_0 + y_0^2)$$

$$= 0.2(0 + 1^2)$$

$$= 0.2.$$

$$K_2 = hf(x_0 + h, y_0 + K_1)$$

$$= 0.2[f(0 + 0.2, 1 + 0.2)]$$

$$= 0.2. f(0.2, 1.2)$$

$$= 0.2[0.2 + (1.2)^2]$$

$$= 0.328.$$

Now $y_1 = y_0 + \frac{1}{2}[K_1 + K_2]$

$$= 1 + \frac{1}{2}[0.2 + 0.328]$$

$$= 1.264$$

Then 2^{nd} ~~order~~ appn is given by

Sona Roopa

$y_2 = y_1 + \frac{1}{2}[k_1 + k_2]$

$x_1 = x_0 + h = 0 + 0.2 = 0.2$

$y_1 = 1.264$ & $h = 0.2$.

Now $k_1 = hf(x_1, y_1)$

$= 0.2[(0.2) + (1.264)^2]$

$= 0.3595$

$k_2 = hf(x_1 + h, y_1 + k_1)$

$= (0.2) f[0.2 + 0.2, 1.264 + 0.3595]$

$= (0.2) f[0.4, 1.6235]$

$= (0.2) [0.4 + (10.6235)^2]$

$= (0.2) [0.4 + (1.6235)^2]$

$= 0.6071$

$y_2 = 1.264 + \frac{1}{2}[0.3595 + 0.6071]$

$= 1.7473$

2) find the approximation solution at $x = 1.2$ of the equation $\frac{dy}{dx} = xy$ given $y(1) = 2$ by R.K method.

Solution:

Here $f(x,y) = xy$; $x_0 = 1$ $y_0 = 2$, $h = 0.2$

The first approximation is given by

$y_1 = y_0 + \frac{1}{2}[k_1 + k_2]$

STUDENT'S NAME		TOTAL MARKS OBTAINED
CLASS	SUBJECT	
ROLL NO.	DATE	

Now $K_1 = h \cdot f(x_0, y_0)$

$= (0.2)[1 \times 2]$

$= 0.4$

$K_2 = hf(x_0+h, y_0+K_1)$

$= (0.2) f[1+0.2, 2+0.4]$

$= 0.2[1.2 \times 2.4]$

$= 0.576$

$y_1 = y_0 + \frac{1}{2}[K_1+K_2]$

$= 2 + \frac{1}{2}[0.4 + 0.576]$

$= 2.488$

3) Solve $\frac{dy}{dx} = x^2+y^2$ $y(0)=1$, $h=0.2$ find $y(0.2)$ by R-K method.

Soln: Here $f(x,y) = x^2+y^2$ $x_0=0$, $y_0=1$
$h=0.2$

The 1st approximation is given by
$y_1 = y_0 + \frac{1}{2}[K_1+K_2]$

Now $K_1 = hf(x_0, y_0)$

$= 0.2(x^2+y^2)$

$= 0.2(0+1)$

$= 0.2$

$K_2 = hf(x_0+h, y_0+K_1)$

$= (0.2) f[0+0.2, 1+0.2]$

$= (0.2)\left[(0.2)^2 + (1.2)^2\right]$

$= 0.296$

$y_1 = 1 + \frac{1}{2}\left[0.2 + 0.296\right]$

$= 1.248$ //

04. find approximation solution at $x = 0.2$, if $\frac{dy}{dx} = x + y^2$, given that $y = 1$ when $x = 0$

Soln: Here $f(x, y) = x + y^2$, $x_0 = 0$, $y_0 = 1$

$h = 0.1$

now we have to find $y(0.1)$ and $y(0.2)$

I^{st} approximation is given by

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$.

Now $k_1 = hf(x_0, y_0) = (0.1) f(0.1)$

$= (0.1)\left[0 + (1)^2\right]$

$= 0.1$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= (0.1) f\left[0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\right]$

$= (0.1) f(0.05, 1.05)$

$= (0.1)\left[0.05 + (1.05)^2\right]$

STUDENT'S NAME		TOTAL MARKS OBTAINED
CLASS	SUBJECT	
ROLL NO.	DATE	

$= 0.11525$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= (0.1)\left[(0.05 + (1.057625)^2\right]$

$= (0.1)\left[0.05 + 1.118570\right]$

$= 0.11686$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= (0.1)\left[0.1 + (1.11686)^2\right]$

$= 0.13474$

$\therefore y_1 = y_0 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$

$y(0.1) = 1 + \frac{1}{6}\left[(0.1) + 2(0.11525 + 0.11686) + 0.134741\right]$

$= 1 + \frac{1}{6}(0.69896) = 1.11649.$

2^{nd} opproximation is given by

$y_2 = y_1 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$

Then $x_1 = 0.1$, $y_1 = 1.11649$ $h = 0.1$

then

$k_1 = h f(x_1, y_1)$

$k_1 = (0.1) f\left[0.1, 1.11649\right]$

$= (0.1)\left[0.1 + (1.11649)^2\right]$

$= 0.13465$

Sona Roopa

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$= (0.1)\left[0.15 + (1.18382)^2\right]$$
$$= 0.15514.$$

$$k_3 = hf\left[x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right]$$

$$= (0.1)\, f(0.15, 1.19406)$$
$$= (0.1)\left[0.15 + (1.19406)^2\right]$$
$$= 0.15758.$$

$$k_4 = hf(x_1 + h, y_1 + k_3)$$

$$= (0.1)\, f(0.2, 1.27407)$$
$$= (0.1)\left[0.2 + (1.27407)^2\right]$$

$$= 0.18233$$

$$y_2 = y_1 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$= 1.11649 + \frac{1}{6}\left[0.13465 + 2(0.15514 + 0.15758) + 0.18233\right]$$

$$y(0.2) = 1.11649 + 0.15707 = 1.27356$$

$$\therefore \quad y_1 = 1.11649$$

$$y_2 = 1.27356$$

//

Sona Roopa

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S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

SEMINAR REPORT: 2018-19

(Odd Semester)

The UG Department of Mathematics has conducted seminar for B.Sc students.

Name of the student	Seminar Topic
1) Praveen Rathod	Trigonometry
2) Sudharani Biradar	Sequence - I
3) Sheela Salodagi	Range kutta method

Head of Department

H. O. D: Department of Mathematics, S. B. Arts & K. C. P. Science College, BIJAPUR,

Principal

Principal, S. B. Arts & KCP Sc. College, Bijapur

IQAC, Co-ordinator
S.B.Arts & K.C.P.Science College, Vijayapur.

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S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

SEMINAR REPORT

2018-19 (EVEN SEMESTER)

BLDE Association's

S.B.Arts And K.C.P Science College, Vijayapur

DEPARTMENT OF MATHEMATICS

NOTICE:

The UG Department of Mathematics is conducting seminar for B.Sc students for the academic year 2018-19(Even Semester)

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Principal,
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S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

Seminar Report: 2018-19

Even Semester

Sl. No	Semester	Name of the Student	Topics	Date
1	I	Rekha Masyal	Shpere	06-03-2019
2	III	Daneshwari Bhatagunaki	Fourier Series	11-03-2019
3	V	Aishwarya Kaladagi	Non Linear and Linear Differential Equations	14-03-2019

Rekha Mashal
U. Reg. No : S1824973

TOPIC - SPHERE

Equation of sphere

Let A(a, b, c) be a fixed point in space and let 'r' be any positive real number and let P(x, y, z) be moving point such that AP = r, a constant

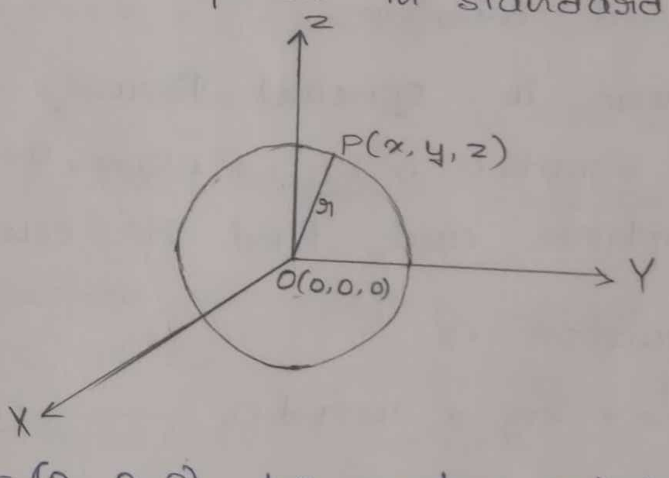
Squaring on both sides,

$$(AP)^2 = r^2$$

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

which is required eq^n of sphere with center point A(a, b, c) and radius r.

Equation of sphere in standard form:



Let O = (0, 0, 0) be center point of sphere.
Let P(x, y, z) be any point on sphere. Join OP.
Since OP is radius of sphere.

$$OP = r$$

By using distance formula

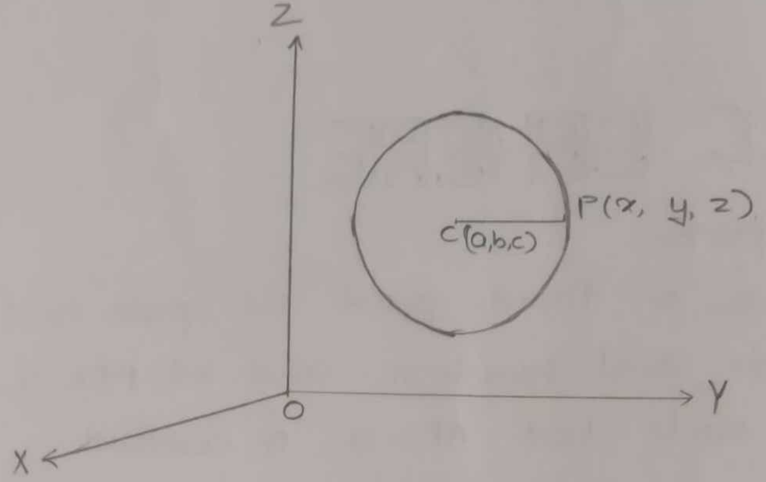
$$\sqrt{x^2 + y^2 + z^2} = r$$

$$x^2 + y^2 + z^2 = r^2$$, which is required eq^n.

Equation of sphere in central form

To find equation of sphere whose center is (a, b, c) and radius a.

Let $C = (a, b, c)$ be the center of the sphere.



Let $P(x, y, z)$ be any point on sphere. Join CP.

Since
$$CP = r$$
$$CP^2 = r^2$$

By distance formula
$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2,$$
which is required equation.

Equation of sphere in general form

To prove that equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ represents a sphere and find its center & radius.

<u>Proof</u>:- Given equation is,

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0 \quad \text{--- } (1)$$

which can be re-arranged as,

$$(x^2 + 2ux) + (y^2 + 2vy) + (z^2 + 2wz) = -d$$

Adding both sides by $u^2 + v^2 + w^2$, we get

$$(x^2 + 2ux + u^2) + (y^2 + 2vy + v^2) + (z^2 + 2wz + w^2) = u^2 + v^2 + w^2 - d$$

$$(x+u)^2 + (y+v)^2 + (z+w)^2 = u^2 + v^2 + w^2 - d$$

$[x-(-u)]^2+[y-(-v)]^2+[z-(-w)]^2=\left[\sqrt{u^2+v^2+w^2-d}\,\right]^2$ — ②

which is of the Form of

$(x-a)^2+(y-b)^2+(z-c)^2=r^2$ —— ③

Hence equation ① represents a sphere.

To determine it's center & radius, compare equations ② & ③.

$a=-u \quad b=-v \quad c=-w$

$\therefore r=\sqrt{u^2+v^2+w^2-d}$

$C=(-u,-v,-w)$

Examples:

1) Find the equation of sphere whose center is $(2,-1,4)$ and radius 3 units.

$C=(2,-1,4)=(a,b,c)$ & $r=3$

WKT, $(x-a)^2+(y-b)^2+(z-c)^2=r^2$

$\therefore (x-2)^2+(y+1)^2+(z-4)^2=9$

$x^2+4-4x+y^2+1+2y+z^2+16-8z-9=0$

$x^2+y^2+z^2-4x+2y-8z+12=0.$

which is our required equation of sphere.

2) Find equation of sphere whose center is $(0,-2,3)$, and $(2,6,-1)$ be a point on sphere.

Let $C(a,b,c)=(0,-2,3)$

$P(x,y,z)=(2,6,-1)$

$CP=r=\sqrt{(2-0)^2+(6+2)^2+(-1-3)^2}$

$r=\sqrt{4+64+16}$

$r=\sqrt{84}$ units.

Required equation of sphere is,

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

$$(x-0)^2 + (y+2)^2 + (z-3)^2 = (\sqrt{84})^2$$

$$x^2 + y^2 + 4 + 4y + z^2 + 9 - 6z = 84$$

$$x^2 + y^2 + z^2 + 4y - 6z - 71 = 0.$$

3> Find the center and radius of sphere

$$2x^2 + 2y^2 + 2z^2 - 8x + 4y - 6 = 0$$

WKT, the general eq^n of the sphere is

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0. \quad \text{--- } (1)$$

Given eq^n is

$$2x^2 + 2y^2 + 2z^2 - 8x + 4y - 6 = 0 \quad \text{--- } (2)$$

~~Comparin.~~

Dividing eq^n (2) by '2'

$$x^2 + y^2 + z^2 - 4x + 2y - 3 = 0 \quad \text{--- } (3)$$

Comparing eq^n (3) with eq^n (1)

$2u = -4$ $2v = 2$ $2w = 0$ & $d = -3$

$u = -2$ $v = 1$ $w = 0$ $d = -3$.

$$\therefore C = (-u, -v, -w)$$

$$= (+2, -1, 0)$$

$$r = \sqrt{u^2 + v^2 + w^2 - d}$$

$$r = \sqrt{4 + 1 + 0 + 3}$$

$$r = \sqrt{8} \text{ units.}$$

Find equation of sphere on joining of (2, -3, 1) and (1, -2, -1) as diameter.

Let $A(x_1, y_1, z_1) = (2, -3, 1)$

$B(x_2, y_2, z_2) = (1, -2, -1)$

WKT, $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0$

$\Rightarrow (x-2)(x-1)+(y+3)(y+2)+(z-1)(z+1)=0$

$\Rightarrow x^2-3x+2+y^2+5y+6+z^2-1=0$

$\Rightarrow x^2+y^2+z^2-3x+5y+7=0.$

which is required eq^n of sphere.

6> Find equation of sphere on the join of points (-1, 3, 2) and (5, 7, -6) as end points of diameter of sphere, also find its center & radius.

Let $A(x_1, y_1, z_1) = (-1, 3, 2)$

$B(x_2, y_2, z_2) = (5, 7, -6)$

Equation of sphere is given by,

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0$

$(x+1)(x-5)+(y-3)(y-7)+(z-2)(z+6)=0$

$x^2-4x+(-5)+y^2-10y+21+z^2+4z-12=0$

$x^2+y^2+z^2-4x-10y+4z+4=0$ —— ①

Comparing eq^n ① with eq^n of sphere in general form we get,

$2u=-4$ $\quad 2v=-10$ $\quad 2w=4$ $\quad d=4$

$u=-2$ $\quad v=-5$ $\quad w=2$ $\quad d=4$

Center is, $C=(-u, -v, -w)=(2, 5, -2)$

radius is, $r=\sqrt{u^2+v^2+w^2-d}$

$r=\sqrt{4+25+4-4}$

$r=\sqrt{29}$ units.

Dhaneshwari . Bhatagunaki

B.Sc IVth sem

Roll. No - 169

Mathematics Seminar.

-: FOURIER SERIES :-

Periodic function :- A function f(x) is said to be provi periodic if it is defined for all real numbers x and ∃ a +ve integer 't' Such that $f(x+t) = f(x)$

The number 't' is called the period of the form f(x). If f(x) is a periodic function with period 't' then $f(x+nt) = f(x)$ where n is any integer.

Ex :- $\sin(x+2\pi) = \sin x$

$\cos(x+2\pi) = \cos x$

A constant function say $h(x)=c$ is also periodic as $h(x+t)=c$ for every +ve integer 't'.

Dirichlet's condition for a fourier expansion

A function f(x) is defined as the interval $[-\pi, \pi]$ can be expressed as in the series as $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ ↳①

Series as $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

where $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\, dx$

To determine the coefficient a_0, a_n & b_n the following definite integrals are used

* $\int_{-\pi}^{\pi} \sin nx \, dx = 0 = \int_{-\pi}^{\pi} \cos nx \, dx$

* $\int_{-\pi}^{\pi} \sin nx \cos nx \, dx = 0$

* $\int_{-\pi}^{\pi} \sin mx \sin nx \, dx = \begin{cases} 0 & m \neq n \\ \pi & m = n \end{cases}$

* $\int_{-\pi}^{\pi} \cos mx \sin nx \, dx = \begin{cases} 0 & m \neq n \\ \pi & m = n \end{cases}$

* To determine the a_0 integrate both side of eq^{n}s w.r.t 'x' bet^n the limit points $(-\pi, \pi)$

$$\int_{-\pi}^{\pi} f(x)\,dx = \frac{a_0}{2}\int_{-\pi}^{\pi} dx + \sum_{n=1}^{\infty} a_n \int_{-\pi}^{\pi} \cos nx \, dx + \sum_{n=1}^{\infty} b_n \int_{-\pi}^{\pi} \sin nx \, dx$$

$$\int_{-\pi}^{\pi} f(x)\,dx = \frac{a_0}{2}[x]_{-\pi}^{\pi} + 0 \quad \left[\because \int_{-\pi}^{\pi} \cos nx \, dx = 0 = \int_{-\pi}^{\pi} \sin nx \, dx\right.$$

$$= \frac{a_0}{2}[\pi + \pi]$$

$$= \frac{a_0}{2} \times 2\pi$$

$$= a_0 \pi$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

* To determine a_n multiply both side of the eq^n (1) by $\cos kx$ & integrate bet^n limit points $[-\pi, \pi]$

$$\int_{-\pi}^{\pi} f(x)\cos nx\, dx = 0 + \sum_{n=1}^{\infty} a_n \int_{-\pi}^{\pi} \cos nx \cos kx\, dx + 0$$

$$\int_{-\pi}^{\pi} f(x)\cos kx\, dx = \sum_{n=1}^{\infty} a_n \int_{-\pi}^{\pi} \cos nx \cos kx\, dx$$

$$n = k \quad \int_{-\pi}^{\pi} \cos nx \cdot \cos kx\, dx = \pi$$

$$\int_{-\pi}^{\pi} f(x)\cos kx\, dx = a_k \pi$$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos kx\, dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx$$

* To determine b_n multiply both sides of equation by $\sin kx$ and integrate between limit points $-\pi$ to π

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where $n = \frac{1}{\pi}$ $\rightarrow$ (1)

$$\int_{-\pi}^{\pi} f(x)\sin kx\, dx = \frac{a_0}{2}\int_{-\pi}^{\pi} \sin kx\, dx + \sum_{n=1}^{\infty} a_n \int_{-\pi}^{\pi} \cos nx\ +$$

$$\sin kx\, dx + \sum_{n=1}^{\infty} b_n \int_{-\pi}^{\pi} \sin nx \sin kx\, dx$$

$$\Rightarrow \int_{-\pi}^{\pi} f(x)\sin kx\, dx = 0 + 0 + \sum_{n=1}^{\infty} b_n \int_{-\pi}^{\pi} \sin nx \sin kx\, dx$$

$$\Rightarrow \int_{-\pi}^{\pi} f(x)\sin kx\, dx = \sum_{n=1}^{\infty} b_n \int_{-\pi} \sin nx \sin kx\, dx$$

Since $n = k$

$$\Rightarrow \int_{-\pi} f(x)\sin kx\, dx = b_k \pi$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx$$

Formulas

1) $\int e^{ax}\cos bx\,dx = \dfrac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx]$

2) $\int e^{ax}\sin bx\,dx = \dfrac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx]$

3) $\sin n\pi = 0$

$\cos n\pi = \pm 1 \quad \begin{cases} (-1)^n & n \text{ is odd} \\ 1 & n \text{ is even} \end{cases}$

Properties of even and odd function

1) The product of 2 even function and that of 2 odd function is always even where as the product of even and odd function is always odd.

$E \times E = E$

$O \times O = E$

$O \times E = O$

2) $\int_{-\pi}^{\pi} f(x)\,dx = \begin{cases} 2\int_0^{\pi} f(x)\,dx & f(x) \text{ is even} \\ 0 & f(x) \text{ is odd} \end{cases}$

Case i : If $f(x)$ is even

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

Case ii : If $f(x)$ is odd

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

Fourier series with period $2l$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n x}{l}$$

a_0, a_n & b_n are called fourier coefficients.

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BSC VI sem (C)

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Roll No :- 49

Seminar on topic

Non- Linear and Linear Differential equations.

INTRODUCTION:-

In previous, we studied about linear differential equation of different forms. But in this chapter we will study about non-linear differential equation. For the solution of these equation, we used one aspect is charpit's method. And we will also study about the homogeneous linear partial differential equations.

CHARPIT'S METHOD:-

A method for finding a complete integral of the general first order partial differential equation in two independent variables, it involves solving a set of five ordinary differential equations.

Method of solving the PDE $f(x,y,z,p,q)=0$ by charpit's method.

Let the given equation be

$$f(x,y,z,p,q)=0 \quad — ①$$

Now, we shall find one or more relation in x,y,z,p,q.

$$g(x,y,z,p,q)=0 \quad — ②$$

when the values of p and q derived from it & the given equation ① are substituted in equation.

$$dz = p\,dx + q\,dy \quad — ③$$

then it becomes integrable. clearly the integral of the equation ③ will satisfy eqn ① because value of p and q are derived from it, are the same as the value of p and q in eqn ①.

Now,

Consider z, p, q expressed as function of x & y such that when these values are substituted in $g=0$ & $f=0$ they are satisfied identically.

It follows that their derivatives wrt x and y will Vanish.

Now. differentiating ① & ② wrt x, we get

$$\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial x}+\frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial x}+\frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial x}+\frac{\partial f}{\partial p}\cdot\frac{\partial p}{\partial x}+\frac{\partial f}{\partial q}\cdot\frac{\partial q}{\partial x}=0$$

and $$\frac{\partial g}{\partial x}\frac{\partial x}{\partial x}+\frac{\partial g}{\partial y}\cdot\frac{\partial y}{\partial x}+\frac{\partial g}{\partial z}\cdot\frac{\partial z}{\partial x}+\frac{\partial g}{\partial p}\cdot\frac{\partial p}{\partial x}+\frac{\partial g}{\partial q}\cdot\frac{\partial q}{\partial x}=0$$

$$\Rightarrow \frac{\partial f}{\partial x}(1)+\frac{\partial f}{\partial y}(0)+\frac{\partial f}{\partial z}(p)+\frac{\partial f}{\partial p}\cdot\frac{\partial p}{\partial x}+\frac{\partial f}{\partial q}\cdot\frac{\partial q}{\partial x}=0$$

and $$\frac{\partial g}{\partial x}(1)+\frac{\partial g}{\partial y}(0)+\frac{\partial g}{\partial z}(p)+\frac{\partial g}{\partial p}\cdot\frac{\partial p}{\partial x}+\frac{\partial g}{\partial q}\cdot\frac{\partial q}{\partial x}=0$$

$$\Rightarrow \frac{\partial f}{\partial x}+\frac{\partial f}{\partial z}(p)+\frac{\partial f}{\partial q}\cdot\frac{\partial q}{\partial x}=-\frac{\partial f}{\partial p}\cdot\frac{\partial p}{\partial x}$$

and $$\frac{\partial g}{\partial x}+\frac{\partial g}{\partial z}(p)+\frac{\partial g}{\partial q}\cdot\frac{\partial q}{\partial x}=-\frac{\partial g}{\partial p}\cdot\frac{\partial p}{\partial x}$$

$$\Rightarrow \frac{\frac{\partial f}{\partial x}+\frac{\partial f}{\partial z}(p)+\frac{\partial f}{\partial q}\frac{\partial q}{\partial x}}{\frac{\partial f}{\partial p}}=-\frac{\partial p}{\partial x} \quad — ④.$$

& $$\frac{\frac{\partial g}{\partial x}+\frac{\partial g}{\partial z}(p)+\frac{\partial g}{\partial q}\cdot\frac{\partial q}{\partial x}}{\frac{\partial g}{\partial p}}=-\frac{\partial p}{\partial x} \quad — ⑤$$

Now. Comparing eqn ④ & ⑤, we get

$$\Rightarrow \frac{\frac{\partial f}{\partial x}+\frac{\partial f}{\partial z}(p)+\frac{\partial f}{\partial q}\cdot\frac{\partial q}{\partial x}}{\frac{\partial f}{\partial p}}=\frac{\frac{\partial g}{\partial x}+\frac{\partial g}{\partial z}p+\frac{\partial g}{\partial q}\cdot\frac{\partial q}{\partial x}}{\frac{\partial g}{\partial p}}$$

$$\frac{\partial f}{\partial x}\cdot\frac{\partial g}{\partial p}+\frac{\partial f}{\partial z}\frac{\partial g}{\partial p}p+\frac{\partial f}{\partial q}\cdot\frac{\partial g}{\partial p}\cdot\frac{\partial q}{\partial x}=\frac{\partial g}{\partial x}\cdot\frac{\partial f}{\partial p}+\frac{\partial g}{\partial z}\cdot\frac{\partial f}{\partial p}p+\frac{\partial g}{\partial q}\cdot\frac{\partial f}{\partial p}\cdot\frac{\partial q}{\partial x}=0.$$

$$\frac{\partial f}{\partial x}\cdot\frac{\partial g}{\partial p}-\frac{\partial g}{\partial x}\cdot\frac{\partial f}{\partial p}+\left(\frac{\partial f}{\partial z}\cdot\frac{\partial g}{\partial p}-\frac{\partial g}{\partial z}\cdot\frac{\partial f}{\partial p}\right)p+\left(\frac{\partial f}{\partial q}\frac{\partial g}{\partial p}-\frac{\partial g}{\partial q}\cdot\frac{\partial f}{\partial p}\right)\frac{\partial q}{\partial x}=0 \quad — ⑥$$

Similarly differentiating eqn ① & ② partially wrt to y, to eliminate $\frac{\partial q}{\partial x}$ we get.

$$\frac{\partial f}{\partial x}\frac{\partial g}{\partial q}-\frac{\partial g}{\partial x}\cdot\frac{\partial f}{\partial q}+\left(\frac{\partial f}{\partial z}\cdot\frac{\partial g}{\partial q}-\frac{\partial g}{\partial z}\cdot\frac{\partial f}{\partial q}\right)q+\left(\frac{\partial f}{\partial p}\cdot\frac{\partial g}{\partial q}-\frac{\partial g}{\partial p}\cdot\frac{\partial f}{\partial q}\right)\frac{\partial p}{\partial y}=0 \quad —⑦$$

But $\frac{\partial p}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)=\frac{\partial q}{\partial x}$

put $\frac{\partial p}{\partial y}=\frac{\partial q}{\partial x}$ in eqn ⑦ the terms $\frac{\partial p}{\partial y}$ & $\frac{\partial q}{\partial x}$ cancels

Therefore adding eqn ⑥ & ⑦, we have.

$$\frac{\partial f}{\partial x}\cdot\frac{\partial g}{\partial p}-\frac{\partial g}{\partial x}\cdot\frac{\partial f}{\partial p}+\left(\frac{\partial f}{\partial z}\cdot\frac{\partial g}{\partial p}-\frac{\partial g}{\partial z}\cdot\frac{\partial f}{\partial p}\right)p+\frac{\partial f}{\partial x}\frac{\partial g}{\partial p}-\frac{\partial g}{\partial x}\cdot\frac{\partial f}{\partial q}+$$

$$\left(\frac{\partial f}{\partial z}\cdot\frac{\partial g}{\partial q}-\frac{\partial g}{\partial z}\cdot\frac{\partial f}{\partial q}\right)q=0 \quad — ⑧$$

Further the total derivaties of

$g(x.y.z.\ p.q)=0$

$$\left(\frac{\partial g}{\partial x}\right)dx+\left(\frac{\partial g}{\partial y}\right)dy+\left(\frac{\partial g}{\partial z}\right)dz+\left(\frac{\partial g}{\partial p}\right)dp+\left(\frac{\partial g}{\partial q}\right)dq=0 \quad —⑨$$

Compare & rearrange eqn ⑧ with ⑨

$$\left(\frac{\partial g}{\partial x}\right)\left(\frac{-\partial f}{\partial p}\right)+\frac{\partial g}{\partial y}\left(\frac{-\partial f}{\partial q}\right)+\frac{\partial g}{\partial z}\left(-p\frac{\partial f}{\partial p}-q\frac{\partial f}{\partial q}\right)+\frac{\partial g}{\partial p}\left(\frac{\partial f}{\partial x}+p\frac{\partial f}{\partial z}\right)+$$

$$\left(\frac{\partial f}{\partial y}+q\frac{\partial f}{\partial z}\right)\frac{\partial g}{\partial q}=0$$

Above is Lagrange's linear equation of the first order with x.y.z.p.q as independent variables & g as dependent variable.

Now we get equation in the form of Lagrangian Method & its integrals are the integral of f

$$\frac{dx}{-\frac{\partial f}{\partial p}}=\frac{dy}{-\frac{\partial f}{\partial q}}=\frac{dz}{-p\frac{\partial f}{\partial p}-q\frac{\partial f}{\partial q}}=\frac{dp}{\frac{\partial f}{\partial x}+p\frac{\partial f}{\partial z}}=\frac{dq}{\frac{\partial f}{\partial y}+q\frac{\partial f}{\partial z}} \quad — ⑪$$

Example:-

* $(p^2+q^2)y = qz$

→ Given differential equation is

$(p^2+q^2)y = qz$ or $(p^2+q^2)y - qz = 0$ — ①

which of the form $f(x, y, z, p, q) = 0$ then

$\frac{\partial f}{\partial x} = 0,\ \frac{\partial f}{\partial y} = p^2+q^2,\ \frac{\partial f}{\partial z} = -q,\ \frac{\partial f}{\partial p} = 2py,\ \frac{\partial f}{\partial q} = 2qy - z$

By charpit's equation, we have.

$$\frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}}$$

$$\frac{dx}{-2py} = \frac{dy}{-2qy+z} = \frac{dz}{-2p^2y+qz-2q^2y} = \frac{dp}{-pq} = \frac{dq}{p^2+q^2+q(-q)}$$

Taking fractions.

$$\frac{dp}{-pq} = \frac{dq}{p^2}$$

$$\int p\,dp = -\int q\,dq$$

On integrating, we get

$$p^2+q^2 = a^2$$

or $p = \sqrt{a^2-q^2}$ in eqn ① we have

$$p = \sqrt{a^2 - \left(\frac{a^2y}{z}\right)^2} = \frac{a}{z}\sqrt{z^2-a^2y}$$

$$(a^2-q^2+q^2)y = qz$$

$$a^2y = qz \text{ or } q = \frac{a^2y}{z}$$

put p and q in $dz = p\,dx + q\,dy$

$$= \frac{a}{z}\sqrt{z^2-(ay)^2}\,dx + \frac{a^2y}{z}\,dy$$

$$z\,dz = a\sqrt{z^2-(ay)^2}\,dx + a^2y\,dy$$

$$z\,dz - a^2y\,dy = a\sqrt{z^2-(ay)^2}\,dx$$

$$\frac{z\,dz - a^2y\,dy}{\sqrt{z^2-(ay)^2}} = a\,dx$$

$$\frac{1}{2}\int \frac{1}{\sqrt{z^2-a^2y^2}}\,2(z\,dz - a^2y\,dy) = 2a\,dx$$

$2\sqrt{z^2 - a^2y} = 2ax + b$

square on b.s, we get

$(z^2 - a^2y^2) = (ax+b)^2$

$z^2 = a^2y^2 + (ax+b)^2$

*) $q = px + q^2$

→ Given eqn is. $q = px + q^2$ or $px + q^2 - q = 0$ — ①

which is of the form $f(x, y, z, p, q) = 0$

$\frac{\partial f}{\partial x} = -p$, $\frac{\partial f}{\partial z} = 0$, $\frac{\partial f}{\partial y} = 0$, $\frac{\partial f}{\partial p} = x$, $\frac{\partial f}{\partial q} = 2q$.

By charpit's method,

$$\frac{dp}{\partial f/\partial x + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}}$$

$$\frac{dp}{-p} = \frac{dq}{0}$$

$\int dq = 0 \Rightarrow q = a$ put in eqn ① we have,

$px + a^2 - a = 0$

$px = -a^2 + a$

$p = \frac{-a^2 + a}{x}$

put p & q in $dz = p\,dx + q\,dy$

$\int dz = \int \left(\frac{-a^2+a}{x}\right)dx + \int a\,dy$

$\therefore z = (a - a^2)\log x + ay + c$

This is required equation.

Homogeneous Linear partial differential equation of order two.

An equation of the type.

$$a_0 \frac{\partial^n z}{\partial x^n} + a_1 \frac{\partial^n z}{\partial x^{n-1} \partial y} + \cdots + a_n \frac{\partial^n z}{\partial y^n} = f(x,y)$$ is called a homogeneous linear partial differential equation of n^{th} order with constant co-efficient.

Second order homogeneous linear partial differential equation.

An equation of the type.

$$a_0 \frac{\partial^2 z}{\partial x^2} + a_1 \frac{\partial^2 z}{\partial x \partial y} + a_2 \frac{\partial^2 z}{\partial y^2} = f(x,y) \quad \text{— ①}$$

is called a homogeneous linear partial differential equation of second order with constant co-efficient where $a_0, a_1, a_2, a_3 \dots a_n$ are Constants.

Now,

putting $\frac{\partial}{\partial x} = D$, $\frac{\partial}{\partial y} = D'$ in eqn ①, we get

$$a_0 D^2 z + a_1 DD' z + a_2 D'^2 z = f(x,y)$$

$$F(D, D')z = f(x,y) \quad \text{— ②}$$

Then the solution of equation ② is

$$G.S = CF + PI$$

where CF of solution is $F(D, D') = 0$

$$\& \; PI = \frac{f(x,y)}{F(D,D')}$$

Rules for finding Complementary function.

Consider the equation:

$$a_0 \frac{\partial^2 z}{\partial x^2} + a_1 \frac{\partial^2 z}{\partial x \partial y} + a_2 \frac{\partial^2 z}{\partial y^2} = 0$$

Put $D = m$ & $D' = 1$

$a_0 m^2 + a_1 m + a_2 = 0$ This Auxiliary equation.

To solve AE. If the roots of the Auxiliary Equation are

Sl.No	Roots of AE	Complementary function
	m_1, m_2	$f_1(y+m_1x)+f_2(y+m_2x)$
	m_1, m_2, m_3	$f_1(y+m_1x)+f_2(y+m_2)+f_3(y+m_3x)$
	$m_1 = m_2$	$f_1(y+mx)+xf_2(y+mx)$
	m_1, m_2, m_3; $(m_2=m_1, m_3\neq m)$	$f_1(y+m_1x)+xf_2(y+m_1x)+f_3(y+m_3x)$
	m_1, m_2, m_3; $m_1=m_2=m_3$	$f_1(y+m_1x)+xf_2(y+m_1x)+x^2f_3(y+m_1x)$
	m_1, m_2 (complex)	$f_1(y+m_1x)+f_2(y-m_2x)$

General Rules for finding the particular Integral.

Given Partial differential equation is

$$F(D,D')z = f(x,y)$$

$$PI = \frac{1}{F(D,D')} f(x,y)$$

If $F(D,D')$ is a homogeneous function of D & D' of degree n & RHS function $\phi(ax+by)$, $e^{(ax+by)}$, $ax+by$, $\sin(ax+by)$.

Then $PI = \frac{1}{f(D,D')}\phi(ax+by)$

$$PI \text{ of } \frac{1}{f(D,D')} f(x,y) = \frac{1}{f(a,b)}\iiint\cdots\int \phi(u)\,du\,du\cdots du.$$

where $u = ax+by$.

In case of failure.

Let $f(D,D')$ is a homogeneous function of degree n,

Differentiating $f(D,D')$ partially wrt D & multiply LHS by x.

we get. $\frac{x}{\frac{\partial}{\partial D}f(DD')}\phi(ax+by)$

If again $f(a,b)$ is zero then, again differentiate the above eqn for 2nd time we get

$$x^2\frac{1}{\frac{\partial^2}{\partial D^2}f(D,D')}\phi(ax+by)$$

If $f(a,b) \neq 0$ then stop the procedure. otherwise if you get $f(a,b) = 0$, then repeat the above procedure. After 'm' times differentiating & multiplying x by 'm' times, we get.

$$x^m \frac{1}{\frac{\partial^m}{\partial D^m} f(D,D')} \phi(ax+by)$$

Let $f(a,b) \neq 0$, then PI is given by,

$$x^m \frac{1}{f(a,b)} \phi(ax+by)$$

★ when $f(x,y) = e^{ax+by}$

$$PI = \frac{1}{f(D,D')} e^{ax+by} = \frac{1}{F(a,b)} e^{ax+by}.$$

put $D = a$ & $D' = b$.

★ when $F(x,y) = \sin(ax+by)$

$$PI = \frac{1}{f(D^2, DD', D'^2)} \sin(ax+by)$$

$$= \frac{1}{f(-a^2, -ab, -b^2)} \sin(ax+by)$$

put $D^2 = -a^2$, $DD' = -ab$, $D'^2 = -b^2$

★ when $F(x,y) = \cos(ax+by)$

$$PI = \frac{1}{f(-a^2, -ab, -b^2)} \cos(ax+by)$$

put $D^2 = -a^2$, $DD' = -ab$, $D'^2 = -b^2$

★ If $F(x,y) = x^m y^n$.

$$PI = \frac{1}{f(D^2, DD', D'^2)} x^m y^n$$

$$= [F(D,D')^{-1}] x^m y^n$$

* If $f(x,y) = e^{ax+by}\phi(x,y)$

$$PI = \frac{1}{f(D,D')} e^{ax+by}\phi(x,y)$$

$$= e^{ax+by}\frac{1}{f(D+a, D'+b)}\phi(x,y)$$

Example:-

*} solve $(2D^2 - DD' - 3D'^2)z = e^{x+y}$

→ Given equation is $(2D^2 - DD' - 3D'^2)z = e^{x+y}$

Auxiliary equation is.

$$2D^2 - DD' - 3D'^2 = 0$$

put $D = m$ & $D' = 1$

$$2m^2 - m - 3 = 0$$

$$2m^2 - 3m + 2m - 3 = 0$$

$$m(2m-3) + 1(2m-3) = 0$$

$$(2m-3)(m+1) = 0$$

$$\therefore 2m - 3 = 0 \quad \& \quad m + 1 = 0$$

$$m = 3/2 \quad \& \quad m = -1$$

$$CF = f_1(y - x) + f_2(y + 3/2\, x)$$

Now to find PI

$$PI = \frac{1}{f(D.D')} e^{x+y}$$

$$= \frac{1}{2D^2 - DD' - 3D'^2} e^{x+y}$$

$$= \frac{1}{2(1) - 1 - 3} e^{x+y}$$

$$= -\frac{1}{2} e^{x+y}$$

$\therefore$ The solution is

$$G.S = CF + PI$$

$$z = f(y - x) + f_2(y + 3/2\, x) - \frac{1}{2} e^{x+y}$$

BLDE ASSOCIATION'S

S.B.ARTS AND K.C.P SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

SEMINAR REPORT: 2018-19

(Even Semester)

The UG Department of Mathematics has conducted seminar for B.Sc students.

Name of the student	Seminar Topic
1) Rekha Mashal	The Sphere
2) Dhaneshwari Bhatagunaki	Fourier Series
3) Aishwarya Kaladagi	Non-linear & linear differential eq^ns

Head of Department

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