On Contra Pre Generalized Pre Regular Weakly Continuous Functions In Topological Spaces

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Abstract- *In this paper we introduce and investigate some classes of functions called contra-pgprw-continuous functions. We get several characterizations and some of their properties. Also we investigate its relationship with other types of functions.*

Keywords- Contra pgprw-continuous,Pgprw-closed sets,Pgprw-open sets.

I. INTRODUCTION

In 1996, Dontchev[1] presented a new notion of continuous function called contra-continuity. This notion is a stronger form of LC-Continuity. In 2015,Wali and Vivekananda Dembre[2] introduced pgprw-closed set in topological spaces. Wali and Vivekananda Dembre[3] introduced pgprw-continuous maps and pgprw-irresolute maps in topological spaces. The purpose of this paper is to define a new class of continuous functions called Contra-pgprw continuous functions and investigate their relationships to other functions.

II. PRELIMINARIES

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c ,P-Cl(A) and P-int(A) denote the Closure of A, Interior of A , Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

Definition 2.1:A subset A of a topological space (X, τ) is called

(i) Generalized closed set(briefly g-closed) [4] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

(ii)Regular generalized closed set(briefly rg-closed)[5] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is regular open in X.

(iii) A pre generalized pre regular ωeakly closed set(briefly pgprω-closed set)[2] if pCl(A)$\subseteq$U whenever A$\subseteq$U and U is rgα open in (X,τ).

The complements of the above mentioned closed sets are their respective open sets.

Defintion 2.2: A map f:(X, τ) → (Y, σ)) is called
(i) Pgprw-continuous function [6]if $f^{-1}(v)$ is pgprw closed in (X,τ) for every closed V in (Y, σ)
(ii) Pgprw-irresolutemap[3]if $f^{-1}(v)$ is pgprw closed in (X,τ) for every pgprw-closed V in (Y, σ)
(iii)Pgprw-closed map[7] if $f^{-1}(v)$ is pgprw closed in (X,τ) for every closed V in (Y, σ)
(iv)Pgprw-open map[7] if $f^{-1}(v)$ is pgprw closed in (X,τ) for every closed V in (Y, σ)

Definition 2.4:[1]A map f:(X, τ) → (Y, σ) is called contra continuous function if $f^{-1}(v)$ is closed in (X,τ) for every every closed V in (Y, σ).

Theorem 2.5:[2](i)Every closed set is pgprw-closed set.

III. ONCONTRA PGPRW-CONTINUOUS FUNCTION

Definition 3.1: A function f: (X, τ) → (Y,σ) is called contra pgprw-continuous function if f^{-1} (V) is Pgprw-closed set in X for each open set V in Y.

Example 3.2: Let X=Y={a,b,c} with topologiesτ={X,∅,{a},{b},{a,b}} andσ = {Y,∅,{b,c}}. Let f: X → Y be a map defined by identity map. Clearly f is contra pgprw-continuous map.

Theorem 3.3: Every contra-continuous function is contra pgprw-continuous function.

Proof: The proof follows from the fact that every closed set is pgprw-closed set[Theorem 2.5(i)]

Remark 3.4: The converse of the above theorem need not be true as seen from the following example.

Example 3.5: Consider X=Y={a,b,c} with topologies τ={X,$\emptyset$,{a},{b},{a,b}} and

σ = {Y,$\emptyset$,{b,c}. Let f: X → Y be a map defined by identity map. Clearly f is contra pgprw-continuous function but not contra-continuous function since f^{-1}({a,c}) = {a,c} which is not closed in X.

Theorem 3.6: If a function f: X → Y is contra pgprw-continuous function, then f is contra-continuous function.

Proof: Let V be an open set in Y. Since f is contra pgprw-continuous function, f^{-1}(V) is closed in X. Hence f is contra-continuous function.

Remark 3.7: The concept of pgprw-continuity and contra pgprw-continuity is independent as shown in the following examples.

Example 3.8 : Let X=Y={a,b,c} with topologies τ={X,$\emptyset$,{a},{b},{a,b}} and

σ = {Y,$\emptyset$,{b,c}. Let f: X → Y be a map defined by identity map. Clearly f is contra pgprw-continuous but not pgprw-continuous since f^{-1}(a) is not pgprw closed in X where{a}is closed in Y.

Example 3.9 : Let X=Y={a,b,c} with topologiesτ={X,$\emptyset$,{a},{b},{a,b}} andσ = {Y,$\emptyset$,{a,b}}. Let f: X → Y be identity map. Clearly f is pgprw-continuous function but not contra pgprw-continuous function f^{-1}{(a,b)} ={a,b} is not pgprw-closed in X where {a,b} is open in Y.

Remark 3.10: The composition of two contra pgprw-continuous functions need not be contra pgprw-continuous as seen from the following example.

Example 3.11: Consider X=Y=Z= {a,b,c} with topologies τ = {X, ϕ,{a},{b,c}}

andσ={Y, ϕ,{b,c}}& μ= {Z ,ϕ,{a}}. Define f: (X, τ)(Y, σ) andg:(Y,σ)(Z, μ) be the identity map. Then f & g are contra pgprw-continuousfunction butg of :(X, τ) (Z, μ)is not contra pgprw-continuousmap since $(gof)^{-1}$(b)= {b} in X is not pgprw closed in X.

Theorem 3.12: If f: (X, τ) →(Y,σ) is contra pgprw-continuous map and g: Y→Z is a continuous function, then gof : X→Z is contra pgprw-continuous function.

Proof: Let V be open in Z. Since g is continuous function,g^{-1}(V) is open in Y. Then f^{-1}(g^{-1}(V)) is pgprw-closed in X since f is contra pgprw-continuous function, Thus $(gof)^{-1}$ (V) is pgprw-closed in X. Hence gof is contra pgprw-continuous function.

Theorem 3.13: If f: X → Y is pgprw-irresolute function and g: Y→Z is contra continuous function
then gof: X→Z is contra pgprw-continuous function.

Proof: Since every contra continuous function is contra pgprw-continuous function, the proof is obvious.

Theorem 3.14: If f: X → Y is contra pgprw-continuous function then for every x$\in$X, each F$\in$ C(Y,f(x)), there exists U$\in$pgprwo(X,x) such that f(U)⊆F (ie) For each x$\in$X, each closed subset F of Y with f(x) $\in$ F, there exists a pgprw-open set U of Y such that x$\in$U and f(U) $\in$F.

Proof: Let f: X → Y be contra pgprw-continuous function Let F be any closed set of Y and f(x) $\in$F where x$\in$X. Then f^{-1}(F) is pgprw-open in X. Also x$\in$ f^{-1}(F). Take U = f^{-1}(F). Then U is a pgprw-open set containing x and f (U) $\in$F.

Theorem 3.15: Let (X,τ) be a pgprw-connected space (Y,σ) be any topological space. If X → Y is surjective and contra pgprw-continuous function then Y is not a discrete space.

Proof: Suppose Y is discrete space. Let A be any proper non-empty subset of Y. Then A is both open and closed in Y. Since f is contra pgprw-continuous function, f^{-1}(A) is both pgprw-open and pgprw-closed in X. Since X is pgprw-connected, the only subset of Y which are both pgprw-open andpgprw-closed are X and φ. Hence f^{-1}(A) = X. Then it contradicts to the fact that f: X → Y surjective. Hence Y is not a discrete space.

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