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On Pre Generalized Pre Regular Weakly Open Sets and Pre Generalized Pre Regular Weakly Neighbourhoods in Topological Spaces

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Abstract. In this paper a new class of sets called pre generalized pre regular weakly open sets(briefly pgprw open sets) and pgprw neighbourhoods are introduced and studied in topological spaces and some properties of new concepts have been studied.

Keywords: pgprw closed sets, pgprw open sets, pgprw -neighbourhoods

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1. Introduction

Stone [1] introduced and studied Regular open sets then Regular semi open sets, Pre-open sets, gspr closed sets, gpr closed sets, gp closed sets,Rg closed sets, rgα-closed sets, π-g-closed sets, pgprw closed sets are introduced and studied by Cameron [2], Mashhour, Abd El-Monsef and El-Deeb [3], Govindappa Navalagi, Chandrashakarappa and Gurushantanavar [4], Gnanambal [5], Maki, Umehara and Noiri [6], Palaniappan and Rao [7], Vadivel and Vairamamanickam [8], Dontchev and Noiri [9], Wali and Vivekananda Dembre [10] respectively.

2. Preliminaries

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c,P-Cl(A) and P-int(A) denote the Closure of A, Interior of A , Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

Definition 2.1. A subset A of a topological space (X, τ) is called

(i) Regular open set [1] if A = int(cl(A)) and a regular closed set if A = cl(int(A)).

(ii) Regular semi open set [2] if there is a regular open set U such that U ⊆A⊆ cl(U).

(iii) Pre-open set [3] if A ⊆ int(cl(A)) and pre-closed set if cl(int(A)) ⊆ A.

(iv) Generalized semi pre regular closed (briefly,gspr-closed) set [4] if spcl(A)$\subseteq$U whenever A$\subseteq$U and U is regular open in X.
(v) Generalized pre regular closed set (briefly, gpr-closed) [5] if pcl(A)$\subseteq$U whenever A$\subseteq$U and U is regular open in X.
(vi) Generalized pre closed (briefly,gp-closed) set [6] if pcl(A)$\subseteq$U whenever A$\subseteq$U and U is open in X.
(vii) Regular generalized closed set(briefly,rg-closed) [7] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is regular open in X.
(viii) Regular-generalized-α closed set [8] if α- cl(A)$\subseteq$U whenever A$\subseteq$U and U is regular α -open in X.
(ix) π-generalized closed set (briefly,πg-closed) [9] if cl (A)$\subseteq$ U whenever A$\subseteq$ U and U is π-open in X.
(x) pre generalized pre regular ωeakly closed set(briefly pgprω-closed) [10] if pCl(A)$\subseteq$U whenever A$\subseteq$U and U is rgα-open in (X, τ).

3. Pre generalized pre regular weakly closed sets in topological spaces

Definition 3.1. [10] A subset A of topological space (X, τ) is called a pre generalized pre regular ωeakly closed sets (briefly pgprω-closed set) if pCl(A)$\subseteq$U whenever A $\subseteq$ U and U is rgα-open in (X, τ).

Results 3.2. From [10]
(i) Every closed set is pgprω-closed set in X.
(ii) Every regular closed set is pgprw-closed set in X.
(iii) Every pgprw-closed set is gspr, gpr, gp,rg, πg closed set.
(iv) The Union of two pgprω -closed subsets of X is pgprω – closed set.
(v) If A is pre generalized pre regular weakly closed set in X and A$\subseteq$B$\subseteq$ pCl(A) then B is also pre generalized pre regular weakly closed set in X.
(vi) If a subset A of topological space X is a pre generalized pre regular weakly closed set in X; then pCl(A) - A does not contain any non empty rgα-closed set in X.

4. Pre generalized pre regular weakly open sets

Definition 4.1. A subset A of a topological space (X,τ) is called pre generalized pre regular ωeakly open (briefly pgprω-open) set in X if A^c is pgprω-closed in X.

The following theorem is the analogue of results 3.2 (i) to (iv).

Theorem 4.2. For any topological spaces (X,τ) we have the following .
(i) Every open set is pgprω-open.
(ii) Every regular open set is pgprw closed set.
(iii) Every pgprω-open set is gspr, gpr,gp,rg, πg-open set.

Theorem 4.3. If A and B are pgprω-open sets in space X, then A$\cap$B is also an pgprω-open in X.
Proof: Let A and B be two pgprω-open sets in X. Then A^c and B^c are pgprω-closed sets in X by Results 3.2 (iv) , $A^c \cup B^c$ is also pgprω-closed set in X. that is $A^c \cup B^c = (A\cap B)^c$ is pgprω-closed set in X.Therefore A$\cap$B is an pgprω-open set in X.

Remark 4.4. The union of pgprω-open set in X is generally not an pgprω-open set in X.

Example 4.5. Let X={a,b,c,d}, τ ={X,ϕ,{a},{c,d},{a,c,d}}. If A={c} B={a} Then A & B are pgprω-open set in X but A ∪B={a,c} is not an pgprω-open set in X.

Theorem 4.6. A subset A of a topological space X is pgprω-open iff U⊆ p-int(A), whenever U is rgα-closed and U ⊆A.

Proof: Assume that A is pgprω-open set in X and U is rgα-closed set of (X, τ) s.t U ⊆A. Then X-A is a pgprω-closed set in (X, τ).Also X−A⊆ X−U and X−U is rgα-open set of (X, τ).This implies that pcl(X−A) ⊆ X−U.But pcl(X−A)=X−p-int(A). Thus, X−p int(A) ⊆ X−U, so U ⊆ p-int(A).Conversely: Suppose U⊆ p-int (A) whenever U is rgα-closed and U ⊆ A. To prove that A is pgprω-open set. Let F be rgα-open set of (X, τ) s.t X−A ⊆ F. Then X−F ⊆ A. Now X−F is rgα-closed set containing A,So;X−F ⊆ p-int(A),X−p-int(A) ⊆ F but pcl(X−A) = X−p-int(A) ⊆ F. Thus pcl(X−A) ⊆ F i.e X−A is pgprω-closed set & hence A is pgprω-open set.

Theorem 4.7. If p-int(A) ⊆ B ⊆A and A is pgprω-open set, then B is pgprω-open set.

Proof: Let p-int(A)⊆B⊆A, Thus X−A ⊆ X−B ⊆ X−p-int(A), i.e. X−A ⊆ X−B ⊆ cl(X−A),Since X−A is pgprω-closed set,then from result 3.2 (v) [10] X−B is pgprω-closed set. Therefore B is pgprω-open set.

Theorem 4.8. If A ⊆ X is pgprω-closed then pcl(A) −A is pgprω-open set.

Proof: Let A be pgprω-closed. Let F ⊆ pcl(A) −A, where F is rgα–closed; then from result 3.2 (vi) [10] we have F=ϕ .Therefore F ⊆ p-int (pcl(A) −A) and Theorem4.6 pcl(A) −A is pgprω-open set.

The reverse implication does not hold good.

Example 4.9. Let X={a,b,c,d}, τ ={X,ϕ,{a},{c,d},{a,c,d}}Let A={a,d}, pcl(A)= {a,c,d} then pcl(A) –A={c} which is pgprω-open set in X, but A is not pgprω-closed.

Theorem 4.10. A set A is pgprω-open set in (X, τ) if and only if U=X whenever U is rgα -open and p-int(A) ∪(X− A) ⊆ U.

Proof: Suppose that A is pgprω-open set in X. Let U be rgα-open and p-int(A) ∪(X-A) ⊆ U ,U^c ⊆ (p-int(A)∪A^c)^c = (p-int(A))^c ∩ A i.e U^c ⊆ (p-int(A))^c − A^c (because A−B = A∩B^c).Thus U^c ⊆ pcl(A^c)–A^c (because (p-int(A))^c = pcl(A^c)).Now A^c is also pgprω-closed and U^c is rgα-closed then from result 3.2 (vi) [10] it follows U^c=ϕ then U=X. Conversely: Suppose F is pgprω-closed and F⊆A.
Then p-int(A) ∪(X-A) ⊆ p-int(A) ∪(X-F). It follows that p-int(A) ∪(X –F)=X.

Theorem 4.11. If A and B be subsets of space (X, τ). If B is pgprω-open and p-int(B) ⊆ A then A∩B is pgprω-open.

Proof: Let B is pgprω-open in X. P-int(B) ⊆ A and p-int(B) ⊆ B is always then p-int(B) ⊆ A∩B and also p-int(B) ⊆ A∩B ⊆ B and B is pgprω-open set by Theorem 4.7 , A∩B is also pgprω-open set in X.

5. pgprw-neıghbourhood

Defintion 5.1. (i) Let (X,τ) be a topological space and Let $x \in X$, A subset of N of X is said to be pgprω-neighbourhood of x if there exists an pgprw-open set G s.t. $x \in G \subseteq N$.

(ii) The collection of all pgprω-neighbourhood of $x \in X$ is called pgprω-neighbourhood system at x and shall be denoted by pgprw-N(x).

Theorem 5.2. Every neighbourhood N of $x \in X$ is a pgprω-neighbourhood of X.

Proof: Let N be neighbourhood of point $x \in X$. To prove that N is a pgprω neighbourhood of x by definition of neighbourhood ∃ an open set G s.t. $x \in G \subset N$ .Hence N is pgprω-neighbourhood of x.

Remark 5.3. In general, a pgprω-nbhd N of x ∈X need not be a nbhd of x in X, as seen from the following example.

Example 5.4. Let X = {a, b, c, d} with topology τ = {X, φ,{a},{b ,{a, b},{a,b,c}}. Then pgprωo(X) = {X, φ,{a}, {b},{c},{a, b},{b,c},{a, c} ,{a, b, c},{a,b,d}}. The set {c,d} is pgprω-nbhd of the point c, since the pgprω-open set {c} is such that c ∈ {c}⊂{c,d}. However, the set {c,d} is not a nbhd of the point c, since no pgprw open set G exists such that c ∈ G ⊂{c,d}.

Theorem 5.5. If a subset N of a space X is pgprω-open, then N is a pgprω-nbhd of each of its points.

Proof: Suppose N is pgprω-open. Let x ∈N. We claim that N is pgprω-nbhd of x. For N is a pgprω-open set such that x ∈N ⊆ N. Since x is an arbitrary point of N, it follows that N is a pgprω-nbhd of each of its points.

Remark 5.6. The converse of the above theorem is not true in general as seen from the following example.

Example 5.7. Let X = {a, b, c, d} with topology τ = {X, ϕ {a},{c,d},{a, c,d}}. Then pgprωo(X) = {X, ϕ, {a},{b},{c},{d},{a,b},{a,c},{a,d},{c,d},{a,b,c},{a,b,d},{a,c,d}}. The set {b,c} is a pgprω-nbhd of the point b, since the pgprω-open set {b} is such that b∈ {b} ⊆ {b,c}. Also the set {b,c} is a pgprω-nbhd of the point {c}, Since the pgprω-open set {c} is such that c ∈ {c} ⊆ {b,c}. That is {b,c} is a pgprω-nbhd of each of its points. However the set {b,c} is not a pgprω-open set in X.

Theorem 5.8. Let X be a topological space. If F is a pgprω-closed subset of X, and x ∈ F^c. Prove that there exists a pgprω-nbhd N of x such that N∩F =ϕ.

Proof: Let F be pgprω-closed subset of X and x ∈F^c. Then F^c is pgprω-open set of X. So by theorem 5.5 F^c contains a pgprω-nbhd of each of its points. Hence there exists a pgprω-nbhd N of x such that N⊂F^c. That is N ∩ F = ϕ.

Theorem 5.9. Let X be a topological space and for each $x \in X$, Let pgprω-N (x) be the collection of all pgprω-nbhds of x. Then we have the following results.

(i) $\forall x \in X$, pgprω-N (x) $\neq \phi$

(ii) $N \in$ pgprω-N (x) $\Rightarrow x \in N$

(iii) $N \in$ pgprω-N (x), $M \supset N \Rightarrow M \in$ pgprω-N (x)

(iv) $N \in$ pgprω-N (x), $M \in$ pgprω-N (x) $\Rightarrow N \cap M \in$ pgprω-N (x)

(v) $N \in$ pgprω-N (x) $\Rightarrow$ there exists $M \in$ pgprω-N (x) such that $M \subset N$ and $M \in$ pgprω-N (y) for every $y \in M$

Proof: (i) Since X is a pgprω-open set, it is a pgprω-nbhd of every $x \in X$. Hence there exists at least one pgprω-nbhd (namely - X) for each $x \in X$. Hence pgprω-N (x) $\neq \phi$ for every $x \in X$.

(ii) If $N \in$ pgprω-N (x),then N is a pgprω-nbhd of x.So by definition of pgprω-nbhd,$x \in N$.

(iii) Let $N \in$ pgprω-N (x) and $M \supset N$. Then there is a pgprω-open set G such that $x \in G \subset N$. Since $N \subset M$, $x \in G \subset M$ and so M is pgprω-nbhd of x. Hence $M \in$ pgprω-N (x).

(iv) Let $N \in$ pgprω-N (x) and $M \in$ pgprω-N (x). Then by definition of pgprω-nbhd there exists pgprω-open sets G_1 and G_2 such that $x \in G_1 \subset N$ and $x \in G_2 \subset M$.
Hence $x \in G_1 \cap G_2 \subset N \cap M$ -- (1). Since $G_1 \cap G_2$ is a pgprω-open set, (being the intersection of two pgprω-open sets), it follows from (1) that $N \cap M$ is a pgprω-nbhd of x. Hence $N \cap M \in$ pgprω-N (x).

(v) If $N \in$ pgprω-N (x), then there exists a pgprω-open set M such that $x \in M \subset N$. Since M is a pgprω-open set, it is pgprω-nbhd of each of its points. Therefore $M \in$ pgprω-N (y) for every $y \in M$.

Theorem 5.10. Let X be a nonempty set, and for each $x \in X$, let pgprω-N (x) be a nonempty collection of subsets of X satisfying following conditions.

(i) $N \in$ pgprω-N (x) $\Rightarrow x \in N$

(ii) $N \in$ pgprω-N (x),$M \in$ pgprω-N (x) $\Rightarrow N \cap M \in$ pgprω-N (x).

Let τ consists of the empty set and all those non-empty subsets of G of X having the property that $x \in G$ implies that there exists an $N \in$ pgprω-N (x) such that $x \in N \subset G$, Then τ is a topology for X.

Proof: (i) $\varphi \in \tau$ by definition. We now show that $x \in \tau$. Let x be any arbitrary element of X. Since pgprω-N (x) is nonempty, there is an $N \in$ pgprω-N (x) and so $x \in N$ by (i). Since N is a subset of X, we have $x \in N \subset X$. Hence $X \in \tau$.
(ii) Let $G_1 \in \tau$ and $G_2 \in \tau$. If $x \in G_1 \cap G_2$ then $x \in G_1$ and $x \in G_2$. Since $G_1 \in \tau$ and $G_2 \in \tau$, there exists $N \in$ pgprω-N (x) and $M \in$ pgprω-N (x), such that $x \in N \subset G_1$ and $x \in M \subset G_2$. Then $x \in N \cap M \subset G_1 \cap G_2$. But $N \cap M \in$ pgprω-N (x) by (2). Hence $G_1 \cap G_2 \in \tau$ .Let $G_\lambda \in \tau$ for every $\lambda \in \wedge$. If $x \in \cup\{G_\lambda : \lambda \in \wedge\}$, then $x \in G_{\lambda x}$ for some $\lambda x \in \wedge$.

Since $G_{\lambda x} \in \tau$, there exists an N∈pgprω-N (x) such that $x \in N \subset G_{\lambda x}$ and consequently $x \in N \subset \cup\{G_\lambda : \lambda \in \wedge\}$. Hence $\cup\{G_\lambda : \lambda \in \wedge\} \in \tau$. It follows that τ is topology for X.

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