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# On Pre Generalızed Pre Regular Weakly Closed Sets in Topologıcal Spaces

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## ABSTRACT

In this paper a new class of sets called pre generalized pre regular weakly closed (briefly pgprw-closed) sets in topological spaces is introduced and studied. A subset A of a topological space (X, τ) is called pgprw-closed if U contains pre closure of A whenever U contains A and U is rgα open in (X, τ). This new class of sets lies between the class of all preclosed sets and the class of all gpr-closed sets and some properties are investigated.

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## 1. INTRODUCTION

In a topological space the concept of closed sets plays an important role. The generalization of closed sets has been studied in different ways in previous years by many topologists leading to several new ideas. In 1970 N. Levine[22] gave the concept and properties of generalized closed (briefly g-closed) sets . In 1982, A.S.Mashhour, M.E.Abd El-Monsef and S.N. El-Deeb[4] introduced and studied the concept of pre-open set. Later H. Maki, J. Umehara and T. Noiri[23], J. Dontechev[16], Y. Gyanambal[18], P. Bhattacharya and B. K. Lahiri[14], introduced and studied the concepts of gp-closed, gsp-closed, gpr-closed, sg-closed sets respectively. P. Sundarm and M. Sheik John[29] defined and studied w-closed sets in topological spaces. S. S. Benchalli and R. S. Wali[13] introduced rw-closed sets. In this paper we define and study the properties of a new set called Pre Generalised Pre Regular Weakly Closed set.

## 2. PRELIMINARIES

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), $A^c$, and P-Cl(A), denote the Closure of A, Interior of A and Compliment of A and pre closure of A in X respectively.

**Definition 2.1:** A subset A of a topological space (X, τ) is called

(i) Semi-pre open set[2] (= β-open[1] if $A\subseteq cl(int(cl(A)))$and a semi-pre closed set(=β-closed )if $int(cl(int(A)))\subseteq A$.
(ii) Regular semi open set [6] if there is a regular open set U such that $U \subseteq A\subseteq cl(U)$.
(iii) α-open set[3] if $A \subseteq int(cl(int(A)))$ and α -closed set if $cl(int(cl(A)))\subseteq A$.
(iv) Semi-open set[10] if $A \subseteq cl(int(A))$ and semi-closed set if $int(cl(A)) \subseteq A$.
(v) Pre-open set[4] if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.
(vi) Regular open set[9] if $A = int(clA))$ and a regular closed set if $A = cl(int(A))$.
(vii)Regular α-open set[30] (briefly,rα-open) if there is a regular open set U s.t $U\subseteq A\subseteq \alpha cl(U)$.
(viii) δ-closed[7] if $A = cl_\delta(A)$ , where $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A\neq \theta, U \in \tau \text{ and } x \in U\}$.

**Definition 2.2:** Let **(**X, $\tau$) be topological space and $A \subseteq X$. The intersection of all semi closed(resp. Pre closed, α-closed and semi-pre closed) subsets of spaces X containing A is called the Semi closure (resp.Pre-closure, α-closure and semi-pre-closure) of A and denoted by sCl(A)(resp. pCl(A),αCl(A),spCl(A)).
It is well know that $sCl(A) =A \cup int(Cl(A))$ , $\alpha Cl(A) =A \cup Cl(int(Cl(A)))$
$pCl(A) =A \cup Cl(int(A))$ , $spCl(A)=A \cup int(Cl(int(A)))$

**Definition 2.3:**[8] Let X be a topological space. The finite union of regular open sets in X is said to be $\pi$ -open. The complement of a $\pi$ -open set is said to be $\pi$-closed.

**Definition 2.4:** A subset A of a topological space (X, $\tau$ ) is called

(i) Pre-generalized-pre-regular closed(briefly pgpr-closed)set[12] if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is rg-open in X.
(ii) Regular ω- closed (briefly rω -closed) set[13] if $cl(A) \subseteq U$ whenever $A\subseteq U$ and U is regular semi- open in X.
(iii) Semi-generalized closed set(briefly sg-closed)[14] if $scl(A)\subseteq U$ whenever $A\subseteq U$ and U is semi-open in X.
(iv) Generalized regular closed (briefl gr –closed) set[15] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(v) Generalized semi-pre closed set(briefly gsp-closed)[16] if $spcl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in X.

(vi) π-generalized closed set (briefly, πg-closed)[17] if cl (A)⊆ U whenever A⊆ U and U is π-open in X.

(vii) Generalized pre regular closed set(briefly gpr-closed)[18] if pcl(A)⊆U whenever A⊆U and U is regular open in X.

(viii) R*- closed (briefly R*-closed) set[19] if rcl(A) ⊆U whenever A ⊆ U and U is regular semi- open in X.

(ix) Weak generalized regular-α closed (briefly wgrα-closed) set[20] if cl(int(A) ⊆ U whenever A ⊆ U and U is regular α-open in X.

(x) Generalized pre regular weakly closed (briefly gprw-closed) set[21] if pcl(A) ⊆ U whenever A ⊆ U and U is regular semi- open in X.

(xi) Generalized closed set(briefly g-closed)[22] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.

(xii) Generalized pre closed (briefly gp-closed) set[23] if pcl(A)⊆U whenever A⊆U and U is open in X.

(xiii) Regular-generalized-weak(briefly-rgw-closed) set[24] if cl(int(A))⊆U whenever A⊆U and U is regular semi open in X

(xiv) Regular generalized closed set(briefly rg-closed)[25] if cl(A)⊆U whenever A⊆U and U is regular open in X.

(xv) Strongly generalized closed set[26] (briefly, g*-closed) if Cl(A) ⊆ U whenever A⊆U and U is g-open in X.

(xvi) Generalized semi pre regular closed (briefly gspr-closed) set[27] if spcl(A) ⊆U whenever A⊆U and U is regular open in X.

(xvii) Regular pre semi –closed (briefly rps-closed) set[28] if spcl(A) ⊆U whenever A⊆ U and U is rg- open in X.

(xviii)W-closed set[29] if cl(A) ⊆ U whenever A⊆ U and U is semi-open in X.

(xix) Regular generalized α-closed set(briefly, rgα-closed)[30] if αcl(A)⊆U whenever A⊆U and U is regular α-open in X.

The compliment of the above mentioned closed sets are their open sets respectively.

## 3. PRE GENERALIZED PRE REGULAR WEAKLY CLOSED SETS IN TOPOLOGICAL SPACES

In this section we introduce pre generalized pre regular weakly closed sets in topological spaces and study some of their properties.

**Definition 3.1:** A subset A of topological space (X, τ) is called a pre generalized pre regular weakly closed set(briefly pgprw-closed set) if pCl(A)⊆U whenever A⊆U and U is rgα open in (X, τ).

First we prove that the class of pre generalized pre regular weakly closed sets properly lies between the class of pre-closed sets and the class of gpr-closed sets.

**Theorem 3.2:** Every pre-closed set in X is pre generalized pre regular weakly closed set but not conversely.

**Proof:** Let A be pre-closed set in topological space X. Let U be any rgα -open set in X s.t A $\subseteq$ U .Since A is pre-closed, we have pcl(A) = A $\subseteq$ U, therefore pcl(A) $\subseteq$ U. Hence A is pre generalized pre regular weakly closed set in X.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.3**: Let X={a, b, c,d} and τ={X,∅,{a},{b},{a,b},{a,b,c}} then the set A={a, d} is pre generalized pre regular weakly closed set but not Pre-closed in X.

**Theorem 3.4:** Every pgprw- closed set is gpr- closed.

**Proof:** Let A be pgprw closed . Let A $\subseteq$ U where U is regular open set, then as every regular open set is rgα open set in X, U is rgα-open in X, Since A is pgprw-closed set then pcl(A) $\subseteq$ U. Therefore A is gpr closed set.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.5**: Let X={a, b, c,d} and τ={X, ∅,{a},{b},{a,b},{a,b,c}} then the set A={a,b} is gpr closed but not pgprw closed set.

**Corollary 3.6:** Every α- closed set is pgprw- closed.

**Proof**: Set A is α- closed ⇒ A is pre–closed ⇒ A is pgprw-closed.

The converse of the above statement need not be true.

**Example 3.7:** Let X ={a,b,c,d,},τ = {X, ∅,{a},{b},{a,b,{a,b,c}}.Here {a,d} is pgprw-closed, but not α -closed.

**Corollary 3.8:** Every closed set is pgprw-closed.

**Proof:** Set A is closed ⇒ A is α- closed ⇒ A is pre –closed ⇒ A is pgprw-closed**.**

The converse of the above statement need not be true.

**Example 3.9:** Let X ={a,b,c} , τ = {X, ∅, {a}, {a,b}}. Here {b} is pgprw-closed but not closed.

**Corollary 3.10:** Every regular closed set is pgprw-closed.

**Proof:** Every regular closed set is closed, from M. Stone[9] and then follows from corollary 3.8.

**Corollary 3.11**

(i) Every π- closed set is pre generalized pre regular weakly closed set in X.
(ii) Every weakly closed set is pre generalized pre regular weakly closed set in X.
(iii) Every g*-closed set is pre generalized pre regular weakly closed set in X.
(iv) Every g-closed set is pre generalized pre regular weakly closed set in X.

**Proof:**

(i) Every π- closed set is closed, from Dontchev & Noiri[8] and then from corollary 3.8.
(ii) Every weakly closed set is closed, from M. Sheik John[29] and then follows from corollary 3.8.
(iii) Every g*-closed set is closed, from A.Pushpalatha,[26] and then follows from corollary 3.8.
(iv) Every g-closed set is closed, from N. Levine,[22] and then follows from corollary 3.8.

**Corollary 3.12:** Every δ-closed set is pre generalized pre regular weakly closed set in X.

**Proof:** Follow from Velicko, Every δ - closed set is closed[7] and then from corollary 3.8.

**Corollary 3.13:** Every δ-g-closed set is pre generalized pre regular weakly closed set in X.

**Proof:** Follow from Dontchev & M.Ganster, Every δ-g closed set is closed[11] and then from corollary 3.8.

**Theorem 3.14 :** Every pgpr-closed set is pgprw-closed .

**Proof**: Let A be a pgpr closed set. Let A⊆U and U is a rgα-open in X.Then as every rgα - open set is rg-open in X, U is rg-open in X since A is pgpr-closed set, pcl(A) ⊆U.
Therefore A is pgprw closed set in X.
The converse of the above statement need not be true.

**Example 3.15:**Let X ={a,b,c,d,} ,τ ={X, ∅,{a},{b},{a,b},{a,b,c}}. Here {a, d} is pgprw-closed, but not pgpr-closed.

**Corollary 3.16:** Every pgprw- closed set is gspr- closed.

**Proof:** Let A be pgprw- closed .Then A is gsp- closed and every gsp- closed is gspr- closed.
Therefore A is gspr-closed.
The converse of the above statement need not be true.

**Example 3.17**: Let X={a,b,c,d},τ ={X, ∅,{a},{b},{a,b},{a,b,c}}.Here {a,b} is gspr- closed, but not pgprw- closed.

**Theorem 3.18:** Every pgprw-closed set is gp-closed.

**Proof:** Let A be a pgprw-closed set . Let A ⊆ U and U is open in X. Then as every open set is rgα open in X, U is rgα open in X, since A is pgprw-closed set hence pcl(A)⊆U. Therefore A is gp closed set in X.

The converse of the above statement need not be true.

**Example 3.19:** Let X = {a,b,c,} , τ ={X, ∅, {a}}. Here {a,b} is gp-closed, but not pgprw-closed.

**Theorem 3.20:** Every pgprw- closed set is rg- closed.

**Proof:** Let A be pgprw closed . Let A ⊆ U where U is regular open set , then as a regular open set is rgα open set and A is pgprw closed set, we have pcl(A)⊆ U then P-Cl(A)⊆Cl(A)⊆U implies Cl(A)⊆U , U is regular open in X. Therefore A is rg is closed set.

The converse of the above statement need not be true.

**Example 3.21**:Let X={a,b,c,d},τ={X, ∅,{a},{b},{a,b,{a,b,c}}. Here{a,b} is rg-closed, but not pgprw- closed.

**Theorem 3.22:** Every pre generalized pre regular weakly closed set is π-g closed set in X.

**Proof:** Let A be pre generalized pre regular weakly closed set in X. Let U be any π-open in X s.t A⊆U. Since every π-open set is rgα open set in X and Since A is pre generalized pre regular weakly closed set in X, it follows that pcl(A)⊆U therefore pcl(A)⊆ U,U is π-open in X. Hence A is π-g closed set in X.

**Example 3.23:** Let X={a, b, c,d} and τ ={X, ∅ ,{a},{b},{a,b},{a,b,c}} then the set A={a,c} is π-g closed set in X , but not pre generalized pre regular weakly closed set in X.

**Theorem 3.24:** The Union of two pre generalized pre regular weakly closed subsets of X is pre generalized pre regular weakly closed set.

**Proof:** Let A and B are the pre generalized pre regular weakly closed sets in X . Let U be rgα open set in X s.t A ∪ B ⊆ U, then A⊆U & B ⊆U . Since A and B are the pre generalized pre regular weakly closed sets , pCl(A) ⊆U & pCl(B)⊆ U and we know that pCl(A) ∪ pCl(B) = pCl(A ∪ B) ⊆ U. Therefore A ∪ B is pre generalized pre regular weakly closed set in X.

**Remark 3.25:** The intersection of two pre generalized pre regular weakly closed sets in X is generally not an pre generalized pre regular weakly closed set in X.

**Example 3.26:** Let X={a, b,c,d}and τ={X, ∅,{a},{c ,d},{a,c,d}}then the set A={a,c,d}& B={ a,b,c} are pre generalized pre regular weakly closed set in X,but A∩B={a,c} is not pre generalized pre regular weakly closed set in X.

**Remark :** The following example shows that pre generalized pre regular weakly closed sets are independent of Rgw-closed, Sg-closed, Gsp-closed , Gr-closed ,R* closed, Rα closed, Rw closed, Gprw closed, Wgrα closed, Rps closed , β –closed sets, Semi closed sets.

**Example :** Let X={a, b, c, d} and $\tau$={X, ∅ ,{a},{b},{a, b},{a, b, c}} Then

- Pgprw closed sets are:{ X, ∅,{c},{d},{a,d},{b,d},{c,d},{a,c,d},{b,c,d},{a,b,d}}
- Rgw closed sets are : {X, ∅ ,{c},{d},{a,b},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}
- Sg closed sets are : {X, ∅,{a},{b},{c},{d},{c,d},{b,c},{a,d},{b,d}, {a,c}, {a,c,d},{b,c,d}}
- Gsp closed sets are : { X, ∅ ,{c},{d},{b,c},{c,d},{a,d},{a,c}, {a,b,d},{a,c,d},{b,c,d}}
- Gr closed sets are : {X, ∅ ,{d},{a,c},{a, d},{ b, d},{c,d},{a,c,d},{b,c,d}, {a,b,d}}
- R* closed sets are : {X, ∅ ,{a,b},{c,d},{a, b,c},{a, b, d},{b,c,d},{a,c,d}}
- Rα closed sets are : {X, ∅ ,{a},{b},{a, c},{a,d},{b,c},{b,d},{a,c,d},{b,c,d}}
- Rw closed sets are : {X, ∅ ,{d},{a,b},{c,d},{a, b, c}{a,c,d},{b,c,d}{a,b,d}}
- Gprw closed sets are:{X, ∅,{d},{a,b},{c,d},{a, b, c}{a,b,d},{a,c,d},{b,c,d}}
- Wgrα closed sets are :{X, ∅,{c},{d},{c,d},{a, b, c}{a,b,d},{a,c,d},{b,c,d}}
- Rps closed sets are :{X, ∅ ,{a},{b},{c},{d}{a,c},{b,d},{c,d},{a,c,d},{b,c,d}}
- β-closed-sets-are: {X, ∅ ,{a},{b},{c},{d},{a,c},{a,d},{b,d},{c,d},{b,c},{a,c,d}, {b,c,d}}
- Semiclosed sets are: {X, ∅,{a},{b},{c},{d},{a,c},{a,d},{b,d},{c,d},{b,c},{a,c,d}, {b,c,d}}

**Remark 3.27:** From the above discussion and know results we have the following implications.

**Theorem3.28:** Let A⊆Y⊆X and suppose that A is pre generalized pre regular weakly closed set in X,then A is pre generalized pre regular weakly-closed relative to Y.

**Proof** : Let A ⊆Y ∩ G, where G is rgα -open. Since A is an pre generalized pre regular weakly closed set in X, then A ⊆G & hence pCl(A) ⊆ G. This implies that Y∩ pCl(A)⊆Y∩G where Y∩pCl(A) is p-closure of A in Y. Thus A is pre generalized pre regular weakly closed relative to Y.

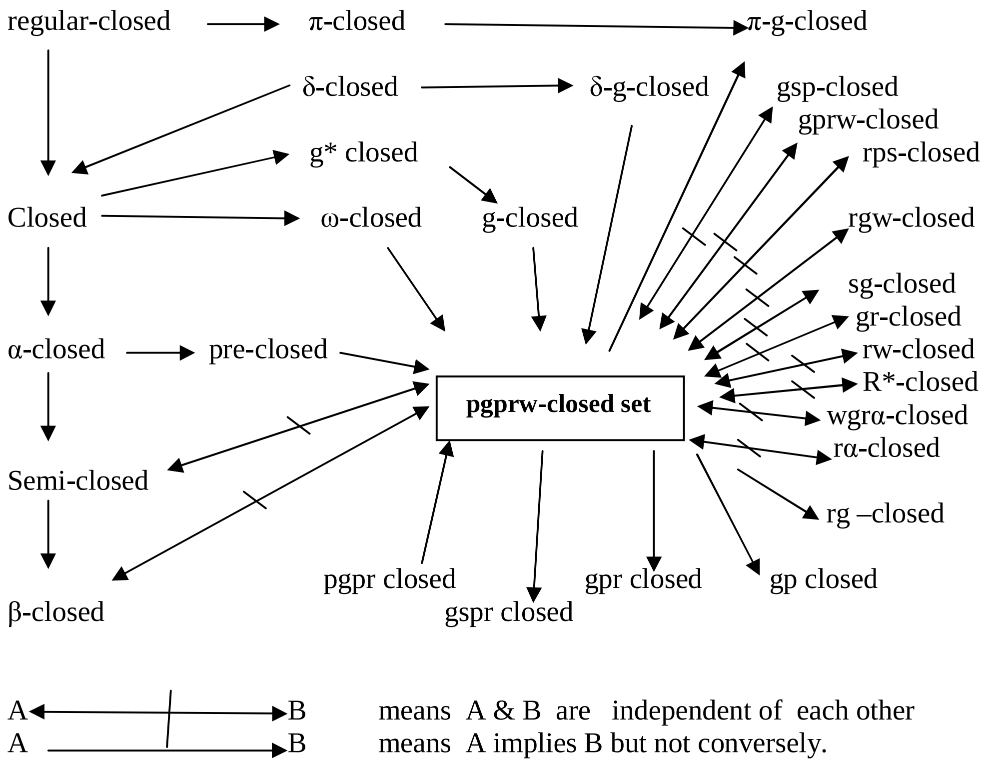


**Fig.1**

**Theorem3.29:** Let (X,τ)be topological space then for each x ∈ X, the set $\{x\}^c$ is pre generalized pre regular weakly closed or rgα open.

**Proof** : x ∈X. Therefore pcl( X-{x})⊆X this implies X-{x} is pgprw-closed.

**Theorem 3.30:** If A is pre generalized pre regular weakly closed set in X and A⊆B⊆ pCl(A) then B is also pre generalized pre regular weakly closed set in X.

**Proof:** If it is given that A is pre generalized pre regular weakly closed set in X then we have to prove that B is also pre generalized pre regular weakly closed set in X. Let U be an rgα-open set of X such that B⊆U, since A⊆B and A is pre generalized pre regular weakly closed set, pCl(A) ⊆U and A⊆U. Now B ⊆ pCl(A) => pCl (B) ⊆ pCl(pCl (A)) = pCl(A) ⊆ U. Therefore pCl(B) ⊆ U. Hence B is pre generalized pre regular weakly closed set in X.

However the converse of the above theorem need not be true as seen from the following example.

**Example 3.31:** Let X={a, b, c} and τ={X, ∅,{a},{b},{a,b}} then the set A={c} , B= {a,c} such that A & B are pre generalized pre regular weakly closed sets in X, but A⊆ B ⊈ pCl(A) Since pCl(A)={c}.

**Theorem 3.32:** If a subset A of a topological space X is both regular open and pre generalized pre regular weakly closed then it is p-closed.

**Proof:** Suppose a subset A of a topological space X is regular open and pre generalized pre regular weakly closed, as every regular open is rgα -open. Now A⊆A then definition of pre generalized pre regular weakly closed, pCl(A)⊆A and also A⊆pCl(A) then pCl(A)=A. Hence A is p-closed.

**Corollary 3.33:** If A be regular open and pre generalized pre regular weakly closed, F is p-closed in X .Then A∩F is an pre generalized pre regular weakly closed set in X.

**Proof:** Let A is regular open and pre generalized pre regular weakly closed by theorem 3.32 A is p-closed , F is p-closed in X So A∩F is p-closed and it follows from theorem 3.2 A∩F is an pre generalized pre regular weakly closed set in X.

**Theorem 3.34:** If A is open and gp-closed set, then A is pgprw-closed.

**Proof:** A is an open and gp-closed set.Let U be a rgα-open set such that A⊆U. A⊆A, an open set and A is gp-closed. Therefore pcl (A) ⊆A⊆U. Thus every rgα -open set U containing A contains Pcl(A). Therefore A is pgprw –closed.

**Remark 3.35:** If A is open and pgprw-closed, then A is not gp-closed.

**Example 3.36:** Let X={a,b,c,d}, τ ={X, ∅,{a},{c,d},{a,c,d}}.Here {a, c, d} is both open and pgprw-closed but not gp-closed.

**Theorem3.37:** If A is rgα -open and pgprw-closed, then A is pgpr-closed.

**Proof:** A is a rgα -open and pgprw –closed set.Let U be a rg-open set such that A⊆U. A⊆A , a rgα -open set and A is pgprw-closed. Therefore pcl(A) ⊆A⊆U. Therefore A is pgpr- closed.

**Remark 3.38:** If A is rgα - open and pgpr-closed, then A is not pgprw-closed.

**Example 3.39:** Let X= {a,b,c} , τ ={X, ∅, {a}}. Here {a, c} is rgα - open and pgpr- closed, but not pgprw-closed.

**Theorem 3.40:** If A is both g- open and g*-closed, then A is pgprw-closed.

**Proof:** A is a g-open and $g^*$-closed set. Let U be a rgα -open set containing A.A ⊆A, which is g-open and g* -open. Therefore cl(A)⊆A then pcl(A)⊆cl(A)⊆A⊆U. Thus every rgα-open set U containing A contains pcl(A). Therefore A is pgprw-closed.

**Remark 3.41** If A is g-open and pgprw-closed, then A is not g*-closed.

**Example 3.42:** Let X={a,b,c,d}, τ ={X, ∅, {a},{b},{a,b},{a,b,c}}. Here {c} is both g- open and pgprw-closed , but not g* -closed.

**Theorem 3.43:** If a subset A of a topological space X is both semi open and ω-closed then it is pre generalized pre regular weakly-closed.

**Proof:** Let A be an semi open and ω-closed set in X. Let A⊆U and U be rgα -open in X. Now A⊆A by hypothesis cl(A) ⊆A then we know that pcl(A)⊆cl(A)⊆A⊆U. Thus A is pre generalized pre regular weakly closed set in X.

**Remark 3.44:** If it is both semi open and pre generalized pre regular weakly closed, then A need not be ω-closed in general, as seen from the following example.

**Example 3.45:** Consider X={a, b, c, d} and τ ={X, ∅,{a},{b},{a,b},{a,b,c}} then the set A={a,d} is both semi open and pre generalized pre regular weakly closed but not ω-closed in X.

**Theorem 3.46:** If a subset A of a topological space X is both regular semi open and gprw-closed then it is pre generalized pre regular weakly closed.

**Proof:** Let A be an regular semi open and gprw-closed set in X. Let A⊆U and U be rgα -open in X. Now A⊆A by hypothesis pcl(A)⊆A then we know that pcl(A)⊆A⊆U, hence pcl(A)⊆U, therefore A is pre generalized pre regular weakly closed set in X.

**Remark 3.47:** If it is both regular semi open and pre generalized pre regular weakly closed, then A need not be gprw-closed in general, as seen from the following example.

**Example 3.48:** Consider X={a, b, c, d} and τ ={X, ∅ ,{a},{b},{a,b},{a,b,c}} then the set A={a,d} is both regular semi open and pre generalized pre regular weakly closed but not gprw-closed in X.

**Theorem 3.49:** If A is both open and g-closed, then A is pgprw -closed.

**Proof:** A is open and g-closed. Let U be a rgα -open set containing A. A ⊆A, an open set and A is g-closed. Therefore cl(A) ⊆A then Pcl(A)⊆cl(A)⊆A⊆U. Thus every rgα -open set U containing A contains Pcl(A) . Therefore A is pgprw-closed.

**Theorem 3.50**: If A is regular- open and gpr-closed, then it is pgprw-closed.

**Proof:** A is regular-open and gpr-closed. Let U be a rgα-open set such that A⊆U. Therefore pcl(A) ⊆A and pcl(A) ⊆U. Thus every rgα -open set U containing A contains pcl(A). Hence A is pgprw -closed.

**Theorem 3.51:** If A is both regular-open and rg-closed, then A is pgprw-closed.

**Proof:** A is a regular- open and rg-closed set. Let A ⊆ U where U is rgα - open. A⊆A, a regular- open set and A is rg-closed . So cl(A) ⊆A then pcl(A) ⊆ cl(A). Therefore pcl(A) ⊆ A, A ⊆U. Therefore pcl(A) ⊆ U. Thus every rgα -open set U containing A contains pcl(A) . Therefore A is pgprw-closed.

**Theorem 3.52:** If a subset A of topological space X is a pre generalized pre regular weakly closed set in X then pCl(A) - A does not contain any non empty rgα-closed set in X.

**Proof:** Let A is a pre generalized pre regular weakly closed set in X and suppose F be an non empty rgα-closed subset of pCl(A) – A. F ⊆ pCl(A) – A => F ⊆ pCl(A) ∩ (X– A) => F ⊆ pCl(A) --(1) & F ⊆ X– A => A ⊆ X– F and X – F is rgα -open set and A is a pre generalized pre regular weakly closed set, pCl(A) ⊆ X– F . => F ⊆ X – pCl(A) --(2) from equations (1) and (2) we get F ⊆ pCl(A)∩(X – pCl(A)) =∅ => F =∅ . Thus pCl(A) – A does not contain any non empty rgα- closed set in X.

However the converse of the above theorem need not be true as seen from the following example.

**Example 3.53:** Let X={a, b, c} and $\tau$ ={X, ∅ ,{a},{b ,c }} then the set A={a,b}, pCl(A) – A= {a,b,c} – {a,b} ={c} does not contain non empty rgα-closed set in X, but A is not pre generalized pre regular weakly closed set in X.

**Corollary 3.54:** If a subset A of topological space X is an pre generalized pre regular weakly closed set in X then pCl(A)-A does not contain any non empty regular open set in X but converse is not true.

**Proof**: Proof of this Corollary follows from Theorem: 3.52 & the fact that every regular open set is rgα-open set.

**Theorem 3.55:**Let A be pre generalized pre regular weakly closed in X, then A is p-closed if & only if pCl (A)-A is rgα -closed.

**Proof:** Necessity : suppose A be p- closed. Then pCl (A)=A and so pCl (A)-A = ∅ which is rgα -closed. Sufficiency: suppose A is pre generalized pre regular weakly closed in X & pCl (A)-A is rgα -closed from Theorem: 3.52 then pCl (A)-A= ∅ => pCl (A)=A .Therefore A is p-closed.

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