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ON PRE GENERALIZED PRE REGULAR WEAKLY IRRESOLUTE AND STRONGLY PGPRW-CONTINUOUS MAPS IN TOPOLOGİCAL SPACES.

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ABSTRACT

The aim of this paper is to introduce a new type of maps called pgprw-irresolute maps,stronglypgprw-continuous maps and study some of these properties.

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Keywords: Pgprw-closed-sets, pgprw-open-sets, pgprw-continuous-maps, pgprw-irresolutemaps, strongly pgprw-continuous maps.

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INTRODUCTION

The concept of irresolute map was introduced byHildebrand[5] and strongly continuous functions was introduced by Levine[6]. Later Wali and Benchalli[2] introduced and studiedrw-irresolute maps and strongly rw-continuous maps.In this section we introduce the concept of pgprw –irresolute maps and strongly pgprw-continuous maps in topological space and investigate some of their properties.

2.PRELIMINARIES

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c,P-Cl(A) and P-int(A) denote the Closure of A, Interior of A , Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

Definition 2.1: A subset A of a topological space (X, τ) is called

(i)Semi-open set [1] if $A \subseteq cl(int(A))$ and semi-closed set if $int(cl(A)) \subseteq A$.

(ii)Regular ω- closed **(**briefly rω -closed)set [2] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi- open in X.

(iii) Pre generalized pre regular ωeakly closed set[3](briefly pgprω-closed set) if $pCl(A) \subseteq U$ whenever $A \subseteq U$ and U is rgα open in (X,τ).

(iv)pre generalized pre-regular ωeakly open(briefly pgprω-open)[4]set in X if A^c is pgprω-closed in X.

Definition 2.2A map f: (X,τ)→(Y,σ) is said to

(i)Semi continuos map[1] if $f^{-1}(V)$ is a semi-closed of (X,τ)for every closed set of (Y,σ).

(ii)irresolute map [5] if $f^{-1}(V)$ is semi- closed in X for every semi-closed subset V of Y.

(iii)Strongly–continuousmap[6]if $f^{-1}(V)$ is Clopen (both open and closed) in X for every subset V of Y.

(iv)A function f from a topological space X into a topological space Y is called pgprw-continuous map(pgprw–Continuous)[7] if $f^{-1}(V)$ is pgprw–Closed set in X for every closed set V in Y.

3.Pgprw–irresolute and Strongly Pgprw–Continuous functions

Definition 3.1: A function f from a topological space X into a topological space Y is called pgprw-irresolute (pgprw–irresolute) map if $f^{-1}(V)$ is pgprw–Closed set in X for every pgprw–closed set V in Y.

Definition 3.2: A function f from a topological space X into a topological space Y is called strongly pgprw continuous (strongly pgprw–continuous) map if $f^{-1}(V)$ is closed set in X for every pgprw–closed set V in Y.

Theorem 3.3: If A map f: (X,τ)→(Y,σ) is pgprw–irresolutemap,if and only if the inverse image $f^{-1}(V)$ is pgprw–open set in X for every pgprw–open set V in Y.

Proof: Assume that f: X→Y is pgprw–irresolute map. Let G be pgprw–open in Y. The $G^c$is pgprw–closed in Y. Since f is pgprw– irresolute, $f^{-1}(G^c)$ is pgprw–closed in X.

But $f^{-1}(G^c) = X–f^{-1}(G)$. Thus $f^{-1}(G)$ is pgprw–open in X. Converserly , Assume that the inverse image of each open set in Y is pgprw–open in X. Let F be any pgprw–closed set in Y. By assumption $f^{-1}(F^c)$ is pgprw–open in X. But $f^{-1}(F^c) = X–f^{-1}(F)$. Thus X– $f^{-1}(F)$ is pgprw–open in X and so $f^{-1}(F)$ is pgprw–closed in X. Therefore f is pgprw– irresolute map.

Theorem 3.4: If A map f: (X,τ)→(Y,σ) is pgprw–irresolute map, then it is pgprw–continuous map but not conversely.

Proof: Let f: X→Y be pgprw–irresolute map. Let F be any closed set in Y. Then F is pgprw–closed in Y. Since f is pgprw–irresolute map, the inverse image $f^{-1}(F)$ is pgprw–closed set in X. Therefore f is pgprw–continuous map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.5: X={a,b,c,d}, Y={a,b,c} τ = {X, ϕ,{a},{b},{a,b},{a,b,c}} σ ={Y, ϕ,{a}},

Let map f: X→Y defined by , f(a)=b , f(b)=a , f(c)=a , f(d)=c then f is pgprw–continuous map but f is not pgprw–irresolute map, as pgprw-closed set F= {b} in Y, then $f^{-1}(F)$={a} in X , which is not pgprw–closed set in X.

Theorem 3.6: Let f: (X, τ)→(Y,σ) and g: (Y, σ) →(Z,η) be any two maps. Then

(i) g o f : (X, τ)→(Z,η) is pgprw–irresolute map if g is pgprw–irresolute map and f is pgprw–irresolute map.

(ii) g o f : (X, τ)→(Z,η) is pgprw–continuous map if g is pgprw–continuous map and f is pgprw–irresolute map.

Proof:

(i) Let U be a pgprw–open set in (Z, η) . Since g is pgprw–irresolute map, $g^{-1}(U)$ is pgprw–open set in (Y, σ). Since f is pgprw–irresolute map, $f^{-1}(g^{-1}(U))$ is apgprw–open set in (X, τ).

Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is anpgprw–open set in (X, τ) and hence gof is pgprw–irresolute map.

(iii) Let U be a open set in (Z, η) . Since g is continuous map, $g^{-1}(U)$ is open set in (Y, σ). As every open set is pgprw–open, $g^{-1}(U)$ is pgprw–open set in (Y, σ). Since f is pgprw– irresolute map,

$f^{-1}(g^{-1}(U))$ is an pgprw–open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is anpgprw–open set in (X, τ) and hence gof is pgprw–continuous map.

Theorem 3.7: Let f: (X,τ) →(Y,σ) be a map. Both (X,τ) and (Y,σ) are Topological-space .Where "every pgprw closed subset is closed" Then the following are equivalent:

(i) f is pgprw-irresolute map

(ii) f is pgprw-continuous map.

Proof :

(i) implies (ii) follows from theorem 3.4

(ii) implies (i) Let F be a pgprw closed set in (Y,σ) then F is a closed set in (Y,σ)

by hypothesis. Since f is a pgprw-continuousmap,$f^{-1}(F)$ is a pgprw closed set in (X,τ), Therefore

f is pgprw-irresolute map.

Remark 3.8: The following examples show that the notation of irresolute maps and pgprw-irresolute maps are independent.

Example: X={a,b,c}, Y={a,b,c} τ = {X, ϕ,{a}} &σ ={Y, ϕ,{a},{b},{a,b}},

thenthe identity map f:(X, τ) → (Y,σ) is pgprw–irresolute map but it is not–irresolute map as inverse image of the semi-open set {b} in (Y,σ) is {b} in X , which is not semi-open set in (X,τ)

Example: X={a,b,c}, Y={a,b,c} τ = {X, ϕ,{a},{b},{a,b}} σ ={Y, ϕ,{a}},

thenthe identity mapf:(X, τ) → (Y, σ) is–irresolutemap but it is notpgprw-irresolute map, as inverse image of the pgprw-closed set {b} in (Y,σ) is {b} in X , which is not pgprw closedset in (X,τ).

Theorem 3.9: Let f: (X, τ)→(Y,σ) is strongly pgprw–continuous map if and only if $f^{-1}(G)$ is open set in X for every pgprw–open set G in Y.

Proof : Assume that f: X→Y is strongly pgprw–continuous map. Let G be pgprw–open in Y. The G^c is pgprw–closed in Y. Since f is strongly pgprw–continuous map, $f^{-1}(G^c)$ is closed in X.

But $f^{-1}(G^c) = X–f^{-1}(G)$, Thus $f^{-1}(G)$ is open in X.

Converserly, Assume that the inverse image of each open set in Y is pgprw–open in X. Let F be any pgprw–closed set in Y. By assumption F^c is pgprw–open in X. But $f^{-1}(F^c) = X– f^{-1}(F)$.

Thus X– $f^{-1}(F)$ is open in X and so $f^{-1}(F)$ is closed in X. Therefore f is strongly pgprw–continuous map.

Theorem 3.10: Let f: (X, τ)→(Y,σ) is strongly pgprw–continuous map then it is continuous map.

Proof: Assume that f: (X, τ)→(Y,σ) is strongly pgprw–continuous map, Let F be closed set in Y. As every closed is pgprw–closed, F is pgprw–closed in Y. Since f is strongly pgprw–continuous mapthen $f^{-1}(F)$ is closed set in X. Therefore f is continuous map.

Example 3.11: X=Y={a,b,c,d}, τ = {X, ϕ,{a},{b},{a,b},{a,b,c}} σ ={Y, ϕ,{a},{b},{a,b},{a,b,c}}.Let map f: X→Y defined by , f(a)=a , f(b)=b , f(c)=b, f(d)=b. then f is continuous but f is not strongly pgprw–continuous, since forpgprw-closed set F= {a,c,d} in Y, then $f^{-1}(F)$={a} in X , which is not closed set in X.

Theorem: 3.12: Let f: (X,τ) →(Y,σ) be a map. Both (X,τ) and (Y,σ) are Topological-space .Where "every pgprw closed subset is closed" Then the following are equivalent:

(i) f is strongly pgprw-continuous map(ii) f is continuous map.

Proof:

(i) =>(ii) Let U be any open set in (Y,σ). Since every open set is pgprw-open, U is pgprw-open in (Y,σ). Then $f^{-1}(U)$ is open in (X,τ). Hence f is continuous map.

(ii) =>(i) Let U be any pgprw-open set in (Y,σ). Since (Y,σ) is a topoogical-space, U is open in (Y,σ). Since f is continuous map.Then $f^{-1}(U)$ is open in (X,τ). Hence f is strongly pgprw-continuous map.

Theorem 3.13: Let f: (X, τ)→(Y,σ) is strongly continuous map then it is strongly pgprw–continuous map.

Proof: Assume that f: X→Y is strongly continuous map. Let G be pgprw–open in Y and also it is any subset of Y since f is strongly continuous map, $f^{-1}(G)$ is open (and also closed) in X. $f^{-1}(G)$ is open in X Therefore f is strongly pgprw–continuous map.

Theorem 3.14: Let f: (X, τ)→(Y,σ) is strongly pgprw–continuous map then it is pgprw–continuous map.

Proof: Let G be open in Y , every open is pgprw–open , G is pgprw–open in Y, since f is strongly pgprw–continuous map, $f^{-1}(G)$ is open in X. and therefore $f^{-1}(G)$ is pgprω–open in X . Hence f is pgprw–continuous map.

Theorem 3.15: In discrete space, a map f: (X, τ)→(Y,σ) is strongly pgprw–continuous map then it is strongly continuous map.

Proof:F be any subset of Y, in discrete space, Every subset F in Y is both open and closed , then subset F is both pgprw–open or pgprw–closed, i) let F is pgprω–closed in Y, since f is strongly pgprω–continuous map, then $f^{-1}(F)$ is closed in X. ii) let F is pgprw–open in Y, since f is strongly pgprw–continuous map, then $f^{-1}(F)$ is open in X. Therefore $f^{-1}(F)$ is closed and open in X. Hence f is strongly continuous map.

Theorem 3.16 : Let f: (X, τ)→(Y,σ) and g: (Y, σ) →(Z,η) be any two functions. Then

(i) g o f : (X, τ)→(Z,η) is strongly pgprw–continuous map if g is strongly pgprw–continuous map and f is strongly pgprw–continuous map.

(ii) g o f : (X, τ)→(Z,η) is strongly pgprw–continuous map if g is strongly pgprw–continuous map and f is continuous map.

(iii) g o f : (X, τ)→(Z,η) is pgprw–irresolute map if g is strongly pgprw–continuous map and f is pgprw–continuous map.
(iv) g o f : (X, τ)→(Z,η) is continuous map if g is pgprw–continuous map and f is strongly pgprw–continuous map.

Proof:

(i)Let U be a pgprw–open set in (Z, η) . Since g is strongly pgprw–continuous map, $g^{-1}(U)$ is open set in (Y, σ). As every open set is pgprw–open, $g^{-1}(U)$ is pgprw–open set in (Y, σ). Since f is strongly pgprw–continuous map $f^{-1}(g^{-1}(U))$ is an open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is strongly pgprw–continuous map.

(ii)Let U be a pgprw–open set in (Z, η) . Since g is strongly pgprw–continuous map, $g^{-1}(U)$ is open set in (Y, σ). Since f is continuous map $f^{-1}(g^{-1}(U))$ is an open set in (X, τ).
Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is strongly pgprw–continuous map.

(iii)Let U be a pgprw–open set in (Z, η) . Since g is strongly pgprw–continuous map, $g^{-1}(U)$ is open setin (Y, σ). Since f is pgprw–continuous map $f^{-1}(g^{-1}(U))$ is anpgprw–open set in (X, τ).
Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is anpgprw–open set in (X, τ) and hence gof is pgprw–irresolute map.

(iv)Let U be open set in (Z, η) . Since g is pgprw–continuous map, $g^{-1}(U)$ is pgprw–open set in (Y, σ).Since f is strongly pgprw–continuous map $f^{-1}(g^{-1}(U))$ is an open set in (X, τ).
Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is continuous map.

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