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Mınımal Weakly Generalızed Closed Sets and Maxımal Weakly Generalızed Open Sets in Topologıcal Spaces

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ABSTRACT

In this paper, a new class of sets called maximal Weakly generalized open sets and minimal Weakly generalized closed sets in topological Spaces are introduced which are subclasses of Weakly generalized open sets and Weakly generalized closed sets respectively. We prove that the complement of maximal Weakly generalized open set is a minimal Weakly generalized closed set and some properties of the new concepts have been studied.

Mathematics subject classification (2010): 54A05.

Keywords: Minimal closed set, Maximal open set, Minimal Weakly generalized closed set, Maximal Weakly generalized open set.

1. INTRODUCTION

In the year 2001 and 2003, F. Nakaoka and N.oda[1,2,3] introduced and studied minimal open (resp.minimal closed) sets which are sub classes of open (resp.closed) sets. The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 1999N. Nagaveni[4] introduced and studied Weakly generalized closed sets and Weakly generalized open sets in topological spaces.

1.1. Definition[1]: A proper non-empty open subset U of a topological space X is said to be minimal open set if any open set which is contained in U is φ or U.

1.2. Definition[2]: A proper non-empty open subset U of a topological space X is said to be maximal open set if any open set which is contained in U is Xor U.

1.3. Definition[3]: A proper non-empty closed subset F of a topological space X is said to be minimal closed set if any closed set which is contained in F is φ or F.

1.4. Definition[3]: A proper non-empty closed subset F of a topological space X is said to be maximal closed set if any closed set which is contained in F is Xor F

1.5. Definition[4]: A subset A of (X, τ) is called Weakly generalized closed set if $Cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

1.6. Definition[4]:A subset A in (X, τ) is called Weakly generalized open set in X if A^c is Weakly generalized closed set in X.

2. MAXIMAL WEAKLY GENERALIZED OPEN SETS

2.1Definition: A proper non-empty Weakly generalized open subset U of X is said to be maximal Weakly generalized open set if any Weakly generalized open set which is contained in U is X or U.

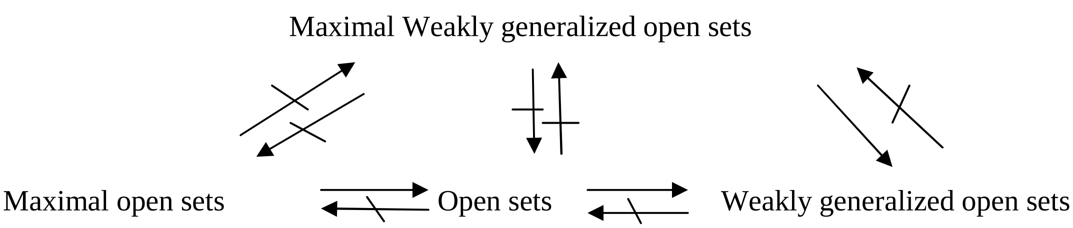
2.2 Remark: Maximal open sets and Maximal Weakly generalized open sets are independent of each other as seen from the following example.

2.3 Example: Let X= {a,b,c} be with the topology $\tau=\{X, \varphi, \{a\}\}$

Open sets are = $\{X,\varphi,\{a\}\}$
Maximal open sets are = {a}
Weakly generalized open sets are = $\{X, \varphi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\}\}$
Maximal Weakly generalized open sets are = {{a,b},{a,c}}

Here the set {a} is a Maximal open set but not a Maximal Weakly generalized open set and the sets {a,b} and {a,c} are Maximal Weakly generalized open sets but not Maximal open sets.

2.4 Remark: From the Known results and by the above example we have the following implication.



2.5 Theorem:

(i): Let U be a maximal Weakly generalized open set and W be a Weakly generalized open set then $U \cap W = \varphi$ or $U \subset W$.
(ii): Let U and V be maximal Weakly generalized open sets then $U \cap V = \varphi$ or U=V

Proof:

(i): Let U be a maximal Weakly generalized open set and W be a Weakly generalized open set. If U∩W= φ, then there is nothing to prove but if U∩W≠ φ then we have to prove that U⊂W . Suppose U∩W≠ φ then U∩W⊂U and U∩W is Weakly generalized open as the finite intersection of Weakly generalized open sets is a Weakly generalized open set. Since U is a maximal Weakly generalized open set, we have U∩W=U therefore U⊂W.

(ii): Let U and V be maximal Weakly generalized open sets suppose U∩V≠ φthen we see that U⊂V and V⊂U by (i) therefore U=V.

2.6 Theorem: Let U be a maximal Weakly generalized open set if x is an element of U then U⊂W for any open neighbourhood W of x.

Proof: Let U be a maximal Weakly generalized open set and x be an element of U. Suppose there exits an open neighbourhood W of x such that U⊄W then U∩W is a Weakly generalized open set such thatU∩W⊂U and U∩W≠ φ. Since U is a maximal Weakly generalized open set, We have U∩W=U that is U⊂W. This contradicts our assumption that U⊄W. Therefore U⊂W for any open neighbourhood W of x.

2.7 Theorem: Let U be a maximal Weakly generalized open set, if x is an element of U then U⊂W for any Weakly generalized open set W containing x.

Proof: Let U be a maximal Weakly generalized open set containing an element x.Suppose there exists an Weakly generalized open set W containing x such that U⊄ W then U∩W is an Weakly generalized open set such that U∩W⊂U and U∩W≠ φ. Since U is a maximal Weakly generalized open set,we have U∩W=U that is U⊂W .This contradicts our assumption that U⊄W.Therefore U⊂W for any Weakly generalized open set W containing x.

2.8 Theorem: Let U be a maximal Weakly generalized open set then U=∩{W: W is any Weakly generalized open set containing x} for any element x of U

Proof: By theorem 2.7 and from the fact that U is a Weakly generalized open set Containing x, We have U⊂∩ {W: W is any Weakly generalized open set containing x }⊂W. Therefore we have the following result.

2.9 Theorem: Let U be a non-empty Weakly generalized open set then the following three conditions are equivalent.

(i) U is a maximal Weakly generalized open set

(ii) U⊂w-cl(S) for any non-empty subset S of U.

(iii) w-cl(U)= w-cl(S) for any non-empty subset S of U.

Proof: (i) $\Longrightarrow$ (ii) Let U be a maximal Weakly generalized open set and S be a non-empty subset of U. Let x∈U by theorem 2.7 for any Weakly generalized open set W containing x,

$S \subset U \subset W$ which implies $S \subset W$. Now $S = S \cap U \subset S \cap W$. Since S is non-empty therefore $S \cap W \neq \varphi$. Since W is any Weakly generalized open set containig x by one of the theorem, we know that, for an $x \in X$, $x \in$ w-cl(A) iff $V \cap A \neq \varphi$. for any every Weakly generalized open set V Containing x that is $x \in U$ implies $x \in$ cl(s) which implies $U \subset$ w-cl(s) for any non-empty subset S of U.

(ii)$\Longrightarrow$(iii) Let S be a non-empty subset of U that is $S \subset U$ which implies
w-cl(S) $\subset$w-cl(U) $\longrightarrow$(a) Again from (ii) $U \subset$w-cl(s) for any non-empty subset S of U,
Which implies w-cl(U) $\subset$w-cl(w-cl(S))=w-cl(S) i.e., w-cl(U) $\subset$w-cl(S) $\longrightarrow$(b), from (a) and (b) w-cl(U)= w-cl(S) for any non empty subset S of U.

(iii)$\Longrightarrow$(i) from (3) we have w-cl(U) = w-cl(S) for any non-empty subset S of U. Suppose U is not a maximal Weakly generalized open set then there exist a non-empty Weakly generalized open set V such that $V \subset U$ and $V \neq U$. Now there exists an element $a \in U$ such that $a \in V$ which implies $a \in V^c$ that is w-cl{(a)} $\subset$ w-cl{(V^c)}= V^c , as V^c is a Weakly generalized closed set in X. It follows that w-cl({a})$\neq$w-cl(U). This is contradiction to fact that w-cl({a})= w-cl(U) for any non empty subset {a} of U therefore U is a maximal Weakly generalized open set.

2.10 Theorem: Let V be a non-empty finite Weakly generalized open set , then there exists at least one (finite) maximal Weakly generalized open set U such that $U \subset V$.

Proof: Let V be a non-empty finite Weakly generalized open set. If V is a maximal Weakly generalized open set, we may set U=V. If V is not a maximal Weakly generalized open set, then there exists a (finite) Weakly generalized open set V_1 such that $\emptyset \neq V_1 \subset V$. If V_1 is a maximal Weakly generalized open set, we may set $U=V_1$. If V_1 is not a maximal Weakly generalized open set then there exists a (finite) Weakly generalized open set V_2 such that $\emptyset \neq V_2 \subset V_1$. Continuing this process we have a sequence of Weakly generalized open sets $V_K \ldots \subset V_3 \subset V_2 \subset V_1 \subset V$. Since V is a finite set, this process repeats only finitely then finally we get a maximal Weakly generalized open set $U=V_n$ for some positive integer n.

2.11Corollary: Let X be a locally finite space and V be a non-empty Weakly generalized open set then there exists at least one(finite) maximal Weakly generalized open set such that $U \subset V$.

Proof: Let X be a locally finite space and V be a non empty Weakly generalized open set. Let $x \in V$ since X is a locally finite space we have a finite open set V_x such that $x \in V_x$ then $V \cap V_x$ is a finite Weakly generalized open set. By theorem 2.10 there exist at least one (finite) maximal Weakly generalized open set U such that $U \subset V \cap V_x$ that is $U \subset V \cap V_x \subset V$. Hence there exists at least one (finite) maximal Weakly generalized open set U such that $U \subset V$.

2.12 Corollary: Let V be a finite maximal open set then there exist at least one(finite) maximal Weakly generalized open set U such that $U \subset V$

Proof: Let V be a finite minimal open set then V is a non-empty finite Weakly generalized open set, by theorem 2.10 there exist at least one (finite)maximal Weakly generalized open set U such that $U\subset V$.

2.13 Theorem: Let U and U_λ be maximal Weakly generalized open sets for any element λ of Λ.
If $U \subset \bigcup_{\lambda\in\Lambda} U_\lambda$ then there exists an element $\lambda \in \Lambda$ such that $U= U_\lambda$.

Proof: Let $U \subset \bigcup_{\lambda\in\Lambda} U_\lambda$ then $U\cap (\bigcup_{\lambda\in\Lambda} U_\lambda) = U$ that is $\bigcup_{\lambda\in\Lambda} (U\cap U_\lambda) = U$ also by Theorem 2.5 (ii)

$U\cap U_\lambda= \varphi$ for any $\lambda \in \Lambda$ it follows that there exist an element $\lambda \in \Lambda$ such that $U= U_\lambda$.

2.14 Theorem: Let U and U_λ be maximal Weakly generalized open sets for any element $\lambda\in\Lambda$. if $U= U_\lambda$ for any element λ of Λ. If $U= U_\lambda$ for any element λ of Λ then $(\bigcup_{\lambda\in\Lambda} U_\lambda) \cap U= \emptyset$.

Proof: Suppose that $(\bigcup_{\lambda\in\Lambda} U_\lambda)\cap U \neq \emptyset$ that is $\bigcup_{\lambda\in\Lambda} (U_\lambda\cap U) = \emptyset$. Then there exists an element

$\lambda\in\Lambda$ such that $U\cap U_\lambda\neq \emptyset$ by theorem 2.5 (ii) we have $U= U_\lambda$, which contradicts the fact that

$U \neq U_\lambda$ for any $\lambda \in \Lambda$ then $(\bigcup_{\lambda\in\Lambda} U_\lambda)\cap U =\emptyset$.

2.15 Theorem: Let U_λ be a maximal Weakly generalized open set for any element $\lambda \in \Lambda$ and
$U_\lambda\neq U\mu$ for any element λ and μ of Λ with $\lambda\neq \mu$ assume that $|\Lambda| > 2$.
Let μ be any element of Λ then $(\bigcup_{\lambda\in\Lambda-\{M\}} U_\lambda)\cap U\mu = \emptyset$.

Proof: Put $U= U\mu$ in theorem 2.14, then we have the result.

2.16 Corollary: Let U_λ be a maximal Weakly generalized open set for any element $\lambda \in \Lambda$ and $U_\lambda\neq U_\mu$ for any element λ and μ of Λ with $\lambda\neq \mu$. If η a proper non-empty subset of Λ then $(\bigcup_{\lambda\in\Lambda-\{\eta\}} U_\lambda) \cap (\bigcup_{\gamma\in\eta} U_\lambda) =\emptyset$.

2.17 Theorem: Let U_λ and $U\gamma$ be maximal Weakly generalized open sets for any element $\lambda \in \Lambda$ and $\gamma \in \eta$ such that $U_\lambda\neq U_\gamma$ for any element λ of Λ then $\cup$ then $\cup U_\lambda \not\subset (\cup U_\lambda)$.

$\gamma\in\eta$ $\gamma\in\eta$ $\lambda\in\Lambda$

Proof: Suppose that an element γ^1 of η satisfies $U_\lambda = U_\gamma^{\ 1}$ for any element λ of Λ.

if $\cup$ U $_\lambda \subset (\cup U_\lambda)$, then we see $U_\gamma^{\ 1} \subset (\cup U_\lambda)$ by theorem 2.13 there exists an element
$\gamma\in\eta$ $\lambda\in\Lambda$
λ of Λ such that $U_\gamma^{\ 1} = U_\lambda$ which is a contradiction it $\lambda\in\Lambda$ follows that $\cup U \not\subset (\cup U_\lambda)$.
$\gamma\in\eta$ $\lambda\in\Lambda$.

3. MINIMAL WEAKLY GENERALIZED CLOSED SETS.

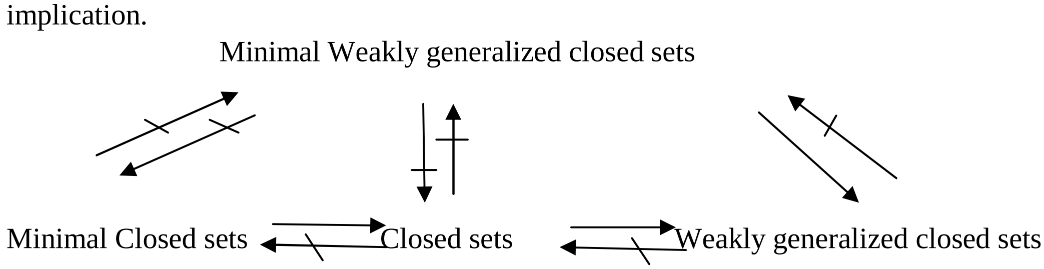
3.1 Definition: A proper non-empty Weakly generalized closed subset F of X is said to be minimal Weakly generalized closed set if any Weakly generalized closed set which is contained in F is ∅ or F

3.2 Remark: Minimal closed sets and Minimal Weakly generalized closed sets are independent each other as seen from the following implication.

3.3 Example: Let X= {a,b,c} and τ = {X, ∅,{a}} be a topological space.

Closed sets are = {X,∅,{b,c}}
Minimal closed sets are = {b,c}
Weakly generalized closed sets are = {X, ∅ ,{b},{c},{b,c},{a,c},{a,b}}
Minimal Weakly generalized closed sets are = {{b},{c}}
Here the set {b,c} is a Minimal closed set but not a Minimal Weakly generalized closed set and the sets {b} and {c} are Minimal Weakly generalized closed sets but not Minimal closed sets.

3.4 Remark: From the known results and by the above example 3.3 we have the following implication.



3.5 Theorem: A proper non-empty subset F of X is minimal Weakly generalized closed set iff X-F is a maximal Weakly generalized open set.

Proof: Let F be a minimal Weakly generalized closed set, suppose X-F is not a maximal Weakly generalized open set then there exists a Weakly generalized open set U≠ X − F such that ∅ ≠U ⊂ X-F that is F⊂X-U and X-U is a Weakly generalized closed set. This contradicts our assumption that F is a maximal Weakly generalized open set. Conversely, let X-F be a maximal Weakly generalized open set. Suppose F is not a minimal Weakly generalized closed set then there exist a Weakly generalized closed set E ≠F such that F⊂E

$\neq$X that is $\emptyset \neq$ X-E $\subset$ X-F and X-E is a Weakly generalized open set. This Contradicts our assumption that X-F is a maximal Weakly generalized open set. Therefore F is a minimal Weakly generalized closed set.

3.6 Theorem:

(i): Let F be a minimal Weakly generalized closed set and W be a Weakly generalized closed set Then F∪W=X or W⊂F.

(ii): Let F and S be minimal Weakly generalized closed sets then F∪S=X or F=S

Proof: (i):Let F be a minimal Weakly generalized closed set and W be a Weakly generalized closed set if F∪W=X then there is nothing to prove but if F∪W≠X then we have to prove that W⊂F. Suppose F∪W≠X then F ⊂F∪W and F⊂W is Weakly generalized closed as the finite union of Weakly generalized closed set is a Weakly generalized closed set we have F∪W=X Therefore F∪W=F which implies W⊂F.

(ii): Let F and S be minimal Weakly generalized closed sets. Suppose F∪S≠X then we see that F⊂S and S⊂F by (i) therefore F=S.

3.7 Theorem: Let F be a minimal Weakly generalized closed set .If x is an element of F then for any Weakly generalized closed set S containing x, F∪S=X.

Proof: Proof is similar to 2.7 theorem.

3.8 Theorem: Let F_α, F_η ,$F\gamma$ be minimal Weakly generalized closed sets such that $F_\alpha \neq F\eta$ if $F_\alpha \cap F\eta \subset F\gamma$. then either $F_\alpha = F\gamma$ or $F\eta = F\gamma$.

Proof: Given that $F_\alpha \cap F\eta \subset F\gamma$, if $F_\alpha = F\gamma$ then there is nothing to prove but if $F_\alpha \neq F\gamma$ then We have to prove $F\eta = F\gamma$.

Now we have $F_\eta \cap F\gamma = F_\eta \cap (F\gamma \cap X)$

$= F_\eta \cap (F\gamma \cap (F_\alpha \cap F\eta)$ (by theorem 3.6 (ii))

$= F_\eta \cap (F_\gamma \cap F_\alpha) \cup (F\gamma \cap F\eta)$

$= (F_\eta \cap F\gamma \cap F_\alpha) \cup (F_\eta \cap F\gamma \cap F_\eta)$

$= (F_\alpha \cap F\eta) \cup (F\gamma \cap F\eta)$ (by $F_\eta \cap F\gamma \cap F_\alpha$)

$= (F_\alpha \cup F\gamma) \cap F_\eta$

$= X \cap F_\eta$ (since F_α, and $F\gamma$ are minimal Weakly generalized closed sets

by thm3.6(ii) $F_\alpha \cup F\gamma = X$

$= F_\eta$

that is $F_\eta \cap F\gamma = F_\eta$ implies $F_\eta \subset F\gamma$, since F_η , $F\gamma$ are minimal Weakly generalized closed sets we have $F\eta = F\gamma$.

3.9 Theorem: Let F_α, F_η ,$F\gamma$ be minimal Weakly generalized closed sets which are different from each other then$(F_\alpha \cap F\eta) \not\subset (F_\alpha \cap F_\gamma)$.

Proof: Let $((F_\alpha \cap F\eta) \subset (F_\alpha \cap F\gamma)$ which implies $(F_\alpha \cap F\eta) \cup (F\gamma \cap F\eta) \subset (F_\alpha \cap F\gamma) \cup (F\gamma \cap F_\eta)$ which implies$(F_\alpha \cup F\gamma) \cap (F\gamma \cup F\eta) \cap (F_\alpha \cup F_\eta)$ since by theorem 3.6 (ii) $F_\alpha \cap F\gamma = X$ and $F_\alpha \cap F\eta = X$ which implies $X \cap F_\eta \subset F\gamma \cap X$ which implies $F_\eta \subset F\gamma$. From the definition of minimal Weakly generalized closed set it follows that $F_\eta = F\gamma$. This is contradiction to the fact that Let F_α, F_η ,$F\gamma$ are different from each other. Therefore $(F_\alpha \cap F\eta) \not\subset (F_\alpha \cap F_\gamma)$.

3.10 Theorem: Let F be a minimal Weakly generalized closed set and x be an element of F then F=∪{S: S is a Weakly generalized closed set containing x such that F∪S ≠X}

Proof: By theorem 3.7 and from the fact that F is a Weakly generalized closed set containing x we have F⊂∪{S:S is a Weakly generalized closed set containing x such that F∪S ≠X}⊂ F therefore we have the result.

3.11 Theorem: Let F be a Proper non-empty co-finite Weakly generalized closed subset then there exists (co-finite) minimal Weakly generalized closed set E such that F ⊂ E.

Proof: Let F be a non-empty co-finite Weakly generalized closed set. If F is a minimal Weakly generalized closed set, we may set E=F. If F is not a minimal Weakly generalized closed set, then there exists a (co-finite) Weakly generalized closed set F_1 such that $F \subset F_1 \neq X$. If F_1 is a minimal Weakly generalized closed set, we may set $E = F_1$ .if F_1 is not a minimal Weakly generalized closed set ,then there exists a (co-finite) Weakly generalized closed set sets F_2 such that $F \subset F_1 \subset F_2 \neq X$ continuing this process we have a sequence of Weakly generalized closed sets $F \subset F_1 \subset F_2 \subset \ldots \subset F_K \subset \ldots$ since F is a co-finite set, this process repeats only finitely then finally we get a maximal Weakly generalized open set $E=E_n$ for some positive integer n.

3.12 Theorem: Let F be a minimal Weakly generalized closed set. If x is an element of X-F then X-F⊂E for any Weakly generalized closed set containing set E containing x.

Proof: Let F be a minimal Weakly generalized closed set and x∈ X–F. $E \not\subset F$ for any Weakly generalized closed set E containing x then E∪F=X by theorem 3.6(ii) .Therefore X–F ⊂E.

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